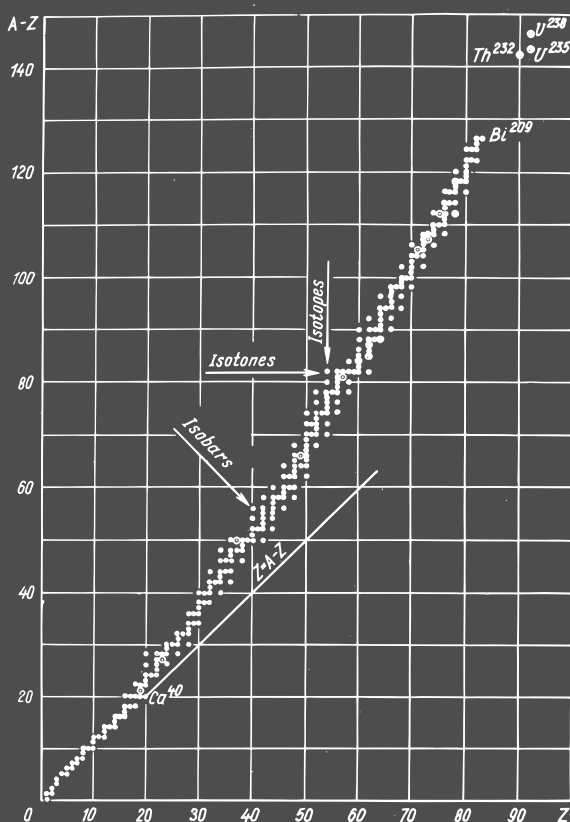


A. Klimov

Nuclear Physics And Nuclear Reactors



MIR PUBLISHERS MOSCOW

A. KLIMOV
NUCLEAR
PHYSICS
AND
NUCLEAR
REACTORS



А. Н. Климов

**ЯДЕРНАЯ ФИЗИКА
И ЯДЕРНЫЕ РЕАКТОРЫ**

МОСКВА • АТОМИЗДАТ

A. Klimov

**NUCLEAR PHYSICS
AND
NUCLEAR REACTORS**

Translated from the Russian
by
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MIR PUBLISHERS · MOSCOW

First published 1975

The Greek Alphabet

| | | | | | |
|-----|---------|----|---------|----|---------|
| Αα | Alpha | Ιι | Iota | Ρρ | Rho |
| Ββ | Beta | Κκ | Kappa | Σσ | Sigma |
| Γγ | Gamma | Λλ | Lambda | Ττ | Tau |
| Δδ | Delta | Μμ | Mu | Υυ | Upsilon |
| Εε | Epsilon | Νν | Nu | Φφ | Phi |
| Ζζ | Zeta | Ξξ | Ksi | Χχ | Chi |
| Ηη | Eta | Οο | Omicron | Ψψ | Psi |
| Θθθ | Theta | Ππ | Pi | Ωω | Omega |

На английском языке

CONTENTS

| | |
|---|------------|
| Constants | 7 |
| Chapter One. ATOMIC STRUCTURE AND ELEMENTARY PARTICLES . . . | 9 |
| 1.1. Atoms and Atomic Electricity | 9 |
| 1.2. Nuclear Structure of the Atom | 13 |
| 1.3. Structure of the Atomic Nucleus | 19 |
| 1.4. Quantum Mechanics | 21 |
| 1.5. Relativity Theory Formulae | 32 |
| 1.6. Nuclear Effective Cross-Sections | 38 |
| 1.7. Elementary Particles | 44 |
| 1.8. Positron | 54 |
| 1.9. Neutrino | 56 |
| 1.10. Mesons | 61 |
| 1.11. Antinucleons | 69 |
| 1.12. Antimatter | 72 |
| Chapter Two. PHYSICAL PROPERTIES OF ATOMIC NUCLEI | 74 |
| 2.1. Nuclear Charge | 74 |
| 2.2. Nucleus Size | 76 |
| 2.3. Nuclear Moments | 77 |
| 2.4. Nuclear Mass and Bond Energy | 78 |
| 2.5. Nuclear Forces | 92 |
| 2.6. Nuclear Models | 100 |
| 2.7. Excited Nuclear States | 107 |
| Chapter Three. NUCLEAR TRANSFORMATIONS | 112 |
| 3.1. Radioactivity | 112 |
| 3.2. Alpha-Decay | 118 |
| 3.3. Beta-Decay | 127 |
| 3.4. Nuclear Gamma-Radiation | 137 |
| 3.5. Nuclear Reactions | 142 |
| 3.6. Heavy Nuclei Fission | 158 |
| Chapter Four. INTERACTION OF MOVING PARTICLES WITH MATTER . | 171 |
| 4.1. Heavy Charged Particles | 172 |
| 4.2. Fission Fragments | 178 |
| 4.3. Electrons | 180 |
| 4.4. Gamma-Quanta | 189 |
| 4.5. Neutrons | 201 |

| | |
|---|------------|
| Chapter Five. MODERATION AND DIFFUSION OF NEUTRONS | 231 |
| 5.1. Moderation of Neutrons | 231 |
| 5.2. Spectrum of Slowing-Down Neutrons | 239 |
| 5.3. Thermal Neutron Diffusion | 243 |
| 5.4. Diffusion of Slowing-Down Neutrons | 256 |
| Chapter Six. NUCLEAR REACTOR | 265 |
| 6.1. Chain Reaction | 265 |
| 6.2. Multiplication Factor for Infinite Medium | 274 |
| 6.3. Number of Neutrons per Absorption Act | 277 |
| 6.4. Thermal Utilization Factor | 279 |
| 6.5. Resonance Escape Probability | 282 |
| 6.6. Fast Multiplication Factor | 293 |
| 6.7. Optimum Parameters of Multiplying Media | 294 |
| 6.8. Reactor Critical State | 296 |
| 6.9. Neutron Reflector | 313 |
| 6.10. Non-Critical Reactor | 320 |
| Chapter Seven. PHYSICAL PROCESSES IN ACTUAL REACTORS | 331 |
| 7.1. Retention of Criticality in Time | 331 |
| 7.2. Nuclear Fuel Conversion | 334 |
| 7.3. Radioactive Poisoning of the Reactor | 347 |
| 7.4. Temperature Coefficient | 356 |
| Chapter Eight. MATERIALS. BIOLOGICAL SHIELDING | 362 |
| 8.1. Materials for Nuclear Reactors | 362 |
| 8.2. Dosimetry and Shielding | 370 |
| Chapter Nine. REACTORS OF ATOMIC POWER PLANTS | 376 |
| 9.1. Water-Cooled Graphite-Moderated Reactors | 376 |
| 9.2. Graphite Gas-Cooled Reactors | 384 |
| 9.3. Heavy-Water Reactors | 388 |
| 9.4. Light-Water Reactors | 390 |
| 9.5. Fast Reactors | 395 |
| Bibliography | 399 |
| Index | 401 |

CONSTANTS

| | |
|--|---|
| Avogadro number | $N_A = 6.02296 \cdot 10^{26} \text{ 1/kmole}$ |
| Faraday constant | $F = 9.64914 \cdot 10^7 \text{ C/kmole}$ |
| Elementary charge | $e = 1.60203 \cdot 10^{-19} \text{ C}$ |
| Planck constant | $h = 6.62517 \cdot 10^{-34} \text{ J} \cdot \text{s}$ $\hbar = 1.05443 \cdot 10^{-34} \text{ J} \cdot \text{s} = 6.5817 \times 10^{-16} \text{ eV} \cdot \text{s}$ |
| Boltzmann constant | $k = 1.3805 \cdot 10^{-23} \text{ J/deg} = 8.617 \times 10^{-5} \text{ eV/deg}$ |
| Velocity of light in vacuo . . | $c = 2.997925 \cdot 10^8 \text{ m/s}$ |
| Electric constant | $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$ |
| Weak interaction constant . . | $g = 10^{-62} \text{ J} \cdot \text{m}^3$ |
| Bohr magneton | $\mu_B = 9.2732 \cdot 10^{-24} \text{ J/T}$ |
| Electron magnetic moment . . | $\mu_e = \mu_B$ |
| Nuclear magneton | $\mu_N = 5.051 \cdot 10^{-27} \text{ J/T}$ |
| Proton magnetic moment . . | $\mu_p = 2.7928 \cdot \mu_N$ |
| Neutron magnetic moment . . | $\mu_n = 1.913148 \cdot \mu_N$ |
| Classical electron radius . . . | $r_e = 2.818 \cdot 10^{-15} \text{ m}$ |
| Radius of action (range) of nuclear forces | $R \approx 1.5 \cdot 10^{-15} \text{ m}$ |
| Hydrogen atom radius (first Bohr radius) | $R(H^I) = 0.529172 \cdot 10^{-10} \text{ m}$ |
| Mass of mass unit | $m_u = 1.6603 \cdot 10^{-27} \text{ kg}$ |
| Energy equivalent of mass unit | $m_u c^2 = 931.4 \text{ MeV}$ |
| Conversion of electron volts into joules | $1 \text{ eV} = 1.60206 \cdot 10^{-19} \text{ J}$ |
| Atomic mass of carbon C^{12} . . | $A_r = 12.000000 \text{ (standard)}$ |

8 Constants

| | |
|--|--|
| Neutron atomic mass | $A_r = 1.008665$ |
| Proton atomic mass | $A_r = 1.007276$ |
| Electron atomic mass | $A_r = 0.00054859$ |
| Atomic mass of hydrogen atom | $A_r = 1.007825$ |
| Mass of hydrogen atom . . . | $M(\text{H}^1) = 1.67 \cdot 10^{-27} \text{ kg}$ |
| Unit of measurement of nuc- lear cross-sections | $1 \text{ barn} = 10^{-28} \text{ m}^2$ |
| Temperature of Maxwellian distribution at average ener- gy of 1 eV | $T = 7737^\circ \text{ K}$ |

CHAPTER ONE

ATOMIC STRUCTURE AND ELEMENTARY PARTICLES

1.1. Atoms and Atomic Electricity

1. **Atomic mass.** The concept of an atom as of a minute indivisible particle of matter arose in the ancient times as an alternative to the concept of the continuous structure of matter. In new times not only has the interest in the atoms of the ancients been retained but atomism has even acquired the feature of a scientific hypothesis. Many naturalists, Newton in particular, adhered to the atomistic concepts. The discovery of real atoms is, however, associated with the name of Dalton, the English scientist, who in 1803 was the first to substantiate a method for determining the relative masses of the atoms of simple substances or *atomic masses* A_r [1]. Dalton's idea was that the ratio of the masses of two simple substances resulting from decomposition of a compound substance is the ratio of the masses of their atoms if the molecule from the compound substance contains one atom of each simple substance. If the mass of the lightest atom is taken as unity, a scale of the atomic masses can be constructed. It was found that the atomic mass A_r is an individual characteristic of a simple substance, that is, the atoms of simple substances can be distinguished according to their mass. When the conjectural atoms were found to have an experimentally measurable characteristic value which, through its numerical expression, allowed atoms of different substances to be distinguished, atoms became reality and an object of natural science research.

Dalton's success was helped by the progress of quantitative analysis in chemistry. The discovery of atoms, in turn, stimulated the progress of chemistry in the nineteenth century, the crown of which was the periodic law discovered in 1869 by the Russian chemist Mendeleyev. The discovered periodicity of the chemical properties of elements, as a function of the increasing atomic mass, allowed Mendeleyev to build up a system of elements which completed the determination of the natural diversity of the atoms and provided a strict order in the arrangement and classification of chemical elements. All the elements missing from the initial version of the periodic system were subsequently either discovered in their natural form or produced artificially. The heaviest naturally found element appeared to be uranium having atomic mass 238.03 and number 92.

At present a number of transuranium elements up to number 104 have been obtained.

The relative masses of atoms A_r were determined in experiments with macroscopic bodies, that is, bodies containing an enormous number of atoms. To obtain the absolute masses and sizes of individual atoms it was necessary to find their number in a unit volume or unit mass of the substance. The solution of this fundamental problem was found in the kinetic theory of gases.

2. The Loshmidt number. According to the kinetic theory, gas is the accumulation of a great number of particles, atoms or molecules in continuous random motion. From the laws of motion and interaction of microscopic particles the kinetic theory explains the physical properties of gases and allows the macroscopic constants of a gas to be expressed as the sizes and masses of individual particles, their concentration (the number of particles per unit volume) and average velocity of motion. The concepts of the kinetic theory were first used for the determination of the number of molecules per unit volume of gas and the molecular radius by the Australian physicist Loshmidt in 1865.

The gas viscosity factor can be expressed in terms of the molecular radius and concentration of molecules, that is, in terms of two unknown quantities. Loshmidt associated the same unknowns with another measurable macroscopic constant, the density ratio of gaseous and liquified gas, assuming certain distributions of the molecules in the liquid. The two relations containing two unknowns enabled Loshmidt to determine the molecular radius of gas and the number of molecules contained in one cubic centimetre (the Loshmidt number).

3. The Avogadro number. As long as the number of particles per unit volume depends on the aggregate state of the substance, its density and temperature, and the number of particles per unit mass is inversely proportional to the molecular or atomic mass of the substance, the number of particles contained in one kilogram-molecule or kilogram-atom of substance is usually taken to be the standard number of molecules or atoms. This number is the same for all substances and is called the Avogadro number, after the Italian scientist who was the first to point out that equal volumes of any gases contain equal numbers of particles if the pressure and temperature of the gases are the same (the Avogadro law). It is true that the Avogadro number depends on the choice of the atomic mass unit as long as the numerical values of atomic masses depend on such a choice. Since 1962 one twelfth of the atomic mass of C^{12} is taken as the atomic mass unit. In the modern atomic mass scale the Avogadro number is

$$N_A = 6.02296 \cdot 10^{26} \text{ 1/kmole} \quad (1.1)$$

It is hardly possible to overestimate the significance of the constant (1.1) for atomic physics. Actually, it is only when this constant is available, that such characteristics of atoms as their absolute masses and sizes can be obtained. The Avogadro number was, therefore, repeatedly measured by a variety of methods, most of them indirect, as is the Loshmidt method. Direct methods are based on the observation of the macroscopic effects which are caused by the random motion of molecules and which are a function of the number of molecules taking part in the motion [2]. But, as is often the case in physics, direct methods are inadequate to determine the constant N_A with high accuracy. The most accurate value of the Avogadro number was obtained when studying X-ray diffraction on crystals [3].

4. Number of atoms in a unit volume. According to the definition of the Avogadro number, the absolute atomic mass is equal to the mass of a kilogram-atom divided by the Avogadro number. The mass of the lightest atom, the hydrogen atom, is $1.67 \cdot 10^{-27}$ kg, and the mass of the heaviest atom found in nature, the uranium atom, is $3.95 \cdot 10^{-25}$ kg. The atomic concentration or the number of atoms per unit volume of substance, N , used in the following formulae is also not difficult to calculate with the use of the Avogadro number:

$$N = \frac{\rho}{A_r} N_A m^{-3} \quad (1.2)$$

where A_r is the mass of a kilogram-atom, kg/kmole, numerically equal to the atomic mass of the substance A_r , and ρ is the density of the substance, kg/m³. If atoms form some chemical compound, then their concentration is

$$N = \frac{\rho}{M_r} N_A x \quad (1.3)$$

where M_r is the mass of a kilogram-molecule numerically equal to the molecular mass, and x is the number of atoms of a given kind per one molecule of the compound.

5. Atom size. Absolute dimensions of atoms are defined on the basis of the kinetic gas theory as well as from researches on the structure of molecules and crystals. The atomic radii do not show any systematic growth with increasing atomic mass and the radii of all atoms have a value of the order of 10^{-10} m with slight divergences due to the periodic dependence of the radius on the atomic mass [4].

6. Atomic electricity. The discovery of the atomic structure of a substance revealed the discrete nature of electricity. In 1834 British physicist Faraday completed his studies of the transport of electricity in electrolyte solutions. He established that during precipitation from the electrolyte solution of one kilogram molecule of

a single valent substance through the solution passes a quantity of electricity equal to $9.649 \cdot 10^7$ C (the Faraday constant). If the transport of electricity is produced by two-valent ions of the substance, the quantity of electricity associated with the same mass of the substance is doubled; with three-valent ions it is trebled, and so on. When the Avogadro number became known, on the basis of Faraday's law, the value of a single electric charge of a minimum charged ion was obtained. The charge of an ion produced in gas proved to be equal to the charge of an ion in the solution, this indicated that the amount of the charge is independent of the way of ionizing the atom. In spite of the fact that the minimum portion of the electric charge was found, the final conclusion on the discrete nature of electricity was only drawn after the discovery of the structural particles of the atom, i.e., the electric charge carriers.

The first of such atomic particles was the electron. Experiments on cathode rays generated during an electrical discharge in a rarefied gas, carried out by a number of scientists and completed by the British scientist J. J. Thomson in 1897, showed that these rays are a flux of particles of a very small mass, each carrying a single electric charge. A single charge already had its own name, *electron* [1], and this term was given to the discovered particle as a carrier of such a charge. According to the definitions of positive and negative electricity made when studying the electrification of macroscopic bodies by friction [1], the electron charge appeared negative as opposed to the positive charge of the hydrogen ion the absolute value of which is also equal to a single charge. The electron charge is an important constant in atomic physics. It has been repeatedly measured and according to the latest data it is

$$e = 1.60206 \cdot 10^{-19} \text{ C}$$

Attempts to find the carrier of positive electricity analogous to the electron were not successful. When ionizing atoms the positive charge was always associated with the whole atomic ion, the mass of which was practically equal to the mass of the neutral atom since the separation from an atom of one or several low mass electrons did not change the mass of the atom. Thus, positive electricity of the atom could be associated with the mass of various values in contrast to negative electricity the single charge carrier of which is the electron having a definite mass and being a component of all atoms.

7. Thomson atom. The data on atoms and atomic electricity accumulated by the close of the XIX century allowed J. J. Thomson to build the first model of the atom (1904). Thomson supposed that the atom is a spherical body of a radius about 10^{-10} m having uniformly distributed positive electricity, which contains such a number of negative electrons that are necessary to compensate for the

positive charge of the atom. Nothing was known of the nature of positive electricity and how it is related to the atomic mass since the number of electrons in each atom was not known. It is true, however, that experiments on the scattering of X-rays discovered in 1895 by the German physicist Roentgen, and some other data indicated that the number of electrons in the atom cannot be very high and is approximately equal to half the atomic mass. Moreover, it was found that for fast electrons ordinary substances are sufficiently transparent, from this it followed that the atomic volume is not completely filled with the substance.

1.2. Nuclear Structure of the Atom

1. **Alpha-particles.** In 1896 the French physicist Becquerel discovered the phenomenon of radioactivity which is the process of spontaneous transformation of atoms of some elements into atoms of neighbouring elements in the periodic table during emission of charged particles. This was followed by a rapid progress in studying the atomic structure. The main stimulus in this respect was the fact that physicists happened to come into possession of an effective instrument for investigating the atomic structure, i.e., the α -particle. Most significant discoveries were made with the help of α -particles radiated by natural radioactive substances: the atomic nuclear structure was established, the first nuclear reactions were produced, the phenomenon of artificial radioactivity was discovered and finally the neutron was found which was of great importance both for explaining the structure of the atomic nucleus and in later investigations which resulted in the discovery of the fission process and generation of free nuclear energy.

α -Particles are doubly ionized helium atoms or helium nuclei moving with high speed. Measurements of the velocities of the α -particles of natural emitters by the deflection in electric and magnetic fields offered values of the order of $(1.5 - 2) \cdot 10^7$ m/s, corresponding to a kinetic energy of 4.5 to 8 MeV *. Such particles move linearly in the substance (Fig. 1.5), quickly lose their energy in ionizing atoms and, after stopping, turn into neutral helium atoms.

2. **α -Particle scattering.** While studying the penetration of a collimated beam of α -particles through a thin metal foil, the British physicist Rutherford noticed that the image of the particle beam on a photoplate was blurred. Rutherford attributed such spreading of the beam to α -particle scattering. Although α -particles freely pass through the volume of atoms, the presence of atomic electricity causes interaction between atoms and charged particles and some

* 1 MeV = $1.602 \cdot 10^{-13}$ J and is the kinetic energy of an electron which fell through the potential difference of 10^6 V.

distortion of their trajectories in accordance with Thomson's model of the atom. The detailed study of α -particle scattering showed, however, that in some rare cases the α -particles scatter at high angles, sometimes greater than 90° , corresponding to the repulsion of fast moving particles in the opposite direction. Such scattering could not be explained within the framework of Thomson's model of the atom.

In one act of collision a heavy α -particle can be scattered backward only through the interaction with a body of a mass larger than that of an α -particle. Therefore, atomic electrons are not responsible for such scattering. Moreover, backward scattering implies strong α -particle deceleration, that is the energy of interaction of the α -particle with the scattering-body should be of the order of the α -particle kinetic energy. But the energy of electrostatic interaction of the α -particle with the Thomson atom having a positive charge distributed in the volume or along the surface of the atom of radius 10^{-10} m and equal, in the units of the elementary charge, to approximately half the atomic mass is much less than this value. However, since the energy of electrostatic interaction is inversely proportional to the distance, it may be arbitrarily great if the charges are at an arbitrarily small distance on condition that the volumes occupied by the charges are always less than the distance between the charges, that is the charges do not overlap. The distance from the α -particle to the centre of the electric charge must be of the order of 10^{-14} m so that the energy of the electrostatic interaction becomes equal to the kinetic energy of the α -particle and the α -particle is stopped and scattered backward. Such a distance is about 10,000 times less than the atomic radius, and the radius of the atomic charge must be even less. The assumption of a small volume of the scattering centre agrees with the very small number of large-angle scattering events.

To explain the results of his observations of the α -particle scattering Rutherford suggested his nuclear model of the atom. According to this model, in the centre of the atom is a nucleus which occupies a very small volume and comprises almost the whole mass of the atom; the nucleus carries a positive electric charge. The main part of the atom volume is filled with moving electrons, the number of which is equal to the number of elementary charges of the nucleus because the atom as a whole is electrically neutral.

3. Theory of α -particle scattering. To substantiate the supposition of the nuclear structure of the atom and to prove that the α -particle scattering is due to the Coulomb interaction with the nucleus of the atom, Rutherford developed his theory of α -particles scattering by point electric charges of great mass and obtained the relationship between the scattering angle θ and the number of particles scattered at angle θ . If the α -particle moves in the direction

of point charge Ze , where Z is the number of elementary charges e and its original trajectory is at distance a (impact parameter) (Fig. 1.1a) from the axis passing through the scattering centre, then, following the Coulomb law, it is possible to calculate by the use of classical mechanics methods the angle θ [3] at which the α -particle will deflect due to the electrostatic repulsion of the like electric charges:

$$\cot \frac{\theta}{2} = 4\pi\epsilon_0 \frac{Mv^2}{Zeze} a \quad (1.4)$$

where M and v are the mass and velocity of the α -particle, ze its charge, $z = 2$; $\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m is the electric constant. The

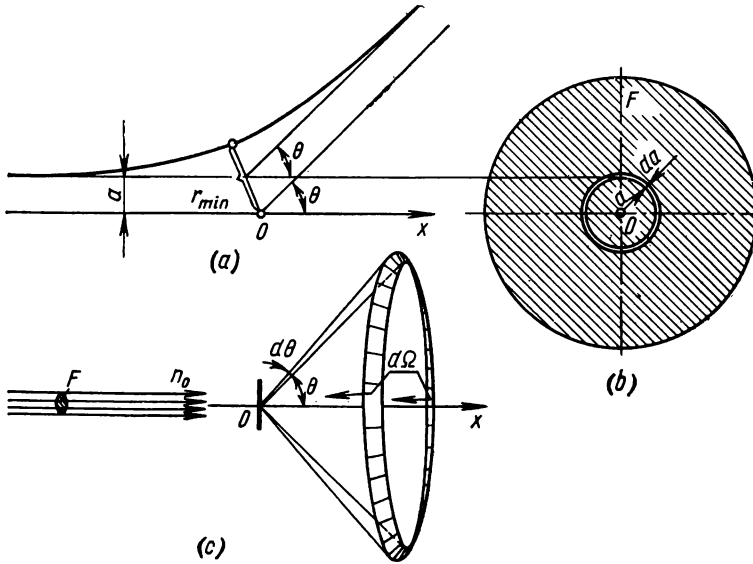


Fig. 1.1. α -Particle scattering by the electric field of the atomic nucleus (a) scattering in the plane of the particle trajectory; (b) the da -ring from which scattering proceeds into $d\theta$ at the angle θ ; (c) scattering into the conical solid angle at the angle θ to the axis

fraction dn/n_0 of particles having impact parameter a in the total number of particles, n_0 , striking the target is equal to the fraction of elementary area, $2\pi a da$, in the total cross-sectional area F of the α -particle beam (Fig. 1.1b). If there are not one but N_F scattering centres in the area F , then the corresponding fraction increases N_F times and, referred to unit a , equals

$$\frac{1}{n_0} \cdot \frac{dn}{da} = 2\pi a \frac{N_F}{F} = 2\pi a N_1$$

where N_1 is the number of scattering centres per unit area of the target. Knowing the relation between a and θ , (1.4), as well as the relation between θ and the solid conical angle $d\Omega = 2\pi \sin \theta d\theta$ into which the particles scatter at angle θ in space (Fig. 1.1c), it is possible to obtain the fraction of particles scattered into a unit solid conical angle $\frac{1}{n_0} \cdot \frac{dn}{d\Omega}$ at angle θ to the axis as $\frac{1}{n_0} \cdot \frac{dn}{da} \cdot \frac{da}{d\theta} \cdot \frac{d\theta}{d\Omega}$:

$$\frac{1}{n_0} \cdot \frac{dn}{d\Omega} = \frac{N_1}{4(4\pi\epsilon_0)^2} \left(\frac{Zze^2}{Mv^2} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} \quad (1.5)$$

Experimental data completely proved the relation (1.5) in scattering α -particles by a substance. The strict fulfilment of the law $1/\sin^4 \frac{\theta}{2}$ shows that scattering is due to electric forces only and that the geometrical sizes of the electric charges of both bodies are at least less than the shortest distance in the act of scattering, r_{\min} (see Fig. 1.1a), since the relation (1.5) holds for either point or final charges but not for charges overlapping in interaction. The larger the scattering angle θ , the shorter the distance r_{\min} . When $\theta = \pi$ ($a = 0$), it is the shortest and is determined by the condition $Mv^2/2 = Zze^2/4\pi\epsilon_0 r_{\min}$ which corresponds to the conversion of the total kinetic energy of the α -particle into the potential energy of electrostatic repulsion of the like charges. To calculate r_{\min} it is necessary to know the nuclear charge Ze which, by the way, is not needed for establishing the electrical nature of scattering, that is, for finding out the law $1/\sin^4 \frac{\theta}{2}$ since the nuclear charge in this case is part of the proportionality constant of relation (1.5).

The nuclear charge may be determined in the same experiments with α -particle scattering. The number of scattered α -particles dn , their total number n_0 as well as the scattering angle θ and the solid angle $d\Omega$ of the reception of scattered α -particle are measured directly in the experiment; N_1 is found by the mass of the scatterer and M , v and z of the α -particles are the known constants. It follows that by measuring the number of α -particles scattered at any angle θ it is possible to calculate the nuclear charge Ze . Such a procedure for defining the nuclear charge is the most straightforward one since it is based on the Coulomb law which was accurately established in experiments with macroscopic bodies and which holds for α -particle scattering (this follows from the experimental confirmation of the dependence $1/\sin^4 \frac{\theta}{2}$). It is true that in this case the straightforward method does not give a precise value of

the measured constant first of all because of the low statistic accuracy in measuring the number of scattered α -particles. Nevertheless, Rutherford's co-workers estimated the values of the atomic nuclear charges of the investigated substances, when experimentally checking the dependence (1.5) by counting the scintillations from α -particles on a screen of sulphurous zink. In agreement with the available data, the values of Z appeared to be approximately equal to half the atomic mass. Furthermore, since 1913, an accurate procedure for determining nuclear charges on the basis of the Moseley law has been known, (Sec. 2.1), according to which the number of elementary nuclear charges is equal to the number of an element in the Mendeleyev Table. Proceeding from the values of the nuclear charges, Rutherford estimated that the radius of nucleus is of the order of 10^{-14} m.

4. Rutherford — Bohr atom. With the discovery of the atomic nucleus there arose the problem of explaining atomic stability. From the point of view of classical electrodynamics the Rutherford atom cannot exist for a long period of time. Since opposite electric charges attract each other, electrons can be at a certain distance from the nucleus only if they move around the nucleus. The motion along a closed trajectory, however, is motion with acceleration, and the accelerated electric charge emits energy into the outer space.

Thus, in a negligible time, any atom must emit its energy stored up due to the electrons being removed to the distance of 10^{-10} m from the nucleus and be constricted to the volume of the nucleus if the electron radii are disregarded and considered, for example, equal to the classical electron radius $2.8 \cdot 10^{-15}$ m. It follows that classical electrodynamics cannot describe the structure of the atom.

The first stationary model of the simplest atom with one electron was proposed by the Danish physicist Bohr (1913). Bohr associated the atomic stability with the quantum nature of radiation. The energy quanta hypothesis developed by the German physicist Planck (1900) to explain the spectrum of an absolutely black body radiation [3] stated that microscopic systems can emit energy only by definite portions, i.e., by quanta, at a frequency ν proportional to the quantum energy E : $E = h\nu$, where $h = 6.625 \cdot 10^{-34}$ J/s is the Planck universal constant. Bohr supposed that the atomic electron energy in the Coulomb nuclear field does not change continuously but takes a number of stable discrete values corresponding to the stationary electron orbits. While moving along such orbits, the electron does not emit energy. The atom emits energy only when the electron makes a transition from the orbit of a higher energy to some other stationary orbit. This radiation is characterized by the only frequency value proportional to the difference of the initial E_{in} and the final E_{fin} of the energy values: $h\nu = E_{in} - E_{fin}$.

On the basis of such conceptions Bohr succeeded in calculating the spectrum and the Rydberg spectroscopic constant for the hydrogen atom. The agreement with experiment was obtained on the assumption that the stationary orbits are characterized by the values of the electron's mechanical moment which are multiples to the value $\hbar = h/2\pi$

$$mvr_n = n \cdot \hbar \quad (1.6)$$

where mv is electron momentum modulus, r_n is the radius of the n th stationary orbit and $n = 1, 2, 3 \dots$ is any given integer. Relation (1.6) is called the condition of circular orbit quantization. The set of energy levels E_n of a one-electron atom, and the radii of the stationary orbits, r_n , are the function of the quantum number n and follow from (1.6) and the Coulomb law:

$$E_n = - \frac{mZ^2e^4}{2(4\pi\epsilon_0)^2 n^2 \hbar^2} \quad (1.7)$$

$$r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{mZe^2} \quad (1.8)$$

When $n = 1$ and $Z = 1$, we obtain the radius of the shortest stationary orbit of the hydrogen atom electron, or the first Bohr radius:

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 0.529172 \cdot 10^{-10} \text{ m} \quad (1.9)$$

The motion of the electron along the orbit may be presented as electric current flowing along a closed circuit and it is possible to calculate the magnetic moment produced by it. The magnetic moment is proportional to the mechanical moment, and as the latter has its minimum value at the first Bohr orbit, the magnetic moment of the first orbit also has the lowest value known as the Bohr magneton:

$$\mu_B = \frac{e\hbar}{2m} = 9.2732 \cdot 10^{-24} \text{ J/T} \quad (1.10)$$

The value of the magnetic moment, (1.10), is inversely proportional to the mass of the particle, but for particles of a given kind, for example, electrons, it has the meaning of unity. It is noteworthy that it is precisely this unity that equals the intrinsic magnetic moment of the electron, associated with its spin (Sec. 1.7-6).

The nuclear model of the atom with its electrons moving along stable orbits is called the Rutherford-Bohr planetary model. It does not lead to correct quantitative results when applied to atoms with more than one electron but is very useful for qualitative interpretation of atomic phenomena. The accurate atomic theory is quantum mechanics,

5. Discrete nature of the microcosm. The discovery of the atomic structure of substance was the first step in the disclosure of the discrete nature of the microcosm. Not only the masses and the electric charges of microbodies are discrete but dynamic values describing the states of microsystems, such as energy, angular momentum, are also discrete and characterized by discontinuous change of their numerical values.

1.3. Structure of the Atomic Nucleus

1. Proton-electron model. Nuclear physics studying the structure and properties of atomic nuclei dates back to the discovery of the atomic nucleus, in 1911. That the masses of the nuclei are almost equal to the atomic masses and that the positive electricity of the atom is associated with the nucleus became known immediately after the discovery of nuclei. The lightest hydrogen atom, as follows from the Moseley law (Sec. 2.1) and the Bohr one-electron theory of the hydrogen atom, and as was confirmed by experiments on its ionization, has one electron and a single charge of the nucleus. After discovery of the stable isotopes of elements, when it became clear that the atomic masses of isotopes are, to a high degree of accuracy, integers if they are expressed in terms of the hydrogen atom mass, the nucleus of the lightest atom was finally given the role of a structural particle for all nuclei. Since 1920, the official term for the nucleus of the hydrogen atom is *proton* [1].

It was not possible to explain the structure of compound nuclei using one structural particle of unit mass and a unit electric charge. Atomic masses of light atoms exceed by twice and of heavy atoms by more than twice the number of unit charges of the nuclei but both these values are equal in the proton. However, already known was another structural particle, the electron, having an electric charge equal to the proton charge and opposite in sign; thus appeared the proton-electron model of the nucleus. In the frame of this model, the value of the atomic mass, rounded off to an integer, indicated the number of protons in the nucleus. The charge of the nucleus was explained by the presence of a certain number of electrons in the nucleus, which neutralize part of the proton charges but which, because of their small mass, do not affect the value of the atomic mass. Although the mass and the charge of the nucleus were satisfactorily explained by the proton-electron model, information being accumulated contradicted the model. It was impossible to explain within the principles of advancing quantum mechanics the presence of such a light particle as an electron in a tiny nucleus. Moreover, the results of the measurement of spins and nuclear magnetic moments also contradicted this model [5].

2. Proton-neutron model. The problem of the structure of atomic nucleus was solved in 1932 when the neutron was discovered. The German physicists Bothe and Becker, continuing Rutherford's experiments on α -particle bombardment of light atoms and observations of nuclear reactions, discovered that when some elements such as lithium, berillium or boron are exposed to radiation there appears a strongly penetrating radiation which passes through lead easier than the most hard γ -radiation emitted by the natural radioactive substance ThC'' . The new radiation interested the French physicists Irene and Fr  d  rick Joliot-Curie who discovered the ability of radiation to form recoil nuclei having high kinetic energies. If the recoil nuclei are considered to appear under the influence of γ -ray, then the γ -ray energy must be so high that it is impossible to fit it to the energy balance of the nuclear reaction activated by the α -particle in the light nucleus. If the γ -ray energy is not high and is comparable to the kinetic energy of the recoil nuclei, then the transfer of almost all the γ -ray energy to the body of very great rest mass appears to contradict the law of conservation of momentum (Sec. 1.5-4).

The nature of the mysterious radiation was discovered by Chadwick, an Englishman. By supposing that this radiation was a stream of heavy particles with no electric charge, Chadwick explained all the observed phenomena. In addition, having measured the energy of hydrogen and nitrogen recoil nuclei he calculated the mass of the conjectural particle which appeared, within the limits of experimental accuracy, to be equal to the proton mass. The heavy particle of no electric charge discovered by Chadwick already had its name, the neutron. The possibility of the existence of a particle similar to the discovered neutron was admitted in connection with the problem of the structure of the atomic nucleus. Rutherford described the properties of such a particle accurately enough and, together with his co-worker Chadwick, endeavoured to produce such particles in a free state, by bombarding aluminium atomic nuclei with α -particles [1].

The analysis of the properties of the new particles led to the conclusion that the neutron, like the proton, should be considered an elementary particle with a half-integral spin (Sec. 1.7-6). It allowed one to conclude that the nucleus consists of protons and neutrons. Theoretical analysis also showed that nuclei of such a composition must be stable. In this case the stability is ensured by a new class of forces, nuclear forces, acting between protons and neutrons. Thus the neutron became the third structural element of the atom.

Subsequently it turned out that the character of the interaction between protons and neutrons in any pair combinations is absolutely the same. It is true that when a pair of protons interact, an

additional factor appears, the electrostatic repulsion of the like charges. This circumstance, however, in no way affects their interaction through nuclear forces. Since both structural particles of the nucleus resemble one another in regard to their nuclear interactions they were given the same name, *nucleon*. Thus, the proton is an electrically charged nucleon, and the neutron is a nucleon with no electric charge.

3. **Nuclear composition.** The supposition of the proton-neutron structure of the nucleus is proved by all the accumulated experimental data obtained for the atomic nuclei. According to the proton-neutron model, in the composition of the atomic nucleus enter a number of protons equal to the number of elementary electric charges contained in the nucleus and a number of neutrons necessary for the sum of neutrons and protons to form the nuclear mass. The total number of nucleons in the nucleus is called the *mass number* A . As the atomic proton and neutron masses differ only slightly from unity and the masses of the bound particles differ only slightly from the masses of the free ones, the atomic masses A , are approximately equal to the mass numbers and, when rounded off to integers, they coincide. The number of protons in the nucleus are equal to the number of its elementary charges and to the number of the element in the Mendeleyev Table designated by Z . The number of neutrons in the nucleus is thus equal to the difference between the mass number and the element number that is, $A - Z$.

1.4. Quantum Mechanics

1. **Diffraction.** It follows from experience that the dynamic properties of particles are more complicated than the properties of macrobodies consisting of a great number of particles, known from classical mechanics. The motion of free particles, as is the motion of free macrobodies, is characterized by a translation in space in a definite direction with some velocity. The fundamental difference in the properties of these and others lies in the fact that during particle interaction a diffraction phenomenon is observed (Fig. 1.2), which is impossible in principle during the interaction of bodies obeying the laws of classical mechanics. Classical bodies move in space along exactly specified trajectories and if many bodies, moving one after another, have the same trajectory, then even after the collision with one and the same quiescent target, the trajectory of motion remains a single one. With slight deviations in the parameters determining the trajectories of the bodies before the collision, after the interaction many trajectories appear but always with differences in the parameters proportional to the initial deviations. The fact that bodies should move after the collision only along some preferable trajectories has no physical foundation in

classical mechanics and, consequently, the spatial picture of clusters and rarefactions of the trajectories of many bodies is impossible. If diffraction is observed, that is, the selectivity of the propagation of the particles in some directions after the collision, then the concept of the trajectory becomes meaningless and the classical mechanics apparatus cannot correctly describe the interaction phenomenon.

It should also be mentioned that the particle interaction is characterized by the selectivity of the values of dynamic quantities.

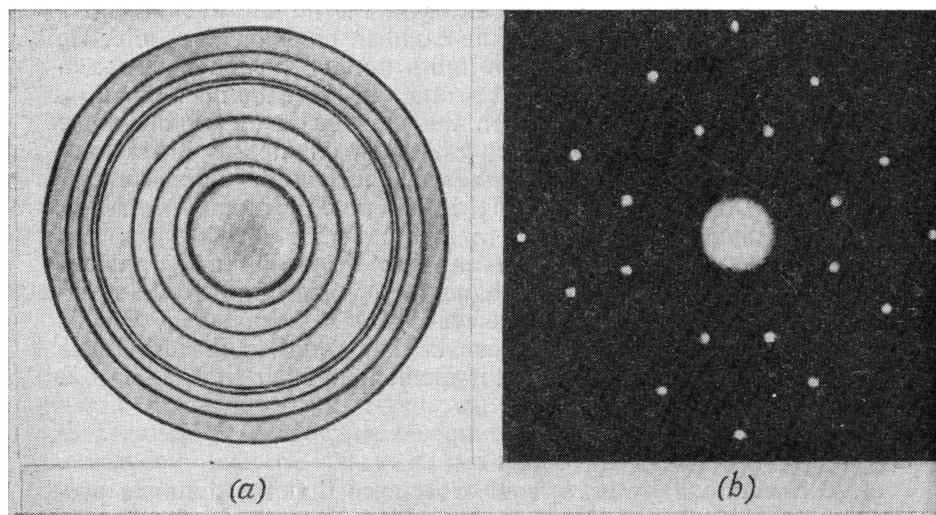


Fig. 1.2. Electron beam diffraction on a thin silver layer (a), and neutron beam diffraction when passing a single crystal (b)

The energy and angular momentum of interacting particles are always discrete, the angular momentum having its own natural unit, $\hbar = h/2\pi$, the integers and half-integers of which express mechanical moments. Interactions of particles possessing the properties in question are described in terms of quantum mechanics, a theory more general than classical mechanics. The concepts of classical mechanics are meaningful only if one neglects the microstructure of bodies and the discrete nature of the dynamic quantities of interacting particles. A very low absolute value of the elementary quantum of action h makes such a neglect in fact inevitable when treating ordinary bodies and the observed properties of the bodies more rough than they really are.

2. Particle and wave. In classical mechanics, two objects exist, a particle and a wave, possessing basically different dynamic

properties. The energy which is carried by a particle or a corpuscle is localized in space. The particle moves along some trajectory, some mathematical curve, and at any moment is characterized by energy E and momentum \mathbf{p} . A typical interaction process of particles is collision if their spatial coordinates coincide and an exchange of energy and momenta takes place, according to the law of energy conservation.

Wave energy, on the contrary, is distributed in space and the energy density carried by the wave decreases as it propagates from the point source. Wave propagation is characterized by frequency ν and wave vector \mathbf{k} oriented along the direction of wave propagation and equal to $1/\lambda$ by modulus, where λ is the wavelength. A typical interaction process is the interference of coherent waves consisting in amplifying or cancelling the wave amplitude, depending on the phase coincidence during the wave superposition. In this instance no energy exchange occurs between the interfering waves. It is precisely the propagating waves that feature the diffraction phenomenon, the wave scattering accompanied by the interference with such spatial arrangement of the maxima and minima of the wave amplitude that is defined only by the wavelength λ and the distance between the objects causing the scattering.

The dynamic properties of particles prove to be such that to describe them with a definite degree of accuracy one has to correlate the classical meanings of "particle" and "wave".

3. De Broglie's postulates. In light propagation experiments, over long period of time, phenomena always showed up which were in agreement with the wave aspect of classical mechanics. The phenomena of interference, diffraction and scattering of light agreed quite well with the concept of light as of propagating light or electromagnetic waves. Meanwhile, it was long known that whereas light of a long wavelength shows typical wave properties, the propagation of light of a short wavelength, neglecting weakly displayed diffraction phenomenon, may be well described in terms of geometrical optics operating with the concept of "ray", analog of the trajectory of a classical material particle. Equations of geometrical optics appeared similar to equations of material point motion. At the end of the last century, the photoeffect phenomenon was discovered, this being the emission of electrons from metallic surfaces under the influence of ultraviolet light, that is, light with a short wavelength. The great probability of the wave energy being transferred to the electron in this process appeared incomprehensible. Moreover, it proved that the energy of the photoelectrons depended not on the light intensity but only on the wavelength. All this meant that the phenomenon progressed as if the light energy were not carried by a wave but by particles of light, i.e., by photons carrying definite portions of energy the value of which is inversely

proportional to the wavelength of the light. Later, other phenomena were discovered which could be explained only by assuming the existence of photons. The Planck quanta hypothesis stimulated the proper interpretation of these phenomena and the understanding of the actual nature of light. Light is a stream of photons possessing both wave and corpuscular properties.

From the point of view of classical physics the photon appeared to be the first particle having properties of a dual nature. Afterwards, these properties turned out to be inherent in all microparticles. However the early experiments on electrons and α -particles revealed only their corpuscular features. The motion of these particles in an electromagnetic field obeys the Lorentz and Newton classical equations. Therefore, when passing particles beams through electric and magnetic fields, it was possible to measure experimentally their velocity and the relationship e/m , and thus to calculate their mass as if they were ordinary charged particles. The α -particles emitted by radioactive substances left rectilinear traces along their flight path in the cloud chamber (see Fig. 1.5). This pointed directly to their corpuscular nature. However, the first hydrogen atom theory faced the necessity of recognizing the singular behavior of the electron when interacting with the proton which did not agree with the behavior of the classical particle. Bohr in his theory postulated the selectivity of the electron orbits, introducing condition (1.6), but this condition was dictated merely by the need for agreement between theory and experiment.

Certain common properties of microparticles which influence the behavior of electrons in the Bohr atom were reported by French physicist de Broglie in 1924. He extended the then known properties of the photon to all the microparticles. De Broglie's idea was that it was impossible to describe the behavior of microparticles using the classical conceptions of the corpuscle alone or of the wave alone. The properties of microparticles combine the properties of these different classical objects. Just as it is impossible to neglect the corpuscular aspect in describing the propagation of light and its interaction with a substance, it is neither possible to neglect the wave aspect in describing the motion and interactions of other microparticles whose corpuscular properties were first known. Just as the light frequency ν was correlated with the energy E carried by the photon, de Broglie in the same way correlated the particle momentum \mathbf{p} with some wavelength $\lambda = 1/k$, where k is the wave vector modulus. These mathematical relations have been named the de Broglie postulates:

$$E = h\nu \quad (1.11)$$

$$\mathbf{p} = \hbar \mathbf{k} \quad (1.12)$$

If the particle velocity v is much less than the light velocity, then the momentum $p = mv$, and the de Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \quad (1.13)$$

that is, the wavelength of a particle with mass m may be directly expressed through the kinetic energy of the particle, E .

4. **Particle wavelength.** Relation (1.11) was verified experimentally when studying the corpuscular properties of the photon. Relation (1.13) was also subjected to experimental verification. The first to do it in 1927 were Davisson and Germer, USA, who observed diffraction of electrons of given energy on nickel crystals. The location of the diffraction maxima enabled them to determine the electron wavelength according to the parameters of the nickel crystal lattice and to compare it with the calculated value of the de Broglie wavelength (1.13). The agreement obtained attested to the validity of the de Broglie formula. Subsequently the de Broglie hypothesis was confirmed many times, and at present the diffraction phenomenon of the X-ray photons, electrons and neutrons is widely used to study the structure of matter [6, 7]. Because of the property (1.13) the distances between atoms and ions in molecules and crystals are determined with high accuracy by that particular particle energy value at which the expected diffraction picture is observed.

The condition for the diffraction is the coincidence of the de Broglie particle wavelength (half wavelength) with characteristic distance between atoms, between other bound particles or crystal planes. The wavelength is unambiguously related to the energy of the particle so that the diffraction is observed at some definite energy value. If the energy increases, which corresponds to a decrease of the particle wavelength, then the diffraction conditions worsen and the diffraction picture of particle scattering quickly vanishes. If the particle energy is lower than the most favourable value, that is, the wavelength is greater than the value of the characteristic distance between the scattering centres, then diffraction becomes impossible. The moving particle interacts with the object as a whole, not consisting of separate elements having spatial structure. A particle with a wavelength greater than the distance between the elements of the interaction object is a very coarse instrument to use for the study of the structure of an object.

$\lambda = \lambda/2\pi$ plays the same role in the interaction of particles with point objects as λ does in their interaction with linear ones, that is, during formation in the process of interaction of bound states with the orbital motion of the particle. According to (1.13)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \quad (1.14)$$

In this case, the wavelength λ has the meaning of the orbit radius [see (1.6)], or the space region radius where the interacting particle is effectively presented. As λ defines the size of the space region in which the interacting particle is found, and as λ depends only on the velocity or the energy of the freely moving particle, it can be considered as some effective size of the moving particle which is revealed only during formation of bound states. In such a case, the cross-sections of particle interactions with other particles during their collisions may be expressed through this effective particle size (Sec. 1.6). The region cross-section with radius λ is merely $\pi\lambda^2$.

5. Uncertainty relation. The de Broglie postulates (1.11) and (1.12) combining the corpuscular and wave properties of microparticles served as the foundation for the theory of the motion and interactions of microparticles, i.e. quantum mechanics. In quantum mechanics, the state of a particle is described by the wave function $\psi(x, y, z)$ which, in the stationary case, depends only on the space coordinates. The specific form of the wave function is determined by solving the Schrodinger equation including the term expressing the particle interaction law. The square of the wave function modulus $|\psi(x, y, z)|^2$ is the distribution of the probability for the particle to have any space coordinates (x, y, z) . The wave function does not indicate the sequence in which the space coordinates are occupied with time, as is required when describing the motion of a classical particle, because this has no meaning for microobjects. For microparticles, the conception of moving along a trajectory analogous to the trajectory of the classical particle does not exist. This circumstance is most clearly indicated by one fundamental corollary of the de Broglie postulates known as the *Heisenberg uncertainty principle*. It appears that for the microparticle possessing corpuscular-wave properties it is impossible to simultaneously indicate its precise coordinate x along a certain axis x , and the projection of the momentum on this axis p_x . The coordinate x can be fixed only in the limits of a certain interval Δx if the momentum projection p_x is fixed with the accuracy Δp_x and

$$\Delta x \cdot \Delta p_x \gtrsim h \quad (1.15)$$

which is the uncertainty relation. For objects possessing the property (1.15) or, in other words, having the corpuscular-wave nature, the conception of trajectory in the classical sense does not really exist when at each point x the particle possesses the precise meaning p_x determined by the motion law.

It follows from relation (1.15) that the precise determination of the space coordinate ($\Delta x \rightarrow 0$) inevitably leads to the full uncertainty of the particle's momentum ($\Delta p_x \rightarrow \infty$), and vice versa. Therefore, if the particle having momentum p is considered to be lo-

cated in a space region of size λ ($\Delta x = \lambda$), which is characteristic of an interacting particle, then since

$$\lambda \cdot \Delta p_x \gtrsim h \quad (1.16)$$

and $h/\lambda = p$, $\Delta p_x \gtrsim p$, and nothing can be said of the momentum projection at any time instant as it may lie between 0 and p . If (1.16) is divided by 2π , we obtain

$$\lambda \cdot \Delta p_x \gtrsim \hbar \quad (1.17)$$

that is, the same conclusion is drawn for the particle interacting with the point object. In this case, the result of the uncertainty relation is as follows

$$\Delta x \cdot \Delta p_x \gtrsim \hbar \quad (1.18)$$

If the particle momentum is fixed precisely enough, so that $\Delta p_x \ll p$, which is characteristic of the freely moving particle, then from (1.15) it follows that $\Delta x \gg \lambda$, and the natural uncertainty of the particle's position in space is much greater than its wavelength. It means that the wavelength defined by the momentum does not manifest itself. Thus, under various conditions, the wave or corpuscular nature of the microparticle may be in the foreground, but relation (1.12) relating the wave characteristics λ to the corpuscular characteristics p always holds.

6. Particle trajectories in tracking devices. It should be pointed out that the formation of charged particle tracks in tracking recording instruments (Sec. 1.7-1) by no means contradicts the conclusion that the classical concept of trajectory is inapplicable to the motion of the microparticle. The particle track becomes visible because it is formed by macroscopic bodies, liquid droplets, vapour bubbles, silver crystallites, and has great width. The track width has nothing to do with the uncertainty of the particle's location along the axis x perpendicular to the direction of its motion. The uncertainty of the particle's location Δx is always much less than the track width while the opposite is in principal impossible. Quite a low value of the action quantum \hbar allows one to simultaneously detect, during experiments momentum projections and the corresponding coordinates with comprehensive accuracy. However an accuracy exceeding (1.15) is naturally unobtainable.

7. Virtual particles. The uncertainty relation (1.18) written in energytime coordinates is

$$\Delta E \cdot \Delta t \gtrsim \hbar \quad (1.19)$$

From this it follows that however accurately the moment of observation is fixed ($\Delta t \rightarrow 0$), the energy of the microparticle or microsystem becomes quite undeterminate. However, if the energy is

precisely fixed, it is impossible to say which moment of time this precise energy value is referred to. It then follows that although the law of energy conservation is always valid, the question of the correspondence of the energy at some particular moment of time to its value observed over a long period does not have sense. Therefore, the supposition of the formation and disappearance of the so-called virtual particle with a high total energy value when the quantity of stored energy is not sufficient for forming an analogous real particle is not contradictory. The existence of the imaginary particle is possible only during a short interval of time Δt such that the energy excess ΔE associated with the appearance of the virtual particle is related to the time interval Δt , as (1.19). The supposition of the emission and immediate absorption of such virtual particles facilitates the description of the interactions of other particles in quantum electrodynamics and in meson nuclear field theories.

8. Energy level widths. Another consequence of uncertainty principle (1.19) relates to the discrete energy levels of microparticle systems. The energy levels of the microsystems are separated by finite intervals and each level is characterized by a definite energy value. The level with the lowest energy value corresponds to the ground energy state of the microsystem. Ever higher levels are excited only when energy is received from the outside. The microsystem, however, quickly gets rid of the acquired surplus energy by emitting a photon or some other particle. This means that all the high energy levels or excitation levels are unstable. The transition of the microsystem from its excited state to the ground state does not occur instantaneously but during some finite time having a mean value τ known as the mean lifetime of the excited level. As the excited state exists for an indefinite time τ , then, by (1.19); it cannot be defined by a definite energy value but must have some spread of permissible values ΔE , that is, the excited level must have width. The level widths Γ are usually small compared to the distances between the levels but they are, however, well observed experimentally during the excitation of nuclear processes. Between the level width Γ and its mean lifetime τ relation (1.19) exists which in a new notation has the form

$$\Gamma \tau \approx \hbar = 6.6 \cdot 10^{-16} \text{ eV} \cdot \text{s} \quad (1.20)$$

The longer the life-time, the narrower the energy level. The lowest, or the ground, level exists infinitely long and, as follows from (1.20), has no width. It is only the ground energy state of a microsystem that is strictly definite as regards the energy.

9. Angular momentum. The relative motion of interacting particles is always characterized by some value of angular momentum

or mechanical moment usually expressed by a non-dimensional number l showing the number of units \hbar the vector-moment comprises. At the same time such an expression is conventional.

The angular momentum vector possesses some peculiarities in quantum mechanics. In the first place, the instantaneous value of the vector \mathbf{l} has no sense, just as the instantaneous value of the vector of the particle's momentum during orbital motion of the particle in its bound state has no sense [see formula (1.17)]. It is only the square of the modulus (l^2), that is the actual absolute value of the angular momentum vector, and one of its spatial projections, usually termed the projection onto the axis z , l_z , that have sense and certain numerical values. Secondly, unlike the classical case, the square modulus of the vector is not equal to $l^2\hbar^2$ but to $l(l+1)\hbar^2$ if by vector value \mathbf{l} is understood not merely a number but a vector having the dimension of angular momentum. And finally, the projection of the angular momentum vector onto the axis z , l_z may take either only integer values $0, \pm 1, \pm 2, \dots, l$ being an integer, or only half-integer values $\pm 1/2, \pm 3/2, \pm 5/2, \dots, l$ being a half-integer in units \hbar where the plus or minus sign means the orientation of the angular momentum vector along or opposite the fixed direction on axis z . The absolute maximum value of the projection l_z of the given vector \mathbf{l} equals $l\hbar$, so that the non-dimensional number l shows the number of units \hbar contained in the maximum value of the projection of the angular momentum vector onto the axis z .

The above properties of the angular momentum vector are usually demonstrated by the following classical model. The vector-moment is presented in the form of an ordinary vector precessing about the axis z , the modulus of which equals $\sqrt{l(l+1)}\hbar$ and which is oriented along or opposite to the direction of the axis z always in such a way that its projection onto the axis z is equal to one of the values from $+l\hbar$ to $-l\hbar$ through the unit \hbar (Fig. 1.3). As the absolute value of the vector is not equal to $l\hbar$, such a vector is never oriented precisely along the axis z and, therefore, apart from its modulus, the value invariable with time is only one vector projection — the projection onto the axis z . The properties of such a model are in certain definite agreement with the properties of the quantum mechanical angular momentum vector.

The total number of possible projections l_z of the mechanical moment l equals $(2l+1)$, with l being an integer or half-integer, as the projection values lie between $+l$ and $-l$ at unit intervals (Fig. 1.3). In fact, the number $(2l+1)$ may be considered to determine the value l of angular momentum. This number is directly observed in the experiment on splitting the atomic spectral lines or particle beams into $(2l+1)$ components in magnetic fields. Splitting is caused by the magnetic moment associated with

angular momentum; the effectiveness of interaction of the magnetic moment with the magnetic field depends on its orientation relative to the magnetic induction vector, that is, is determined by the value of the projection \mathbf{l} onto the direction of the magnetic induction vector. The realization of any of the possible mechanical moment projections is equally probable. Therefore, in the interaction of many particles with one and the same moment the number of cases corresponding to each of the possible projections is the same; this follows directly from the similar intensities of the components of the splitted lines or particle beams.

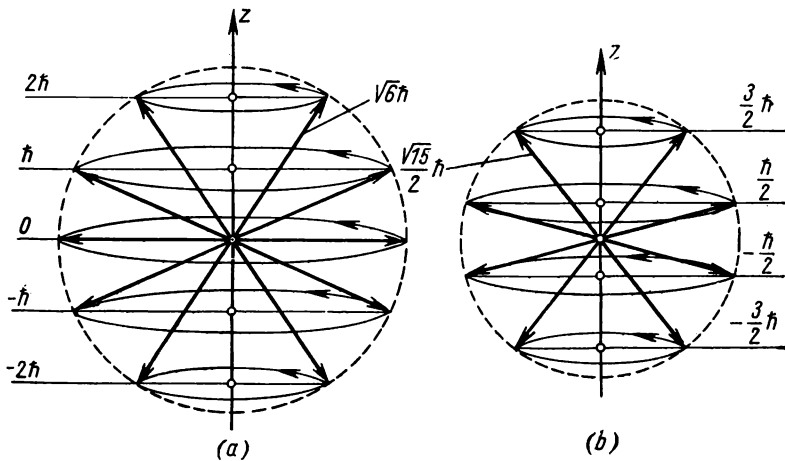


Fig. 1.3. Model of the momentum vector
(a) total angular momentum, $l=2$; (b) half-integer angular momentum, $l=3/2$

The interaction of two microparticles having intrinsic mechanical moments l_1 and l_2 is characterized by some total moment. Vectors \mathbf{l}_1 and \mathbf{l}_2 are added up so that each projection of the sum onto the axis z is also expressed in integers (or half-integers), with the difference between the neighbouring values being only unity. Therefore, the addition of the vectors \mathbf{l}_1 and \mathbf{l}_2 is the addition of the values of their projections as algebraic numbers. As any projection is equally probable all the pair combinations of the added projections are probable in adding. If each projection of one vector is added to each projection of another vector, one has a large set of possible values of the resulting projection in which $(l_1 + l_2)$ and $(-l_1 - l_2)$ are presented once, $(l_1 + l_2 - 1)$ and $(-l_1 - l_2 + 1)$ twice and so on ..., with thus $(2l_1 + 1) \times (2l_2 + 1)$ values of the resulting projection in all. This set of projections determines several vector values from $(\mathbf{l}_1 + \mathbf{l}_2)$ to $(\mathbf{l}_1 - \mathbf{l}_2)$ if $l_1 > l_2$ or to $(\mathbf{l}_2 - \mathbf{l}_1)$ if $l_2 > l_1$. Thus, the sum of the mechanical mo-

ments l_1 and l_2 may be in the range from $(l_1 + l_2)$ to $|l_1 - l_2|$. The total number of the possible resulting moments will be $(2l_i + 1)$, where l_i equals the smallest of l_1 and l_2 , and the probability of realizing any of them is determined by the statistical weight of a state with a given resulting mechanical moment.

10. Statistical weight. The statistical weight characterizes the probability of some state compared to the probability of the least probable state. Thus, the statistical weights are relative numbers the least of which may always be taken as unity. With such a choice, the statistical weight of the state with the mechanical moment l is merely equal to $(2l + 1)$. Each state with the moment l is, in fact, a group of $(2l + 1)$ equally probable states with differing values of projection l_z . The number of possible states in the group may be taken as the statistical weight of the state if among all the possible groups there is at least one with the number of states equal to unity. Such a state is exactly the state with the least value of the mechanical moment $l = 0$, which may be considered to have the only value of projection $l_z = 0$.

It should be kept in mind that the total probability of forming a state with some value of mechanical moment l may strongly depend on the interaction energy corresponding to this or that value of the mechanical moment. However, all other things being equal, when only statistical properties of states or their multiplicity are of importance, the correlation of the realization probabilities of states having different values of l is determined only by the relation of their statistical weights. When particles with the mechanical moments l_1 and l_2 interact, the resulting moment may be any from $(l_1 + l_2)$ to $|l_1 - l_2|$ but in each particular case only one value l' from the range of all possible values is realized. As mentioned above, the total number of possible values of the resulting projection of the vectors l_1 and l_2 equals $(2l_1 + 1)(2l_2 + 1)$ and this number is the statistical weight of the state being summed up, $l_1 + l_2$. The statistical weight of one of the possible values of the resulting vector-moment l' is $(2l' + 1)$. Therefore, the probability of the formation of a state with moment l' is

$$g = \frac{(2l' + 1)}{(2l_1 + 1)(2l_2 + 1)} \quad (1.21)$$

The value g is called a statistical factor and shows which part of the particles interaction with the mechanical moments l_1 and l_2 leads to the formation of the state with the mechanical moment l' .

11. Parahydrogen and orthohydrogen. The greater the mechanical moment of the microsystem state, the greater its statistical weight and the more probable the state. This can be illustrated by an example of molecular hydrogen. Each hydrogen molecule consists of two atoms whose nuclei, protons, have their own

mechanical moments equal to $1/2$ (Sec. 1.7-6). Following the addition rules, the resulting vector of the nuclear moments in one molecule may be either 1 or 0. A molecule with a nuclear mechanical moment $l = 1$ is called an orthohydrogen molecule, and with $l = 0$ a parahydrogen molecule. At normal temperature, the difference between the energies of the lowest energy states of ortho- and parahydrogen molecules is negligibly small compared to the energy of thermal motion and, therefore, both states with $l = 1$ and $l = 0$ are realized. But as the statistical weight of the orthohydrogen molecules is three times the statistical weight of the parahydrogen molecules (statistical factor g , as follows from (1.21), is correspondingly equal to $3/4$ and $1/4$), the number of the orthohydrogen molecules is also three times greater than the number of the parahydrogen molecules in ordinary hydrogen gas. At a very low temperature, about 20°K , all the hydrogen is transformed into parahydrogen because the lowest energy state of the hydrogen molecule taking part in the energy exchange with other molecules is feasible with the total spin of the nuclei of the hydrogen atoms equal to zero. Orthohydrogen differs from parahydrogen only in the phenomena where the nuclear spin of the hydrogen molecule is of significance (Sec. 2.5-6).

1.5. Relativity Theory Formulae

1. **Mass, energy, momentum.** The mass of the particle depends on the velocity of motion, v :

$$m' = \frac{m}{\sqrt{1 - \beta^2}} \quad (1.22)$$

where $\beta = v/c$, and c is the velocity of light; m is the rest mass, and m' , the relativistic mass, although the relativistic mass is the mass increment to the rest mass. When $v \ll c$, $m' = m$, consequently the relativistic mass is worth speaking about only when the velocities of motion are close to the velocity of light. Particles moving with such velocities and the velocities themselves are also called *relativistic*.

According to the relativity theory, the total particle energy is equal to

$$\mathcal{E} = m'c^2 \quad (1.23)$$

where m' is the relativistic mass of the particle. Another expression of the total energy, equivalent to (1.23), is

$$\mathcal{E} = \sqrt{p^2c^2 + m^2c^4} \quad (1.24)$$

where p is the particle momentum

$$p = m'v \quad (1.25)$$

If the particle is at rest, ($v = 0$), its total energy, as follows from (1.24) and (1.23), equals the rest energy:

$$\mathcal{E}_0 = mc^2 \quad (1.26)$$

As follows from (1.24), the total particle energy includes both the energy of motion, or *free energy* due to momentum p , and the rest energy, or *bound energy* corresponding to the rest mass m .

(1.26) and (1.23) are the relations of equivalence between mass and energy. In this case (1.26) refers to the particular case of the particle at rest with rest mass distinct from zero while (1.23) is a general case of the mass and energy equivalence. If the particle has mass m , it possesses energy $\mathcal{E}_0 = mc^2$. And conversely mass $m' = \mathcal{E}/c^2$ is equivalent to any energy \mathcal{E} . For example in a nuclear reactor during fission of uranium new nuclei appear more tightly bonded than the original ones having therefore lower rest masses. If Δm denotes the mass reduction of the nuclei with a certain number of fissions it will correspond to the release of free energy $\Delta\mathcal{E} = \Delta mc^2$, which is transformed into the energy of thermal motion of the environmental atoms. If, however, all the fission energy $\Delta\mathcal{E}$ remains within the volume of the nuclear reactor, then both the total energy and the total mass of the substance comprising the reactor volume do not undergo changes due to this number of fissions. However, if all the released heat is removed from the reactor volume, then the total energy and, hence, the total mass of the reactor substance will be reduced by the values $\Delta\mathcal{E}$ and Δm , respectively. Part of the removed heat may be transformed into electric energy. Thus electric energy, in the long run, appears because of the rest mass reduction of the atomic nuclei. If the electric energy is produced by burning chemical fuel, it also results from the reduction of the mass of the substance which has reacted, to be more exact, of the masses of atoms bound to form molecules because the free energy can appear only with an equivalent reduction of the bound energy or rest mass.

Another example concerns the photon. The photon, or electromagnetic quantum of energy, has no rest mass. However, by (1.23), it possesses relativistic mass $m' = h\nu/c^2$, since the photon energy is equal to $h\nu$ and consequently possesses momentum

$$p = m'c = h\nu/c \quad (1.27)$$

because light velocity c is exactly the velocity of the photon. Expression (1.27) is in agreement with de Broglie relation (1.12) as $k = 1/\lambda = \nu/c$. It also follows from (1.24) on condition that $m=0$. In general, any particle having rest mass equal to zero has momentum [see formula (1.24)]

$$p = \frac{\mathcal{E}}{c} = \frac{m'c^2}{c} = m'c \quad (1.28)$$

that is, like the photon it always moves with the velocity of light.

2. Kinetic energy. The kinetic energy E of the relativistic particle is the difference between the total energy and the rest energy

$$E = m'c^2 - mc^2 = mc^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) \quad (1.29)$$

When $v \ll c$, expression (1.29) changes, as it should, into the kinetic energy formula of classical mechanics, $E = mv^2/2$. If the kinetic energy, E^0 is expressed in terms of rest energy mc^2

$$E^0 = \frac{E}{mc^2} = \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) \quad (1.30)$$

then

$$\beta = \frac{\sqrt{E^0(E^0 + 2)}}{E^0 + 1} \quad (1.31)$$

and

$$\frac{m'}{m} = E^0 + 1 \quad (1.32)$$

which directly follows from (1.29) and (1.30). Table 1.1 gives some values E^0 and the corresponding values of β and m'/m .

Table 1.1

Kinetic Energy, Velocity and Mass of Particle

| $E^0 = E/mc^2$ | $\beta = v/c$ | m'/m | $E^0 = E/mc^2$ | $\beta = v/c$ | m'/m |
|----------------|---------------|--------|----------------|---------------|--------|
| 0.001 | 0.045 | 1.001 | 1.0 | 0.87 | 2.0 |
| 0.005 | 0.1 | 1.005 | 2.0 | 0.94 | 3.0 |
| 0.01 | 0.14 | 1.01 | 5.0 | 0.986 | 6.0 |
| 0.1 | 0.42 | 1.1 | 10 | 0.996 | 11 |
| 0.5 | 0.75 | 1.5 | 100 | 0.99995 | 101 |

If the velocity-dependent mass increment is considered only when it exceeds 1% of the rest mass, then the particles moving with velocities exceeding 14% of the light velocity, or more than $4.2 \cdot 10^7$ m/s should be thought of as relativistic particles. The kinetic energy of such particles is 1% of the particle rest energy. As the electron rest energy is 0.5 MeV and that of nucleons is about 1 GeV (see Table 1.2), with the chosen criterion the electron will be relativistic at the energy greater than 5 keV, the nucleons — at the energy greater than 10 MeV, and α -particles at the kinetic energy greater than 40 MeV. Heavy particles, therefore, are to be

considered relativistic only at energies as great as tens of mega-electron volts while the electron can practically always be thought of as a relativistic particle. Relativistic particles are always particles having rest masses equal to zero.

3. Atom recoil in electron emission. If two particles with masses M and m are formed in the process of the decay of a body at rest, then, according to the momentum conservation law, they fly apart with equal oppositely directed momenta, $p_M = p_m$. In the nonrelativistic case, the kinetic energy is $p^2/2m$, and with equal momenta, the kinetic energies are inversely proportional to the masses:

$$\frac{E_M}{E_m} = \frac{m}{M} \quad (1.33)$$

that is, the main part of the decay energy, $E_{dec} = E_M + E_m$, is carried away by the lighter particle. In β -decay, the nuclei of the atoms emit electrons and neutrinos with no rest mass, thus the recoil energy of the atom is quite small. The atom accepts the greatest recoil when the electron is emitted with the maximum possible energy (the neutrino energy, $E_\nu = 0$). If the kinetic energy of the atom is denoted by E_a , that of the electron by E_e and their masses by M and m , respectively, then, since it is only the electron that will be relativistic, the ratio of the kinetic energies, on condition that $p_a = p_e$, will be

$$\frac{E_a}{E_e} = \frac{p_a^2}{2M} \frac{1}{(m'c^2 - mc^2)} = \frac{1}{2M} \frac{(m'v)^2}{m'c^2 - mc^2} = \frac{m}{2M} (E_e^0 + 2) = \frac{m + m'}{2M} \quad (1.34)$$

where E_e^0 is the electron's kinetic energy in terms of mc^2 . For the neutrino emitted with maximum energy $E_\nu (E_e = 0)$, (1.34) is the ratio E_a/E_ν , when $m = 0$. If one takes as an example $E_e = 1$ MeV, then $E_e^0 = 2$, and the recoil energy of the atom compared to the energy of the electron is twice as great as in the relativistic case (1.33), and so on ... It is essential, however, that in this case the electron relativistic mass exceeds the rest mass not by two but by three times (1.32).

It follows that the mere substitution of the electron mass by its relativistic mass in expression (1.33) is not justifiable.

4. Specific momentum. The latter circumstance is due to the fact that the transfer in space of that part of the mass which is formed by the energy of motion or of the relativistic mass proper is associated with a smaller momentum than the transfer of the rest mass is. The relativistic mass inertia is less than the rest mass inertia. The momentum accounted for by one unit of kinetic energy is the smaller, the larger the contribution of the relativistic addition to the mass of the body. The smallest momentum, in

the above sense, is possessed by particles having rest mass equal to zero, for example, photons. The ratio of the photon's momentum to its energy equals

$$x_\gamma = \frac{h\nu}{c} \cdot \frac{1}{h\nu} = \frac{1}{c} \quad (1.35)$$

while the same relation, or momentum per unit kinetic energy of the particle having rest mass not equal to zero is

$$x_m = \frac{m'v}{E} = x_\gamma \sqrt{\frac{E^0 + 2}{E^0}} = x_\gamma \chi \quad (1.36)$$

where $\chi > 1$. When $v \ll c$, χ turns to $2c/v$ and arbitrarily increases with decreasing velocity. On the contrary, as $v \rightarrow c$, $\chi \rightarrow 1$ and $x_m \rightarrow x_\gamma$, that is, the momentum of the particle with $m \neq 0$ and the momentum of the photon practically coincide at a very high equal energy because in this instance the masses of both particles have only the relativistic origin ($m' \gg m$). However, if this extremely relativistic case is excluded, then the momentum carried by the photon is much less than the momentum of particles with the rest mass not equal to zero and with the same kinetic energy (when $\beta = 0.5$, $\chi \approx 4$; $\beta = 0.98$, $\chi \approx 1.2$).

5. Particle production thresholds. Particle collisions may be followed by the formation of new particles with rest mass not equal to zero. Usually, one of the particles taking part in the collision does not move and a part of the kinetic energy of the second bombarding particle is transformed into the rest energy of the new particles. In the general case, the total kinetic energy cannot be transformed into the rest energy of the new particles because the momentum of the bombarding particle must remain after the collision and the momentum is associated with the kinetic energy. Hence, the kinetic energy must exceed the value of the rest energy of the new particles. Its minimum value at which the production process is feasible is called the *threshold energy*. The threshold value does not only depend on the rest energy of particles being formed but also on the mass of the bombarding particle because at the same kinetic energy the momentum is the less, the lower the rest mass. The threshold kinetic energy in a laboratory coordinate system is such an energy at which in the coordinate system of the inertia centre, where the total momentum is always equal to zero, the energy of the two particles relative motion is wholly transformed into rest energy, so that all the particles are at rest relative to each other after production of the new ones. To calculate the threshold energy it is convenient to use the invariant of the relativity theory:

$$\mathcal{E}^2 - c^2 p^2 = \text{inv} \quad (1.37)$$

Here \mathcal{E} and p are respectively the total energy and momentum of an arbitrary system of bodies which interact only with each other. Expression (1.37) has one and the same meaning in any coordinate system, which is seen by the example of one particle. Invariant (1.37) for one particle directly follows from (1.24) and is the square of the particle's rest energy. The rest energy is an inherent property of the particle and does not depend on the reference system. If M_1 and M_2 are the masses of the bombarding and the target particles, respectively, E_{thr} and p_1 are the threshold kinetic energy and the momentum of the particle with mass M_1 , and \mathcal{M} is the sum of the masses of the new particles, then writing (1.37) in the laboratory coordinate system before the collision and in the inertia centre system after the collision, one obtains the equation

$$(M_1c^2 + M_2c^2 + E_{thr})^2 - c^2p_1^2 = (M_1c^2 + M_2c^2 + \mathcal{M}c^2)^2 \quad (1.33)$$

Before the collision, the total energy of the two particle system, besides the rest energies, includes the kinetic energy of the first particle, and the total momentum of the system is p_1 because the particle with mass M_2 is at rest. After the collision of particles having masses M_1 and M_2 and the formation of the new ones having mass \mathcal{M} , the kinetic energies and momenta of all the particles in the laboratory reference system, of course, differ from zero but in the inertia-centre system they are equal to zero, since, according to the condition, E_{thr} is the threshold kinetic energy. If p_1 is expressed by M_1c^2 and E_{thr} , then we shall have

$$E_{thr} = \mathcal{M}c^2 \left(\frac{M_1 + M_2 + \frac{\mathcal{M}}{2}}{M_2} \right) \quad (1.39)$$

and

$$E_{thr} - \mathcal{M}c^2 = \mathcal{M}c^2 \left(\frac{M_1}{M_2} + \frac{\mathcal{M}}{2M_2} \right) \quad (1.40)$$

that is, the excess of E_{thr} over $\mathcal{M}c^2$ increases linearly with the increase of $\mathcal{M}c^2$ when $\mathcal{M} \ll M_1$ and, in general, it increases faster than (1.40).

6. Threshold of endoergic nuclear reaction. Ordinary nuclear reactions occur during the collision of two nuclei with masses M_1 and M_2 , where M_2 is a target nucleus at rest. As a result of the reaction, instead of the original nuclei, two new ones with masses M_3 and M_4 are formed. If the mass sum $M_3 + M_4$ is greater than the mass sum $M_1 + M_2$, the mass excess can occur only at the expense of the kinetic energy of the nucleus M_1 . In this case, the reaction is also threshold and the value of the threshold energy

E_{thr} may be determined by a method analogous to the above. From the equality of the invariants

$$(M_1c^2 + M_2c^2 + E_{thr})^2 - c^2p_1^2 = (M_3c^2 + M_4c^2)^2 \quad (1.41)$$

it follows that

$$E_{thr} = \Delta\mathcal{M}c^2 \left[\frac{M_1 + M_2}{M_2} + \frac{\Delta\mathcal{M}}{2M_2} \right] \quad (1.42)$$

where $\Delta\mathcal{M} = (M_3 + M_4) - (M_1 + M_2)$. The quantity $Q = [(M_1 + M_2)c^2 - (M_3 + M_4)c^2]$ is called the nuclear reaction energy and in this case it is negative. Further, instead of positive factor $(-Q)$, the modulus Q is taken and

$$E_{thr} = Q \frac{M_1 + M_2}{M_2} + \frac{Q^2}{2M_2c^2} \quad (1.43)$$

The second term in (1.43) is usually negligibly small compared to the first term and, therefore,

$$E_{thr} = Q \frac{M_1 + M_2}{M_2} \quad (1.44)$$

Formula (1.44) can be derived simply from the laws of energy and momentum conservation of classical mechanics. However, the minor second term in (1.42) or in (1.43) is derived only from the precise relativistic relations for energy and momentum. This minor addition to E_{thr} is due to the fact that the transfer of a larger rest mass requires a greater momentum.

1.6. Nuclear Effective Cross-Sections

1. **Definitions.** Two macroscopic spheres with radii R_1 and R_2 collide with each other if the centre of one of them, while moving, crosses the region described by the radius $R_1 + R_2$ around the centre of the second sphere. The cross-sectional area of this region is equal to $\pi(R_1 + R_2)^2$ and its absolute value determines the probability of the collision of the two spheres when moving in the given region of the space. Although both spheres have finite dimensions, the above cross-section may be ascribed to a body-target and then only the centre is left of the first sphere, or, in other words, the bombarding body is considered to be a point.

Nuclear effective cross-sections, or the effective cross-sections of the particles interaction, are of the same meaning: the effective cross-section is the area of the cross-section of such a space region near the particle-target in the crossing of which by the bombarding point-particle interaction appears with 100% probability, followed by scattering or a reaction. At the same time, there are some distinctions between the effective and the classical cross-sections,

First, neither in the limits of the volume of the nucleus nor in the vicinity of the elementary particle is there such a region in the crossing of which by some other particle interaction is certain to take place. The value of the effective cross-section merely gives the same number of interactions as is really observed when in some cases even when the bombarding particle crosses the region of the effective cross-section no interaction occurs, while in other cases interaction occurs in spite of the particle's flight beyond the limits of the effective cross-section.

Second, the effective cross-sections are not so much defined by the geometrical size of complex microparticles or by the radii of action of forces as by the wave properties of the particles. When bound states are formed, the space region occupied by the interacting particle has a radius of the order of the de Broglie wavelength λ (Sec. 1.4-4), and, hence, the cross-section of the order of $\pi\lambda^2$. As λ , (1.14), is inversely proportional to the velocity, the cross-section increases with the energy decrease. However, bound states are formed at strict energy relationships and the relevant cross-sections are observed only at chosen energy values. This leads to a very complicated picture of the behaviour of the cross-sections *versus* energy (Sec. 4.5). Thus the effective cross-section is a value averaged from many cases of interaction, which, first of all, determines the interaction effectiveness of colliding particles and only under certain specific conditions it gives an idea of their sizes or the radii of action of forces.

Nuclear effective cross-sections which in neutron physics are also called *neutron effective cross-sections* are found experimentally and are measured in barns:

$$1 \text{ barn} = 10^{-28} \text{ m}^2 \quad (1.45)$$

Nuclear reaction rates or the number of the reacted particles are simply calculated if the effective cross-section values are known. The procedure of using the cross-sections in calculations and the technique of measurement will be clear from the following discussion.

2. Flat target. If σ is the effective cross-section of the neutron-nucleus interaction, or as it is called the nucleus cross-section, there is a nucleus on surface $F(m^2)$, and a point neutron (in accordance with the definition of σ), when moving in a direction perpendicular to F , is certain to intersect surface F with equal probability at any point, then the probability of its interaction with the nucleus is equal to the ratio of the areas σ and F :

$$w' = \frac{\sigma}{F} \quad (1.46)$$

With N_F nuclei not overlapping each other, the probability of the neutron colliding with any of them increases N_F times:

$$\omega = \frac{\sigma}{F} N_F = \sigma N_1 \quad (1.47)$$

where N_1 is the mean number of nuclei in unit area. Thus, the probability of the neutron colliding with the nucleus is only determined by the cross-section value and the density of nuclei distribution on the plane of the bombarded target and does not depend on the total number of the particles-targets. In fact, ω is the fraction of the target area overlapped by the cross-sections of nuclei, which is, on the average, the same for both the unit area and the total area. In this sense, expression (1.46) is a specific case of (1.47), if instead of the condition of the compulsory intersection of area F by the neutron it is implied that one nucleus also exists in any neighbouring section of the plane with the same area F (m^2), that is, on condition that $1/F = N_1$.

If n_0^F neutrons pass through area F , the fraction of neutrons subjected to interaction is a fraction of the target plane covered by the cross-sections of nuclei, that is, ω . If the number of neutrons which come into interaction with each other in area F is denoted by Δn^F , then

$$\frac{\Delta n^F}{n_0^F} = \sigma N_1 \quad (1.48)$$

This relation is the fundamental one in determining cross-sections for the case of a flat target. The cross-section is the fraction of particles which have come into interaction over the *total* target area referred to the number of nuclei in *unit* target area.

When neutrons collide with nuclei, different processes are possible, for example, scattering, capture and others. If Δn_i^F is related to some particular i th type of interaction, then cross-section σ_i found from (1.48) is the cross-section describing only this interaction and it is called *partial*. If Δn^F is the sum of all the partial interactions, $\Delta n^F = \sum \Delta n_i^F$, that is, is equal to the total number of neutrons which have collided with nuclei and have initiated any of the possible processes, then the corresponding cross-section is called *total*. It is evident that the total cross-section σ_t is equal to the sum of the partial ones:

$$\sigma_t = \sum \sigma_i \quad (1.49)$$

Particle fluxes are usually related to unit time and to unit surface being irradiated. Such a normalized flux Φ is the number of particles intersecting in a unit time the unit area perpendicular to the direction of the moving particles motion and is the flux den-

sity. Nevertheless, in the foregoing, Φ will always be called simply a flux. It will not lead to misunderstanding as the fluxes not related to unit area are not used. If n is the number of moving particles in unit volume and v is the velocity of their motion then, as each particle in unit time traverses path v , the flux will be

$$\Phi = nv \quad (1.50)$$

For the given flux of neutrons, Φ , and the known cross-section σ , it is possible to find the collision velocity per target unit area. If Δn^F is also related to unit area and unit time, and v , which is now the collision velocity, is denoted, then the fraction of neutrons that experienced collisions in unit time in unit area will be v/Φ . As this fraction, by (1.48), is equal to σN_1 , then

$$v = \Phi \sigma N_1 \quad (1.51)$$

The collision rate in area F is naturally F times greater.

Formulae (1.48) and (1.51) relate to a particular case of a flat target on which nuclei or atoms are arranged in one row. The real targets are always three-dimensional. Nevertheless, if the number of nuclei through the total target depth in unit area N_1 is not very great and the cross-section is small so that $N_1\sigma \ll 1$, then the random overlap of nuclei is of no significance, and the target may be considered flat. However, if $N\sigma \gtrsim 1$, the target cannot be considered flat, and determining the cross-section by formula (1.48) will lead to an error.

3. Volume target. Figure 1.4 shows a body with the number of atoms in unit volume N . The neutron flux Φ_0 with the coordinate $x = 0$ and the area equal to unity is incident onto the front face of the body. If the flux, at an arbitrary point x in the body depth, is denoted by $\Phi(x)$, then with the help of expression (1.51) one can find the rate of collisions in the layer dx in the vicinity of point x , as the flat target condition is always met in an infinitely thin layer:

$$dv = \Phi(x) \sigma N dx \quad (1.52)$$

where $Ndx = dN_1$ is the number of atoms in layer dx . As the number of collisions is the number of neutrons falling out of the neutron flux, or $dv = -d\Phi$, (1.52) is a differential equation with respect to Φ :

$$-d\Phi = \Phi \sigma N dx \quad (1.53)$$

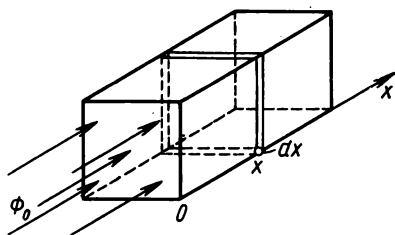


Fig. 1.4. To the definition of the neutron flux in the depth of the body

the solution of which, under the boundary condition $\Phi(0) = \Phi_0$, has the form

$$\Phi(x) = \Phi_0 e^{-N\sigma x} \quad (1.54)$$

Thus, the directed neutron flux in an extended body changes according to an exponential law and relation (1.54) should be considered when determining cross-sections experimentally. When $N\sigma x = N_1\sigma \ll 1$, expression (1.54) reduces to (1.48), since in this assumption $(\Phi_0 - \Phi)/\Phi_0 \approx N\sigma x$. The following value is the distribution constant of (1.54):

$$\Sigma = N\sigma \quad (1.55)$$

which, when describing the penetration of photon fluxes through the substance, is called the *linear coefficient of attenuation*, and in neutron physics the *macroscopic cross-section*. The latter name is associated with the fact that $N\sigma$ is the would-be cross-section of all the nuclei contained in a unit volume of substance. Such a treatment is conventional because, as follows from (1.55), the macroscopic cross-section is not the cross-section proper and is measured in 1/m.

The macroscopic cross-section is convenient to use in denoting or calculating the process rates in a unit volume of substance. Indeed, from expression (1.52) it is possible at once to write the number of collisions in unit volume and in unit time in the vicinity of the point x with flux $\Phi(x)$ as dv/dx since the derivative in question for a body with the cross-sectional area equal to unity is, in fact, related to unit volume V , that is, it is dv/dV :

$$\frac{dv}{dV} = \Phi\Sigma \quad (1.56)$$

Thus, the rate of collisions in a unit volume is the product of the flux (1.50) by the macroscopic cross-section (1.55). If the macroscopic cross-section is the product of the concentration of nuclei (1.2) by the partial microscopic cross-section σ_i , for example, the cross-section of scattering or capture, then it will also be partial, and the product (1.56) in this case is the rate of specific processes in a unit volume of the substance, the number of scattering events or the number of neutron adsorption events, for example. Expression (1.56) enables one to calculate the process rates with the known Φ and to compose the balance of neutrons when deriving equations for finding Φ . Although (1.56) is obtained from an example of a directed flux, it holds for the case of chaotic neutron diffusion in large volumes of substance. The actual flux direction has nothing to do with calculation of (1.56) which has its meaning at a *point* of the volume with flux Φ . The flux itself (1.50) is understood to be a value merely proportional to the neutron concen-

tration n at this point, with a proportionality coefficient equal to the mean neutron velocity v .

If the number of interactions ν_0 , when the microparticle penetrates through a body of volume V_0 , is small so that the flux is practically not attenuated in the body volume, then

$$\nu_0 = \frac{dv}{dV} V_0 = \Phi \Sigma V_0 = \Phi \sigma N_0 \quad (1.57)$$

where $N_0 = NV_0$ is the total number of target particles in the volume V_0 .

4. Mean path length. A simple relationship relates the macroscopic cross-section to the mean neutron path length before excitation of the process. If Σ is the total macroscopic cross-section, the mean neutron path length before the first collision with a nucleus can be derived from the distribution of the neutron flux (1.54) in an infinitely thick layer of substance. By definition the mean path is the sum of paths before the collision of each of the neutrons being investigated divided by the number of neutrons. The distance x is covered by those neutrons which have experienced collision at x and which have fallen out of the initial flux, that is, $-d\Phi(x)$ neutrons. The integral of all the $x[-d\Phi(x)]$ referred to the initial flux Φ_0 , gives l , the mean path before the collision:

$$l = \frac{1}{\Phi_0} \int_0^{\infty} x \Phi N \sigma dx = \Sigma \int_0^{\infty} x e^{-\Sigma x} dx = \frac{1}{\Sigma} \quad (1.58)$$

Thus, the mean path length before the first collision is the value inverse to the total macroscopic cross-section. If Σ_i is the macroscopic cross-section of exciting the i th process, the mean path length traversed by the neutron before the excitation of this process also equals $l_i = 1/\Sigma_i$. Scattering and absorption are the two most typical processes in the interaction of neutrons with atomic nuclei. If Σ_s and Σ_a are the macroscopic cross-sections of scattering and absorption, respectively, then the path lengths before scattering and absorption are

$$l_s = \frac{1}{\Sigma_s} \quad (1.59)$$

$$l_a = \frac{1}{\Sigma_a} \quad (1.60)$$

As the partial cross-sections are summed, and in this case $\Sigma_t = \Sigma_s + \Sigma_a$ then when calculating the total path length from partial path lengths their inverse values are to be summed, that is,

$$\frac{1}{l_t} = \frac{1}{l_s} + \frac{1}{l_a} \quad (1.61)$$

During the discussion of the cross-sections neutrons were implicitly described as bombarding particles and atomic nuclei as target-particles. However, all the above discussed can equally be applied to other microparticles.

1.7. Elementary Particles

1. Detection of microparticles. Observation of moving microparticles appears possible thanks to their electric charges. Most detection methods are based on the interaction of moving charged particles with atomic electrons followed by excitation and ionization of the atoms of the surrounding medium. The probability of such interaction is quite great. The appearance in the substance of a great number of ion pairs along the path of the moving particles may be used for detecting the transmitted particle both directly by collecting negative electrons and positive atomic ions and with the help of other phenomena occurring in the substance in the presence of ions or in the process of their recombination, that is, during the inverse transformation of ion pairs into neutral atoms. The effects observed in this case allow one not only to determine the fact of the charged particle transmission through the sensitive volume of the detector but also in a number of cases to measure many of the particle's characteristics, such as the direction of its motion, its energy, momentum, electric charge and mass. Neutral particles may only be detected under the condition of the process excitation accompanied by the appearance of secondary charged particles. Thus, neutrons must either cause a nuclear reaction accompanied by emitting a charged particle or form a recoil nucleus during nuclear scattering. Photons are detected by fast electrons appearing at their absorption or their scattering. As the probability of these processes is much less than the probability of ionizing the atom by the moving charged particle, the neutral particle detection is accordingly much more complicated than the charged particle detection.

Of particular importance among microparticle detectors are the tracking devices. These allow one to directly observe the traces of some charged particles which they leave along the path of their motion (Figs 1.5, 1.6, 1.8 to 1.10). The study of the trace, or track, can give much information about the particle so that a single successfully detected track may suffice to reliably identify the particle. If the track is obtained in a constant magnetic field in which charged particles of opposite signs move along circular trajectories curved in opposite sides (Fig. 1.6), then one can simultaneously determine the particle momentum (2.10) and its charge sign, respectively. Tracking devices can also detect neutral particles leaving no traces, if, within the limits of the sensitive volume of

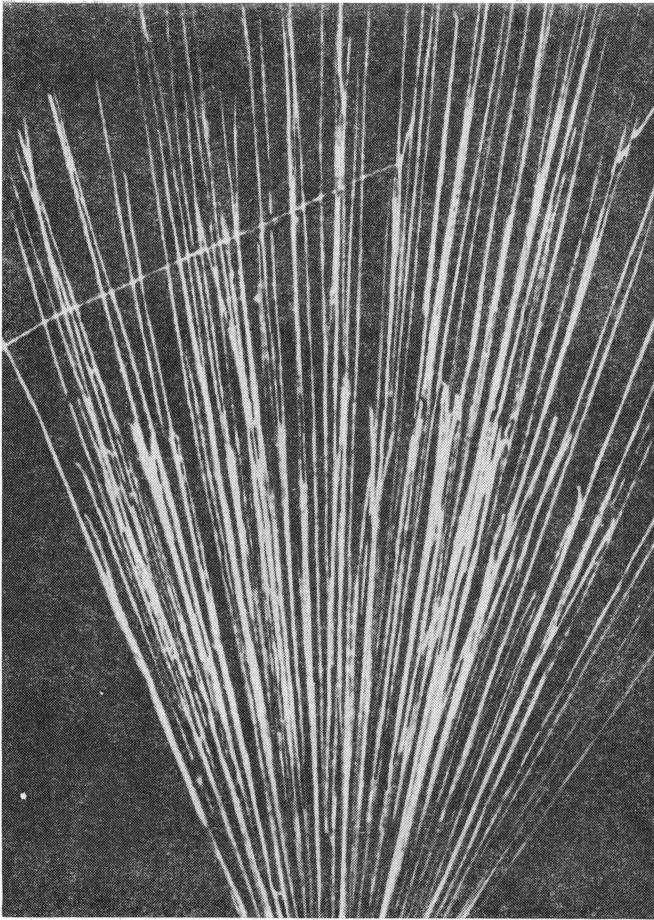


Fig. 1.5. α -Particle traces in a cloud chamber. Two groups of traces are formed by particles of different radioactive substances. The picture shows the (α, p) -reaction in the case of the nitrogen nucleus. The long and the short traces emerging from the end of the trace of an α -particle are those of the neutron and of the recoil nucleus

the device, they undergo transformation or interact with environmental particles. These interactions are accompanied by the appearance of charged particles. At the same time, some physical parameters of neutral particles can be measured. Many references report on detection methods, for instance [8].

2. Variety of particles. The structural particles of the atom namely, electron, proton and neutron, enter the class of elementary

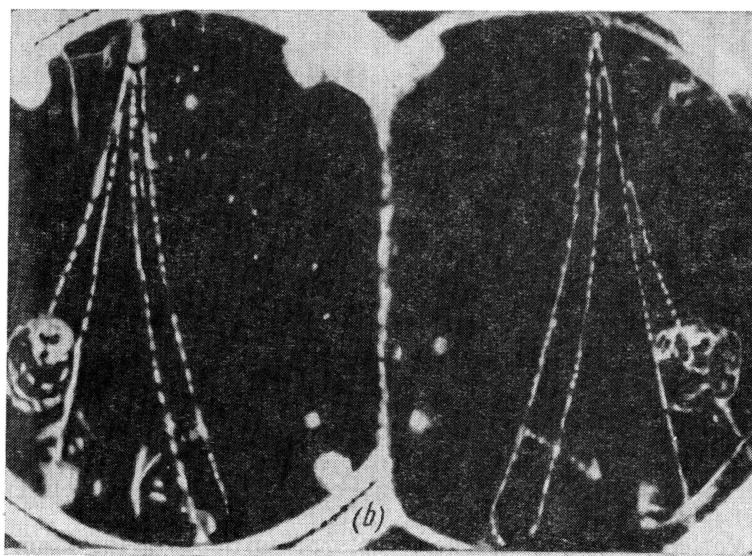
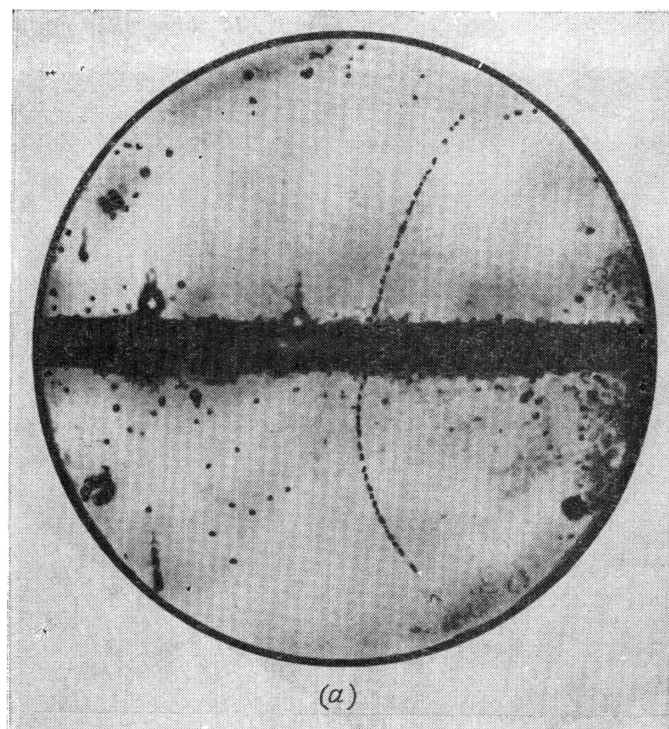


Fig. 1.6. Pictures made inside a cloud chamber in magnetic field
 (a) positron detection; the direction of motion is determined by the increase of the track curvature upon slowing-down in substance layer; the charge sign is in the directions of motion and deflection in the magnetic field; (b) electron-positron pair production

particles. Elementary particles are those which have no structure. This means that they do not consist of other particles, they cannot be split into parts and, hence, they should be considered as point particles. The elementary particle always behaves as a single whole. Such a particle preserves its individuality in a microsystem, such as nucleus, atom, molecule. It means that any compound particle, for instance the atom, should be thought of as consisting of other particles, elementary, in the long run.

Besides the three structural particles of the atom, at the present time about two hundred more elementary particles are known. Most of them are unstable and soon after formation from other particle collisions they spontaneously decay, transforming into stable elementary particles. The life-time of such particles is very short and is 10^{-6} to 10^{-22} s. Unstable particles include the neutron which undergoes beta-decay, (1.65), and has the mean life-time 1013 s. Neutrons become stable in bound states with protons in the atomic nuclei because their bond energy in nuclei (2.4) greatly exceeds the energy released in the beta-decay of the free neutron. Stable particles are relatively scarce. Their total is nine: one photon, four neutrinos (two neutrinos and two antineutrinos), two electrons (an electron and an antielectron, or positron) and two nucleons (a proton and an antiproton) (Table 1.2).

3. Pair nature of elementary particles. The existence of particle-antiparticle pairs is the main feature of the microcosm. It is characteristic of both stable and unstable elementary particles. Except the photon and π^0 -meson, any elementary particle exists in two modifications one of which is an antiparticle relative to the other. The physical parameters of a particle-antiparticle pair are absolutely equal, except for some specific characteristics, charges, which show the antagonism of the constituents of a pair and which, therefore, have opposite algebraic signs. The electric charges of a pair of charged antiparticles are also opposite in sign. The character of the interaction of the pair particles with other particles is, generally speaking, different while interaction with each other is accompanied by *annihilation*, i.e., the disappearance of two antiparticles and the appearance of new particles with lower rest mass. In contrast with the annihilation process, such particles may be produced in pairs in the interaction of other microbodies. One of the pair is called a particle and the other an antiparticle.

4. Interconvertibility. Another fundamental feature of elementary particles is their ability to interconvert in collisions, spontaneous decays and annihilation. Such conversions are not arbitrary. First, they always proceed in compliance with the energy conservation laws of classical physics, that is, the total energy, total momentum and total angular momentum of all the participating particles are conserved. Second, the conversions of elementary par-

Elementary

| Groups and names of particles | | Particle and antiparticle symbol | Electric charges, e units | Mass, electron mass units | Rest energy, MeV |
|-------------------------------|--|----------------------------------|-----------------------------|---------------------------|------------------|
| Photon | | γ | 0 | 0 | 0 |
| Leptons | <i>Neutrinos</i> | | | | |
| | Electron neutrino and anti-neutrino | $\nu_e, \bar{\nu}_e$ | 0 0 | 0 | 0 |
| | Mu-meson neutrino and anti-neutrino | $\nu_\mu, \bar{\nu}_\mu$ | 0 0 | 0 | 0 |
| | <i>Electrons</i> | | | | |
| | Electron and positron | $e^- e^+$ | -1 +1 | 1 | 0.511 |
| | <i>Mu-mesons</i> | | | | |
| | Mu-minus- and mu-plus-mesons | $\mu^- \mu^+$ | -1 +1 | 206.8 | 105.7 |
| Mesons | <i>Pi-mesons</i> | | | | |
| | <i>Pi</i> -plus- and <i>pi</i> -minus-mesons | $\pi^+ \pi^-$ | +1 -1 | 273.2 | 139.6 |
| | <i>Pi</i> -zero meson | π^0 | 0 | 264.2 | 135 |
| | <i>K-mesons</i> | | | | |
| | K-plus- and K-minus-mesons | $K^+ K^-$ | +1 -1 | 966.3 | 494 |
| | K-zero- and anti-K-zero-mesons | $K^0 \bar{K}^0$ | 0 0 | 974.5 | 498 |
| <i>Meson resonances</i> | | | | | |
| Barions | <i>Nucleons</i> | | | | |
| | Proton and antiproton | $p \bar{p}$ | +1 -1 | 1836.1 | 938.26 |
| | Neutron and antineutron | $n \bar{n}$ | 0 0 | 1838.6 | 939.55 |
| | <i>Hyperons</i> | | | | |
| | Λ^0 - and anti- Λ^0 -hyperon | $\Lambda^0 \bar{\Lambda}^0$ | 0 0 | 2182.8 | 1115.4 |
| | Σ^+ - and anti- Σ^+ -hyperon | $\Sigma^+ \bar{\Sigma}^-$ | +1 -1 | 2327.6 | 1189.4 |
| | Σ^- - and anti- Σ^- -hyperon | $\Sigma^- \bar{\Sigma}^+$ | -1 +1 | 2342.6 | 1197.1 |
| | Σ^0 - and anti- Σ^0 -hyperon | $\Sigma^0 \bar{\Sigma}^0$ | 0 0 | 2333.4 | 1192.3 |
| | Ξ^- - and anti- Ξ^- -hyperon | $\Xi^- \bar{\Xi}^+$ | -1 +1 | 2584.7 | 1320.8 |
| | Ξ^0 - and anti- Ξ^0 -hyperon | $\Xi^0 \bar{\Xi}^0$ | 0 0 | 2572 | 1314.3 |
| | Ω^- - and anti- Ω^- -hyperon | $\Omega^- \bar{\Omega}^+$ | -1 +1 | 3278 | 1675 |
| | <i>Barion resonances</i> | | | | |

Table 1.2

Particles

| Mean life-time, s | Spin, \hbar units | Lepton charge | Barion charge | Stran-geness | Isoto-pic spin | Decay products |
|-----------------------|---------------------|---------------|---------------|--------------|----------------|---|
| Stable | 1 | 0 | 0 | | | |
| Stable | $1/2$ | +1 -1 | 0 0 | | | Electrons and neutrinos |
| Stable | $1/2$ | +1 -1 | 0 0 | | | |
| Stable | $1/2$ | +1 -1 | 0 0 | | | |
| $2.2 \cdot 10^{-6}$ | $1/2$ | +1 -1 | 0 0 | | | |
| $2.55 \cdot 10^{-8}$ | 0 | 0 0 | 0 0 | 0 0 | 1 | Mu-mesons and neutrinos Photons |
| $1.8 \cdot 10^{-16}$ | 0 | 0 | 0 | 0 | 1 | |
| $1.23 \cdot 10^{-8}$ | 0 | 0 0 | 0 0 | +1 -1 | $1/2$ | Pi-, Mu-mesons, neutri- nos, electrons |
| $0.92 \cdot 10^{-10}$ | 0 | 0 0 | 0 0 | +1 -1 | $1/2$ | |
| $5.62 \cdot 10^{-8}$ | 0 | 0 0 | 0 0 | +1 -1 | $1/2$ | |
| 10^{-22} | Integer | | | | | Mesons |
| Stable | $1/2$ | 0 0 | +1 -1 | 0 0 | $1/2$ | Protons, electrons, neut- rinos |
| 1013 | $1/2$ | 0 0 | +1 -1 | 0 0 | $1/2$ | |
| $2.62 \cdot 10^{-10}$ | $1/2$ | 0 0 | +1 -1 | -1 +1 | 0 | Nucleons, mesons, pho- tons |
| $0.79 \cdot 10^{-10}$ | $1/2$ | 0 0 | +1 -1 | -1 +1 | 1 | |
| $1.58 \cdot 10^{-10}$ | $1/2$ | 0 0 | +1 -1 | -1 +1 | 1 | |
| $< 10^{-14}$ | $1/2$ | 0 0 | +1 -1 | -1 +1 | 1 | |
| $1.74 \cdot 10^{-10}$ | $1/2$ | 0 0 | +1 -1 | -2 +2 | $1/2$ | |
| $3.06 \cdot 10^{-10}$ | $1/2$ | 0 0 | +1 -1 | -2 +2 | $1/2$ | |
| $0.7 \cdot 10^{-10}$ | $3/2$ | 0 0 | +1 -1 | -3 +3 | 0 | Barions and mesons |
| 10^{-22} | Half-integer | | | | | |

ticles proceed following specific rules discovered while observing particle conversions in experiment. To systematize these conversion rules, elementary particles are attributed some dimensionless values having discrete quantitative expression, for example, *special charges* (a *lepton* one and a *barion* one), *parity*, *strangeness*. After that the particle conversion rules are interpreted as laws of conservation of these values.

Such a method allows the separation of groups of particles with similar properties as possessing the same numerical values of the quantities involved, that is, it allows the classification of elementary particles. At the same time, the main physical characteristic determining the particle classification is the mass, the value of which served as a basis of the classification of elements by Mendeleev. It is true, mass is now used in a new capacity as an equivalent to the rest energy of the particle. Elementary particles arranged in the increasing order of their rest energy or mass are immediately distributed among the above groups characterized by the same values of specific quantities. The less the difference in the mass of the particles, the more similar, as a rule, are their properties. Particles with very close mass values and, correspondingly, similar properties are united into families of one, two or three particles, characterized by the same *isotopic spin* value equal to 0, $\frac{1}{2}$ or 1, respectively. Analogous to an ordinary spin, the number of isotopic spin "projections", gives the number of particles in a family, from which originates the name isotopic spin. The elementary particles listed in Table 1.2 are arranged according to the current classification [5].

It should be pointed out that the interconversion of elementary particles in corresponding reactions does not testify in favour of their compound structure. Secondary particles originate anew in the reaction or decay, while original particles exciting the reaction may disappear as a result of the reaction, and they always disappear in decay. Furthermore, any elementary particle may form simply as a consequence of the lowering of the kinetic energy of colliding particles, following, naturally, the conservation laws. If the particle spin is equal to $\hbar/2$, such a particle may be born only in pair with an antiparticle at the cost of the kinetic energy of other particles, since the mechanical moments of the relative particle motion are always expressed in integers \hbar . The half-integer values of the mechanical moments are inherent only in some elementary particles as their own mechanical moments or spins.

5. Classes of interaction. The rates of the transformation processes in time or, which is the same, the probabilities of realizing the corresponding processes in separate acts of collision display a sharp difference which depends on which particles take part in the phenomena. This fact testifies to the existence in nature of forces,

differing in the value, which determine the development of the phenomena and to a different attitude of elementary particles to these forces. Besides the *gravitational* (Newton) and *electric* (Coulomb) forces, at present are known strong, or *nuclear*, and *weak* forces. Gravitational and electromagnetic forces are longrange and displayed at any distance from the source of forces. They have long been known in physics. Nuclear and weak forces are shortrange. Their sphere of action is limited to distances of the order of $1.5 \cdot 10^{-15}$ m and less. The existence of these forces became known only from experiments on the microparticle interaction at short distances. The first of these were Rutherford's experiments on α -particle scattering by atomic nuclei.

Gravitational forces are so weak that in the microparticle interaction their existence hardly influences the phenomena taking place. In the particle interaction only nuclear, electromagnetic and weak forces are of significance. The greatest of these are the nuclear forces and the smallest are the weak ones. The force magnitude is estimated by the value of the interaction constant for a given class of forces. This value is determined experimentally. The interaction constant may be compared to the time characteristic which determines the mean time interval during which the phenomena are developing under the influence of the forces in question. The characteristic time of the processes proceeding under the influence of the nuclear forces is about 10^{-23} s. Of the same order is the time Δt of the particle flight over the region limited by the radius of action of the nuclear forces, $R = 1.5 \cdot 10^{-15}$ m, with a velocity close to the velocity of light: $\Delta t = 2R/c = (2 \times 1.5 \cdot 10^{-15} \text{ m}) / (3 \cdot 10^8 \text{ m/s}) = 10^{-23}$ s. That means that the nuclear forces are so great that each approachment of even the most fast moving particles at a distance of the radius of action of nuclear forces is accompanied by the realization of a possible phenomenon. This realization is not observed under the influence of shortrange weak forces. The electromagnetic forces are about 100 times weaker than the nuclear ones, and the characteristic time of the electromagnetic interactions is estimated as 10^{-21} s. The weak forces are negligibly small compared to the nuclear and electromagnetic ones. Under their influence, the phenomena develop relatively slowly for about 10^{-10} s. If the characteristic time constant of the gravitational forces is considered, it will be of the order of 10^{16} s, or about 10^9 years. It proves that the quantity of the gravitational forces is extremely small compared to other forces in Nature. The above process realization times correspond to the most favourable conditions but can be much greater if some factors hinder the development of the possible phenomena.

The elementary particles differ in their participation in interactions under the influence of some forces or other. Heavy particles —

mesons, barions — participate in all the interactions occurring under the influence of the nuclear, electromagnetic and weak forces. On the contrary, light particles, or leptons, do not take part in the nuclear interactions. Those leptons which have an electric charge participate in the electromagnetic and weak interactions while neutrinos take part only in the processes caused by the weak forces. Thus, absorption or formation of neutrinos in a process always shows that the phenomenon takes place due to the action of the weak forces. Photons, or electromagnetic quanta, take part only in electromagnetic interactions and the absorption or emission of a γ -quantum unambiguously indicates the electromagnetic nature of the process. The photons play the part of electromagnetic interaction carriers.

The electron does not take part in strong interactions. The electron-proton interaction may serve as an example. Although the electron-proton collision may, in principle, bring about the formation of a neutron which will by all means produce a neutrino, such a process is, however, hardly possible, as it takes place under the influence of the weak forces. Therefore, the electric attraction of these oppositely charged particles does not result in the mutual annihilation of electric charges in each interaction act in which other particles are formed. In stable atoms, such a process is impossible because it is not profitable from the point of view of energy. With the free energy gain, however, it takes place in β^+ -radioactive atoms (Sec. 3.3-2), and is called the *K*-capture process. Atomic electrons for an arbitrarily long time are at a short distance, of the order of the de Broglie wavelength, from the atomic nucleus. Therefore, it is in the β^+ -radioactive atoms and not in the collision of free protons and electrons that such a process is most probable.

6. Spin and magnetic moment. The spin of a particle is the intrinsic mechanical moment. The mechanical moment of a body rotating about its axis is the classical analogue of spin. Spin was first discovered in the electron when explaining the thin structure of the atomic optical spectra. The following arguments are the foundation of the electron spin hypothesis. The electron having a spin and an electric charge must have a magnetic moment the vector of which is directed along the spin vector axis. The interaction of the intrinsic magnetic moment with the magnetic field formed by the orbital motion of the electron about the nucleus results in some increase or decrease of the electron's energy in the electric field of the nucleus, depending on the orientation of the intrinsic magnetic moment of the electron in, or opposite to, the direction of the magnetic induction vector of the orbital magnetic field, that is, it results in two close frequencies instead of one when the atom emits light, as observed in experiment. Direct measurement of the

magnetic moment and the spin of the electron showed that the intrinsic magnetic moment of the electron is equal to the electron magnetic moment unit of the Bohr atom, the Bohr magneton (1.10), and the spin is equal to half the smallest mechanical moment of the orbital electron, or $\hbar/2$.

Subsequently, it was found that many elementary particles, among them the proton and neutron, have the spin $1/2$ in the \hbar units. The magnetic moment of the proton, contrary to the expected analogy with the electron, differs from the value of the nuclear magneton

$$\mu_N = \frac{e\hbar}{2m_p} = 5.051 \cdot 10^{-27} \text{ J/T} \quad (1.62)$$

which is the unit magnetic moment of the particle with the proton mass m_p (1.10), and is equal to

$$\mu_p = 2.7928\mu_N \quad (1.63)$$

In measuring the magnetic moments of the atomic nuclei, it was found that the neutron with no electric charge must also have a magnetic moment. Special experiments proved the existence of the magnetic moment in the neutron which appeared to be equal to

$$\mu_n = -1.913148\mu_N \quad (1.64)$$

The minus sign corresponds to the direction of the magnetic moment vector opposite to the spin vector, which is true for the electron as well, while the plus sign, along the direction of the spin. For a classical charged body, the minus sign means the magnetic moment formed by the rotation of the negative electric charge, and the plus sign, by the rotation of the positive charge. The numerical values of the nucleon magnetic moments as well as the presence of the neutron magnetic moment are not simply explained. They may be due to the compound structure of these particles (but then these particles are not elementary). The antinucleons have the same magnetic moment values as the nucleons but of opposite signs: an antiproton is negative, and an antineutron is positive.

7. Statistics. Both elementary and compound particles have either half-integral ($1/2, 3/2, 5/2, \dots$) or integral ($1, 2, 3, \dots$) spin, or no spin at all, i.e., their spin is zero, which corresponds to the integral spin value. Particles with half-integral and integral spins differ in their statistical properties, in the behaviour of particle ensembles. Particles having half-integral spin obey Pauli's principle which does not allow two identical particles to be found in identical states described by the same set of quantum dynamic values.

This property of the particles having a half-integral spin pre-determines a strict order of the distributions of many electrons in

atomic electron shells. The presence of nucleon shells in atomic nuclei is due to this property. Only elementary particles with the $1/2$ spin can form a substance. Such particles are described by the Fermi — Dirac statistics and are called *fermions*. Particles with integral spins do not obey Pauli's principle. On the contrary, they have a tendency to accumulate in a single quantum state with the lowest energy. These particles are described by the Bose — Einstein statistics and are called *bosons*. It is possible that the elementary particles, bosons, are interaction carriers.

1.8. Positron

1. **Discovery.** The positron was discovered in 1932 by American physicist Anderson when observing cosmic radiation with the help of a cloud chamber placed into a magnetic field (see Fig. 1.6a). Anderson photographed the traces of particles which strongly resembled electron traces but were curved due to the magnetic field. As the curvature was opposite to the electron tracks it showed that the discovered particles had a positive electric charge. Soon after the discovery, also with the help of the cloud chamber, photographs were obtained which shed light upon the genesis of the positrons: under the influence of γ -quanta of secondary cosmic radiation the positrons produced in pairs with ordinary electrons (see Fig. 1.6b). Such properties of the newly discovered particle were in striking agreement with the already available Dirac relativistic electron theory. The Dirac theory did not only describe the electron with a negative electric charge but also an analogous particle with a positive charge. The absence of such a particle in nature was considered as pointing to ambiguous solutions of Dirac's equations. But the discovery of the positron was the triumph of the theory.

In compliance with the Dirac theory, the electron and positron may produce in pairs, and this process requires energy consumption equal to the rest energy of these particles, 1.02 MeV (see Table 1.2). As natural radioactive substances were known emitting γ -ray quanta with energy higher than 1 MeV, it seemed possible to produce positrons in a laboratory, and that was done. Experimental comparison of the properties of the positrons and electrons showed that all the physical characteristics of these particles except the electric charge sign, coincide. In 1934, Irene and Frédéric Joliot-Curie, in France, discovered one more source of positrons — β^+ -radioactivity.

2. **Annihilation.** From the Dirac theory it follows that the electron and positron, in colliding, must annihilate, releasing energy equal to the total energy of the colliding particles. The process turned out to take place mainly after the positron slowed down in the substance, when the total energy of two particles equals their

rest energy 1.02 MeV. Pairs of γ -ray quanta, each with energy 0.5 MeV, were detected in experiments. They flew in strictly opposite directions from the target irradiated by positrons. The necessary production, during the electron and positron annihilation, of not one but two γ -ray quanta arises from the law of momentum conservation. The resulting momentum of the stopped positron and electron before the conversion process, is equal to zero but it cannot be zero during the production of only one γ -ray quantum due to the annihilation.

It has been known since 1951 that in some amorphous bodies, liquids, and gases, the positron after being slowed down does not readily annihilate in a great number of cases, but forms for a short time a system bound with the electron. This system is called positronium. The positronium, as regards its chemical properties, is analogous to the hydrogen atom because it is a system consisting of single positive and negative electric charges, and can enter into chemical reactions. As the electron and positron are different particles, they may be in their bound state with the lowest energy not only with antiparallel spins but with parallel spins as well. In the first case, the total spin of the positronium is $s = 0$, which corresponds to the parapositronium, and in the second, $s = 1$, which corresponds to the orthopositronium. It is interesting to note that the annihilation of the electron-positron pair when it forms part of the orthopositronium cannot be followed by the production of two γ -ray quanta. Two γ -ray quanta carry away, relative to each other, their mechanical moments equal to 1, (see Table 1.2), and can make up a total moment equal to zero but not to unity. Therefore, in this case, the annihilation is accompanied by the emission of three γ -ray quanta with the total energy of 1.02 MeV. The production of the positronium is three times more probable than of the parapositronium because the relation of the statistical weights $(2s + 1)$ (Sec. 1.4-10) of both states of the positronium is 3:1. Even in bodies with a high percentage (up to 50%) of annihilation of a pair in a bound state, that is, after the production of the positronium, two γ -ray quanta predominantly appear, and very rarely three. The point is that the life time of the positronium is about 10^{-10} s and that of the orthopositronium is about 10^{-7} s. The long-lived positronium, continuously interacting with the environmental atoms, has no time to annihilate with the emission of three γ -ray quanta before the positron, being its part, annihilates with an extraneous electron in a state with antiparallel spins and with the emission of two γ -ray quanta.

3. Consequences of positron discovery. The positron was the first antiparticle discovered. The existence of the electron antiparticle and the conformity of the total properties of two antiparticles with the Dirac theory concepts, which could be extended to other par-

ticles, pointed to the probability of the pair nature of all the elementary particles and oriented subsequent physical studies. Such an orientation appeared quite fruitful, and at present the pair nature of elementary particles is a strictly established law of nature confirmed by a great number of experimental facts.

1.9. Neutrino

1. Neutrino hypothesis. The neutrino has no electric charge. Its magnetic moment and rest mass probably equal zero. As it interacts very weakly with the substance this particle is extremely difficult to observe. Therefore, originally, the existence of the neutrino was postulated only on the basis of indirect data. The neutrino produces in a number of decay processes under the action of weak forces. Among such processes the β -decay of atomic nuclei is the best known. This decay is a spontaneous transformation of a neutron into a proton or of a proton into a neutron in the interior of the atomic nucleus:



Such transformations are possible in those nuclei the change of the proton-neutron composition of which due to decays, (1.65) or (1.66), is profitable from the standpoint of energy, that is, results in the formation of nuclei of lower mass (Sec. 3.3-3). As the sum of the masses of the proton and electron is somewhat lower than the mass of the neutron, process (1.65), as opposed to (1.66), proceeds spontaneously also in the case of a free neutron not bound in the nucleus.

The assumption of the neutrino emission in processes (1.65) and (1.66) is based on the following experimental facts. If only two particles arose as a result of the β -decay, then, as follows from the law of energy and momentum conservation, in each case of the decay of nuclei of the same kind the electron would have the same kinetic energy. The electron energy, however, is not constant and may have values from zero to some maximum value equal to the decrease of the rest energy of the decaying particles. The mean kinetic energy of the electrons is approximately one third of the above mentioned maximum energy so that, in the absence of the third particle, it is not clear where more than half of the energy released in the β -decay disappears. In addition, in the decay of a particle having a half-integral spin, two particles with a half-integral spin cannot be formed (see Table 1.2). To bring the observed facts in agreement with the laws of energy conservation, Pauli, the Swiss physicist, made an assumption (1931), that emitted in the β -decay is one more particle having a half-integral spin, no

electric charge and it does not give its energy to detectors. Fermi named this particle a neutrino.

2. Experimental detection. Pauli's assumption was checked many times in experiments. The early experiments on the neutrino detection were based on the law of momentum conservation. If during the β -decay, no neutrino is formed, the electron and the nucleus emitting it or, to be more precise, the atom as a whole, must have equal and oppositely directed momenta and, hence, at least proportional quantities of energy. If, in addition to the electron, the nucleus emits also a neutrino, the electron and recoil-atom momenta may have arbitrary orientation in space and different absolute values, all this leading to nonproportionality in the energies of the atom and electron. Comparison of the energy spectra of recoil atoms with the energy spectra of electrons, as well as observation of divergence angles of an electron and an atom much less than 180° bear witness to the existence of the neutrino emission. The most convincing data were obtained when registering recoil atoms in the K -capture process, the variety of the β^+ -decay (1.66). In this process, the nucleus absorbs the atomic electron instead of emitting a positron



In this case, with no neutrinos the atom does not obtain any recoil at all and, if the neutrinos are emitted, the recoil energy must be the same in each decay because two and not three microparticles, a new atom and a neutrino, are formed during the K -capture. An experiment with Be^7 decaying according to the scheme



proved the neutrino hypothesis and the recoil atom energy measurement showed good numerical agreement with the calculation of the distribution of the energy emitted during the K -capture, between the atom being formed and the neutrino. The main difficulty of such experiments is associated with the detection of the recoil atoms because nearly all β -decay energy is carried away by the light particles (1.34) and the fraction of energy for the very heavy atom is not greater than a hundred electron-volts. Nevertheless, the formation of the neutron in the β -decay has been reliably detected.

Pauli's assumption of the neutrino emission in β -decay was, however, sponsored by the attempt to keep unshakable the laws of energy and angular momentum conservation. Experiments also based on the conservation law did not, therefore, irreproachably prove the existence of the neutrino. In addition, the main property of elementary particles is that they are interconvertible, and if in β -decay a neutrino, an elementary particle, is emitted, it must cause processes in which the neutrino itself vanishes and produces new

particles. Thus, only observation of these processes could finally prove the existence of the neutrino in nature.

The experimental difficulties arising in detecting immediate interaction of the neutrino with substance are obvious. Experimental data on the life-times of beta-active nuclei show that the processes accompanied by the neutrino emission proceed extremely slowly. It follows that the inverse process of the neutrino absorption by the nucleon must also proceed very slowly, and keeping in mind that the free neutrino moves at the velocity of light and, having approached the nucleon, interacts not longer than 10^{-23} s, one expects negligible probability of neutrino absorption by the nucleons. The estimate of the cross-section of the interaction of antineutrinos emitted in the beta-decay with protons gave a value of the order of 10^{-48} m², immeasurably lower than the values of the effective nuclear cross-sections ($\approx 10^{-28}$ m²). As the proton concentration, (1.3), in ordinary condensed media is about 10^{30} 1/m³, the path in the substance, (1.60), before absorption of the antineutrino by the proton turns to be equal to 10^{18} m, or 100 light years. However, no matter how small the cross-section is, the required number of reaction events can be registered in a reasonable time if one has an intense neutrino source and a large sensitive volume of the reaction detector, which can enclose as many target particles as possible. The advent of nuclear fission reactors and liquid scintillation counters made this unique experiment possible.

3. Observation of reaction caused by antineutrino.

The experiment was made by Reines and Cowan, USA in 1953 and was subsequently repeated for a number of times. Observations were made of a process inverse to the β -decay of free neutron (1.65). As in neutron decay, in addition to an electron, a proton and an antineutrino are formed, the interaction of the antineutrino with the free proton must lead to an inverse process, the formation of a free neutron and a positron:



Particles arising in the antineutrino absorption were detected with a scintillation counter of a special design (Fig. 1.7). Three tanks with a liquid scintillator were placed behind a lead shield. The two upper tanks served to reduce as much as possible the background while taking measurements in the lower tank. In particular, the second tank and the third tank were connected into an anticoincidence circuit; this enabled elimination of those events in the third tank that were accompanied by light flashes in the second tank. Such simultaneous events occurred when a fast charged particle, for example a μ -meson, was passing through the two tanks. The antineutrino has no electric charge, does not ionize the substance, and, therefore, it excited light flashes only in the tank

in which it was absorbed and transformed into other particle. The sensitive volume of the antineutrino detector, that is, the volume of the third tank, was 1.4 m^3 . Protons necessary for the interaction with the antineutrinos entered into the composition of the hydrogenous substance of the scintillator. To facilitate registering of arising neutrons, cadmium salt was introduced into the scintillator of the third tank. Cadmium effectively absorbs neutrons slowed down to the energy of thermal motion, emitting γ -rays. The antineutrino absorption was detected by two sequential light flashes in the third tank. The positron generated in process (1.69) is practically instantly slowed down and annihilates with the emission of two γ -ray quanta. Positron moderation and the absorption of annihilation γ -rays in the scintillator substance are accompanied by ionization and, hence, by a light flash proportional to the energy of the formed positron and γ -rays. Neutron moderation due to elastic collisions with the environmental protons takes from 1 to $25 \text{ } \mu\text{s}$, after which the neutron is absorbed by the cadmium. The γ -rays thus emitted are absorbed by the scintillator, thus producing the second flash of light proportional to the γ -ray total energy 9.1 MeV . The volume of the third tank was viewed by 110 photomultipliers whose electric pulses, after an appropriate amplification, were registered by a recorder. If the successive electric pulses did not meet the requirement for the pulse height proportional to the initial ionization or for the time shift between the first and the second pulses, such pulse-pair-events were not taken into account as being of random occurrence.

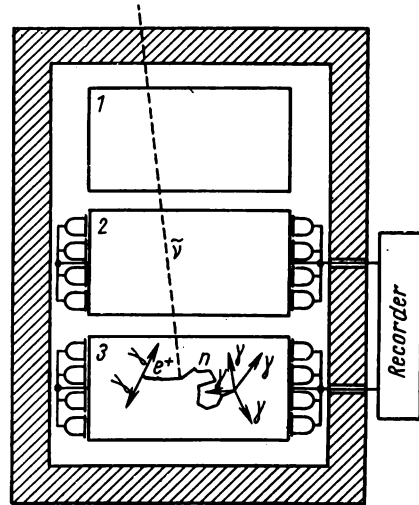


Fig. 1.7. Schematic diagram of Reines and Cowan experiment

The scintillation counter was positioned under the base of the nuclear reactor. Uranium fission products are β -radioactive and the nuclear reactor is an effective source of antineutrinos. Proceeding from the thermal power value of the reactor and the mean number of β -decays per one act of fission, Reines and Cowan estimated the antineutrino flux where the counter was located to be equal to $1.3 \cdot 10^{17} \text{ 1/(m}^2 \cdot \text{s)}$. Measurements were carried out for hundreds of hours, with the reactor operating and shut down. This allowed the

elimination of external effects, for example, reaction due to cosmic antineutrinos. The reactor efficiency equalled 36 ± 4 events per hour, which corresponds to the absorption cross-section of the antineutrino by the proton:

$$\sigma = (11 \pm 2.6) \cdot 10^{-48} \text{ m}^2 \quad (1.70)$$

The efficiency in this experiment was $1/5$ of the background. The authors considered that nearly half of the detected background events were due to cosmic radiation. If all these events were caused by cosmic antineutrinos with an energy close to that of the antineutrinos emitted by the nuclear reactor, then the cosmic antineutrino flux to the Earth proves comparable with the flux from the reactor in Reines' and Cowan's experiments, that is, of the order of $10^{17} \text{ 1/(m}^2 \cdot \text{s)}$ or less if one keeps in mind that with an increase of the antineutrino energy the probability of its interaction with the proton increases.

4. Proof of neutrino and antineutrino nonidentity. The experimental identification of the process cross-section, (1.69), allowed an experiment to be made to prove that the neutrino and antineutrino emitted in β^+ - and β^- -decays are really different particles. The proof boiled down to that the reaction inverse to the β^+ -decay must not proceed under the influence of the antineutrino from the nuclear reactor. That is, the antineutrino must not be absorbed by neutrons:

$$\bar{\nu} + n \not\rightarrow p + e^- \quad (1.71)$$

The experiment consisted in prolonged irradiation of a large volume of carbon tetrachloride (about 4 m^3) with antineutrinos emitted by the nuclear reactor. If reaction (1.71) were possible, one of the nuclear neutrons of a Cl^{37} atom could be transformed into a proton with the formation of an Ar^{37} radioactive nuclide because the antineutrino energy is not sufficient to knock this proton out of the nucleus. The expected quantity of the radioactive gas was calculated, proceeding from the flux $\bar{\nu}$ and the cross-section (1.70). The experiment proved that the amount of Ar^{37} was much smaller than should have been in the equally probable interaction of the antineutrinos, either with the protons or neutrons, that is, the antineutrino and the neutrino are different particles.

The observed formation of a small amount of Ar^{37} may be explained by the influence of cosmic rays and in particular of cosmic neutrinos. Under their influence, a reaction inverse to β^+ -decay, (1.66) or (1.67), is naturally possible:

$$\nu + n \rightarrow p + e^- \quad (1.72)$$

The source of stellar energy as well as solar energy is thermonuclear synthesis in which four hydrogen atoms produce one helium

atom. As the nucleus of the helium atom consists of two protons and two neutrons, when it is being formed the two protons must become neutrons as a result of the β^+ -conversion, (1.66) or (1.67), with the emission of a neutrino. Thus, stars are neutrino sources. The solar neutrino flux to the Earth is determined by the sun luminescence and has a value 10^{14} — 10^{15} $1/(m^2 \cdot s)$. As the neutrino and antineutrino are different particles, solar neutrinos, in turn, cannot cause process (1.69) observed by Reines and Cowan. Cosmic antineutrinos are of some other origin, and if antistars exist, antineutrinos are emitted by them in thermonuclear synthesis like the neutrinos of ordinary stars.

5. Discovery of μ -meson neutrino. Neutrinos produce not only in β -decay but in some other decay processes as well. Neutrinos emitted in the π -meson decay turned out to be not identical to the β -decay neutrinos. This was confirmed in an experiment with a large spark chamber in 1962. The irradiation of the chamber by neutrinos and antineutrinos formed in π^+ - and π^- -meson decay showed that the interaction of these particles with the nucleons of the chamber substance is always accompanied by production of μ -mesons and never electrons, that is, processes (1.69) and (1.72) are in this case impossible. Neutrinos forming μ -mesons in the interaction with nucleons were called μ -meson neutrinos to differentiate them from electron neutrinos forming in the β -decay. It follows from the experiment that electron and μ -meson neutrinos differ in the character of their interaction with other elementary particles, while the other properties of all the neutrinos are similar. After this discovery, however, the lepton charges of electrons and electron neutrinos should be considered differing from the lepton charges of μ -mesons and μ -meson neutrinos to the same extent as the lepton charges differ from the baryon charges.

1.10. Mesons

1. Prediction of mesons. The discovery of the positron began the era of predicting particles. Thus, the probability of the existence of mesons, particles with mass values intermediate between the electron and proton masses, was theoretically predicted. For the short radius of action of nuclear forces to be explained it was necessary to assume the existence of nuclear interaction carriers having rest mass not equal to zero. In 1935, Yukawa, the Japanese physicist, estimated the mass of such particles to be 200-300 electron masses. He explained that these particles were not known to physics because of their probable instability resulting in fast decay after appearing in a free state. As the meson rest energy is equal to 100-150 MeV, the formation of these particles is possible only un-

der the action of high-energy particles with energies at least exceeding the rest energy (1.39). In the thirties such high energy accelerators did not exist. Therefore, mesons could be detected only in cosmic rays. Particles of the energy up to 10^4 MeV and higher enter into the composition of the primary cosmic ray component consisting in the main of protons.

2. **Discovery.** In 1938, Anderson and Noddermeir (USA), studying cosmic radiation with the help of a cloud chamber in a magnetic field, photographed the trace of a charged particle of mass $\approx 200 m_e$. These particles were subsequently called μ -mesons. They have either a positive or a negative electric charge, are unstable and decay in about 10^{-6} s, according to the schemes:

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \quad (1.73)$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad (1.74)$$

The μ -meson decay is accompanied by a release of a large quantity of energy (about 100 MeV). The formed electrons, however, carry away the smaller part of this energy, not constant from one decay event to another and not exceeding 50 MeV. It shows that in addition to the electron, in the μ -meson decay at least two more particles are produced which do not leave traces in tracking devices and hence have no electric charges. They are not γ -rays since the high-energy γ -ray quanta most probably form electron-positron pairs, never observed in the vicinity of the μ -meson decays. These particles are a neutrino and an antineutrino. One must be an electron and the other, a μ -meson as the μ -meson and electron lepton charges must be preserved independently (Ses. 1.9-5).

The negative μ^- -meson which has stopped in the substance approaches the atomic nucleus due to electrostatic attraction. In the vicinity of the atomic nucleus it behaves like an electron, occupying the corresponding Bohr orbits. The μ^- -meson performs radiation transitions and quickly gets to the inner K -orbit, forming a μ -mesonic atom, that is, an atom in which one electron is substituted by the μ^- -meson. As the μ -meson mass is 200 times greater than the electron mass, the radii of the Bohr orbits of the μ -mesons are also 200 times smaller than those of the electron orbits (1.8). Therefore, the radius of the μ -meson K -orbit in a heavy atom, for instance, in the Pb atom ($Z = 82$), appears even smaller than the nuclear radius. In spite of the fact that the μ^- -meson in the heavy atom is, in fact, inside the nucleus in direct contact with nucleons, no immediate interaction between μ^- -mesons and nucleons is observed. The μ^- -meson life-time in lead is only 30 times shorter than that of the free μ^- -meson and is $7 \cdot 10^{-8}$ s, which is much greater than the time of traversing the action range of nuclear forces by a fast moving particle. During this time, the μ^- -meson as

part of the μ -mesonic atom either undergoes its ordinary decay, (1.74), or is absorbed by the proton:



In the latter case almost the entire rest energy of the absorbed μ -meson is carried away by the light neutrino so that the nucleus formed having a new proton-neutron composition obtains only a small recoil momentum (1.34).

The very long time necessary for process (1.75) implies that the μ -meson is not Yukawa's nucleus-active particle. The nuclear interaction carrier must enter into the reaction with the nucleon each time they come closer together at a distance of the radius of action of the nuclear forces, $\sim 10^{-15}$ m. Otherwise stated, the time required for the interaction must be of the order of the time the fast moving meson is in the vicinity of the nucleon, or $\sim 10^{-23}$ s. The μ -meson appeared to have the properties more like the electron or the neutrino, which places it into the group with the light particles, or leptons. Nevertheless, this particle retained its name meson, though the μ -meson is a heavy electron rather than a meson in the modern sense of the word. Mesons are nucleus-active particles with masses intermediate between the electron and the proton masses and having an integral spin.

3. Pi-mesons. Cosmic radiation μ -mesons are not new-comers from space because they are unstable. They produce as a result of processes initiated by primary cosmic protons in the Earth's atmosphere. However, the nuclear passivity of μ -mesons is in contradiction with their large number in the secondary component of cosmic radiation because the probabilities of their formation in nucleon interactions and absorptions by nucleons must not differ. Powell, Okkialini and Lattes (Britain, 1947) found an explanation for the extraordinary number of μ -mesons. They established that μ -mesons form during the decay of some other, primary π -mesons which strongly interact with nucleons. Therefore, they produce most probably due to cosmic protons. The π -meson life-time is much shorter than the life-time of μ -mesons (see Table 1.2), and cosmic radiation decay products are greater in number than the original π -mesons. In addition, π -mesons mainly form in the upper layers due to the absorption of primary cosmic radiation by the atmosphere and few of them reach the Earth.

π -Mesons were discovered when exposing nuclear photoemulsions at great heights above sea level. Analysing tracks in emulsions, the Powell group drew a conclusion that in addition to the familiar μ -mesons, there also existed the primary or π -mesons having mass of about 300 m.

The decay of these particles gives birth to μ -mesons (Fig. 1.8). In addition, stars were observed at the end of the traces of some

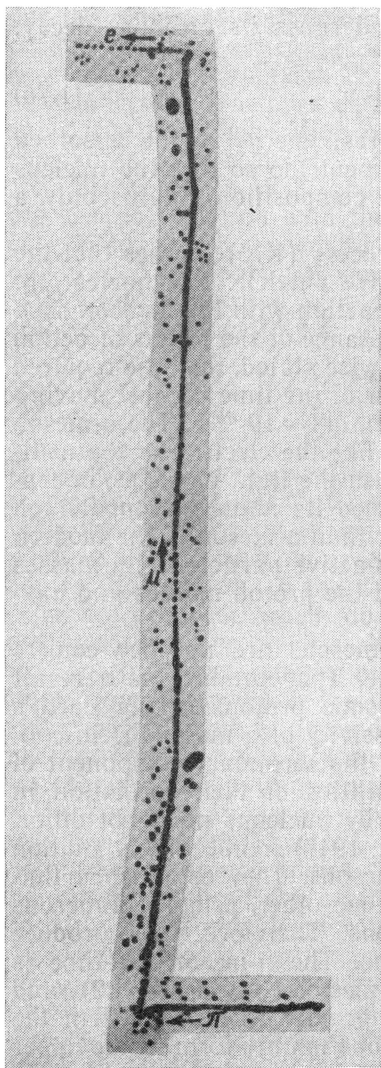


Fig. 1.8. Sequential scattering of π^+ - and μ^+ -mesons in nuclear photoemulsion

π -mesons (Fig. 1.9). The stars were indicative of the effective absorption of π -mesons by atomic nuclei. μ -Mesons arising in the π -meson decay always have one and the same track length in photoemulsion, this is only possible at one and the same initial energy of μ -mesons. This fact shows that the π -meson splits into two particles. As the second particle does not leave any trace in the emulsion, it has no charge. Its mass must either be low or equal to zero because this particle carries away the greater part of the π -meson decay energy. The total energy of decay is determined by the difference of the π - and μ -meson rest energies equal to 33 MeV, while the direct measurement of the kinetic energy of the μ -meson gives about 4 MeV. This second particle is not a γ -quantum because in the vicinity of the π -meson decays electron-positron pairs were never observed. Therefore, the second particle should be considered to be a neutrino, and, as is known at present, it must be the μ -meson neutrino:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad (1.76)$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (1.77)$$

The decay of π -mesons of both signs, following schemes (1.76) and (1.77), is equally probable. However, after stopping in the substance, they behave differently towards atomic nuclei. π^- -Mesons are attracted by nuclei and react with protons. One of the nuclear

protons transforms into a neutron and the total rest energy of the π^- -meson, into nuclear excitation energy. The release of the excess energy from the nucleus is accompanied by ejection of nucleons

some of which, as protons, leave traces in the photoemulsion, thus forming a star (see Fig. 1.9). Consequently, in a substance, the absorption of π^- -mesons by nuclei effectively competes with decay (1.76). Due to the electrostatic repulsion π^+ -mesons stopped in a medium do not approach the nuclei and undergo decay (1.77).

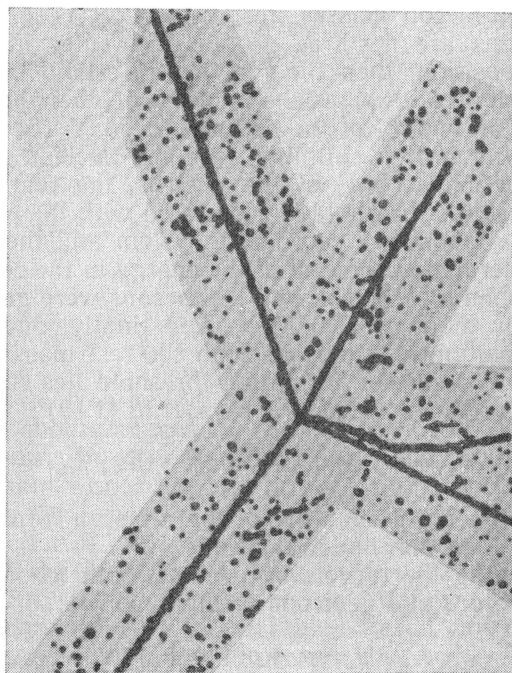


Fig. 1.9. A star in photoemulsion, produced in the capture of π^- -meson by the nucleus

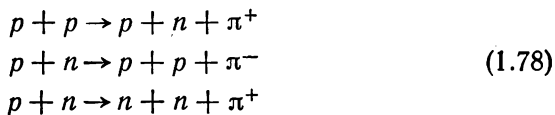
4. π -Meson production. The formation of stars by π -mesons shows the nuclear activity of these particles or that the π -mesons are the sought-for carriers of the Yukawa nuclear interactions. Therefore, the interest in π -meson experiments, which could shed light on the nature of nuclear forces, is quite understandable. However, for successful experiments it was necessary to produce intensive fluxes of these particles under laboratory conditions; this became possible in 1948 when charged particle accelerators of energies 350-400 MeV were put into operation. Just as in stopping an electron in the Coulomb nuclear field there appears a free photon, a carrier of electromagnetic interaction, a nuclear field quantum, the π -meson, appears in its free state when slowing down a nucleon

in a nuclear force field of some other nucleon. Since the rest energy of the π -meson differs from zero, the above process of the π -meson production is not possible at any value of the bombarding proton energy, but only beginning from some threshold value. The proton kinetic threshold energy may be calculated from (1.39) if one assumes $M_1 = M_2 = m_p$ and $\mathcal{M} = m_\pi$ where m_p and m_π are the proton and the π -meson masses, respectively. It is 290 MeV. If the target nucleons are not free protons but enter into the atomic nuclear composition, then the reaction threshold of the π -meson production appears lower as the nucleons in their bound state are in motion at an energy of the order of 25 MeV (Sec. 2.5-3). With the bombarding proton colliding with the nucleon moving in the opposite direction as part of the nucleus, the reaction threshold lowers because the threshold is associated with the proton's kinetic energy in the laboratory coordinate system and the π -meson rest energy is generated at the cost of the energy in the centre-of-inertia system. Thus, on a carbon target, the mesons were generated at the proton energy of about 200 MeV. And finally, due to the small specific momentum of a particle with the rest mass equal to zero, (Sec. 1.5-4), the π -meson formation threshold lies still lower when the nucleons are bombarded by γ -rays. If in (1.39)

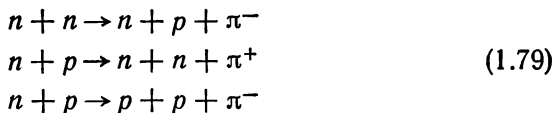
$$M_1 = 0, \quad M_2 = m_p \quad \text{and} \quad \mathcal{M} = m_\pi, \quad \text{then } E_{thr} = 150 \text{ MeV}$$

But in this case the probability of the π -meson formation is about 10 times less than that due to nucleons.

In bombarding the targets by protons, both π^+ - and π^- -mesons form from protons and neutrons (atomic nuclei):



Bombarding by neutrons also gives rise to both π^+ - and π^- -mesons:



If the bombarding particles are γ -rays with energy higher than 150 MeV, the charged π -mesons form in the following reactions:



From the above formation and decay schemes (1.76), (1.77), it follows that the π -meson spin must be integral. The direct measurement has shown that the π -meson spin equals zero.

5. Description of nuclear interactions. The interaction between nucleons through the agency of mesons is thought of as an exchange of these virtual particles (Sec. 1.4-7). Thus, the neutron-bonded proton emits a virtual π^+ -meson being then absorbed by the neutron, this explains the $(p - n)$ -interaction:

$$p \rightarrow n' + \pi^+; \quad \pi^+ + n \rightarrow p' \quad (1.82)$$

or

$$(p - n) \xrightarrow{\pi^+} (n' - p')$$

The $(n - p)$ -interaction is analogous:

$$n \rightarrow p' + \pi^-; \quad \pi^- + p \rightarrow n' \quad (1.83)$$

or

$$(n - p) \xrightarrow{\pi^-} (p' - n')$$

The proton-neutron bound system has no energy reserve to form an actually unbound π -meson. The emergence of conventional, or virtual, π -mesons in processes (1.82) and (1.83), is, however, feasible in the framework of uncertainty relation (1.19) because under this condition the formation of virtual mesons is not in contradiction with the law of energy conservation. The π -meson rest energy is $m_\pi c^2$ and if ΔE in (1.19) is taken to be equal to this value, the time uncertainty of the virtual meson existence is $\Delta t \approx \hbar/m_\pi c^2$. During this time interval the virtual π -meson can traverse a distance not greater than $r_0 = c\Delta t$ or

$$r_0 = \frac{\hbar}{m_\pi c} = 1.4 \cdot 10^{-15} \text{ m} \quad (1.84)$$

It follows that the processes accompanied by the emission and absorption of the virtual mesons (1.82) and (1.83) are only possible at distances of the order of r_0 or that the interaction between nucleons is only possible at a short finite distance not exceeding r_0 . Similar arguments given in a reverse order and proceeding from the observed radius of action of nuclear forces led Yukawa to the estimation of the meson rest mass. The effect of the nucleon exchange by quantum states (1.82) and (1.83) was observed in an experiment (Sec. 2.5-5) and this phenomenon was used to advantage to detect antineutrons (1.93).

The interaction between the proton-neutron and neutron-proton nucleon pairs is explained with the help of charged π -mesons. However, the $(p - p)$ and $(n - n)$ pair interaction does not at all differ from the $(p - n)$ pair interaction. Hence if all that has been

said above about the charged π -mesons is valid, neutral π -mesons must also exist, the exchange of which in the virtual processes explains the interaction of the like nucleon pairs:

$$p \rightarrow p' + \pi^0; \quad \pi^0 + p \rightarrow p' \quad (1.85)$$

$$n \rightarrow n' + \pi^0; \quad \pi^0 + n \rightarrow n' \quad (1.86)$$

or

$$(p - p) \xrightarrow{\pi^0} (p' - p') \text{ and } (n - n) \xrightarrow{\pi^0} (n' - n')$$

6. Pi-zero-meson. Soon after the discovery of the charged π -mesons a neutral π^0 -meson was detected. Its properties are quite similar to those of the π^+ - and π^- -mesons. The difference lies in the absence of the electric charge, its somewhat lower mass, and very short life-time, compared to that of the charged π -mesons, governed by the decay mechanism. The π^0 -meson decay takes place without the neutrino taking part but with the formation of γ -rays, that is, due to electromagnetic forces and not weak interaction:

$$\pi^0 \rightarrow \gamma + \gamma \quad (1.87)$$

γ -Rays formed in decay carry away both the rest energy and the kinetic energy of the π^0 -meson since its decay due to the absence of the ionizing energy losses and the very short life-time takes place in flight. The emergence of two γ -ray quanta in process (1.87) follows from the law of momentum conservation.

π^0 -Mesons like charged π -mesons produce in nucleon collisions:

$$\begin{aligned} p + p &\rightarrow p + p + \pi^0 \\ n + n &\rightarrow n + n + \pi^0 \\ p + n &\rightarrow p + n + \pi^0 \end{aligned} \quad (1.88)$$

or in the interaction of γ -quanta with nucleons:

$$\gamma + p \rightarrow p + \pi^0 \quad (1.89)$$

$$\gamma + n \rightarrow n + \pi^0 \quad (1.90)$$

Due to the lower mass of the π^0 -meson the thresholds of these reactions are somewhat lower than the thresholds of the formation reactions of the charged π -mesons, and are 280 MeV for (1.88), and 145 MeV for (1.89) and (1.90).

7. Multiple production of mesons. Both charged and neutral π -mesons may originate in other reactions in elementary particle collisions as well as in the decays of some unstable particles heavier than π -mesons. If the energy of the relative motion of colliding nucleons is very high and exceeds several π -meson rest energy

values, then multiple origination of π -mesons may be expected, i.e., production of several charged and neutral π -mesons simultaneously in one act of collision.

1.11. Antinucleons

1. Production of nucleon pairs. The probability of the existence of antinucleons was theoretically predicted. Like the electron-positron pairs, the nucleon-antinucleon pairs were expected to form in collisions of particles of sufficiently high energy interacting by means of nuclear forces, i.e., in the nucleon-nucleon or nucleon- π -meson collisions. If the nucleon is denoted by N and the antinucleon by \bar{N} , the production processes of nucleon pairs may be written in the form

$$N + N \rightarrow N + N + N + \bar{N} \quad (1.91)$$

$$\pi + N \rightarrow N + N + \bar{N} \quad (1.92)$$

One particle with $1/2$ spin cannot form at the cost of the kinetic energy of colliding particles. The origination of nucleons in pairs only with antinucleons is interpreted as the law of the baryon charge conservation. The nucleon is assigned the baryon charge $+1$ and the antinucleon, the baryon charge -1 . In processes (1.91) and (1.92) any pairs of nucleons and antinucleons, (p, \bar{p}) , (n, \bar{n}) , (p, \bar{n}) , (n, \bar{p}) , can originate, just as the production of nucleon pairs is possible in the collision of any nucleon or π -meson with any of the nucleons, p or n , naturally, with the preservation of the electric charge in these processes.

Reaction threshold (1.91) may be obtained from (1.39) if one takes $M_1 = M_2 = m_p$ and $\mathcal{M} = 2m_p$, since the proton and the antiproton masses are equal, amounting to $6 m_p c^2$, or about 5.6 GeV. Reaction threshold (1.92) is lower (3.6 GeV) for in this case the mass of the bombarding particle is lower and the rest energy of the nucleon pair is produced on account of the kinetic energy of the colliding particles as well as the rest energy of the π -meson. If the target-nucleons form part of the atomic nuclei, the reaction thresholds are lowered to 4.3 and 2.85 GeV, respectively, on account of possible collisions with nucleons moving towards the bombarding particle in the nucleus. As the reaction thresholds of the antinucleon formation are very high, attempts were initially made to discover antiprotons in the composition of secondary cosmic rays. In cosmic rays, however, antiprotons failed to be identified owing to the very low intensity of the flux of primary cosmic protons and the difficulty of selecting the events of interest from a lot of side phenomena.

2. Antiproton. The attempt to obtain antinucleons in experiment stimulated the construction of proton accelerators to energies of

some gigaelectron-volts. In 1955, a group of American physicists, headed by Segre were the first to detect antiprotons. Antiprotons were obtained during bombardment of a copper target by protons accelerated up to energies of 6.2 GeV in the Berkeley proton synchrotron. With the help of a system of magnetic analysers and lenses a beam of negatively charged particles of momentum corresponding to that of the expected antiprotons was selected from among all possible reaction products and focused. In addition to antiprotons, the beam contained π^- -mesons of the same momentum. The π^- -mesons were produced in abundance by bombarding the target with protons of such a high energy. Owing to their much lower mass the π^- -mesons, however, had a much higher velocity than the protons and this fact allowed one to select from the beam and register only particles of the velocity corresponding to the antiproton velocity, i.e., to select particles by their mass. The first experiment registered 60 particles with a single negative charge and with the proton mass, i.e., antiprotons (see Table 1.2).

Although the charge and the mass of the discovered particles fully corresponded to the characteristics of the antiproton, the observation of the inverse process, i.e., the process of the antiproton and nucleon annihilation could alone give the final conclusion on the identity of these particles with antiprotons. For this purpose nuclear photoemulsions were irradiated by antiprotons and stars were observed at the end of the antiproton tracks (Fig. 1.10). Antiprotons like π^- -mesons after stopping in a medium are attracted by the atomic nuclei and annihilate with one of the nucleons of the nucleus, emitting energy equal to $2m_p c^2$. On account of this energy on the average three charged π -mesons appear of energy 200-250 MeV, sometimes K -mesons (one in 20 π -mesons), and the remaining energy is distributed among the nucleons of the nucleus, which leave the nucleus at high energy, forming jointly with the charged mesons a star in the photoemulsion. Neutrons and possible π^0 -mesons do not leave any tracks. The analysis of the stars in the photoemulsion allows one to obtain the total energy of the charged products of the reaction. If the investigated particles were not antiprotons but resembled π -mesons, unstable nuclear-active particles, then in colliding with the atomic nuclei, they would originate some other particles and excite the nuclei only at the cost of their rest energy. Hence, the total energy of the electrically charged reaction products in the stars would always be lower than $m_p c^2$ since part of the energy is carried away by neutral particles.

However, in the stars being examined, the total energy of only charged particles is much higher than the rest energy of the antinucleons. This points to the fact that in the reaction with the star origination it is not only the antiproton that disappears but one particle more which may only be either a proton or a neutron of the

nucleus since there are no other heavy particles in the substance. From this it follows that the discovered particles are antiprotons. Measurement of the magnetic moment of the antiproton, when scattering on protons in a bubble hydrogen chamber in a magnetic field, gave a value which, within the limits of the measurement error, coincided with the value of the proton's magnetic moment, with a magnetic moment sign opposite to that of the proton.

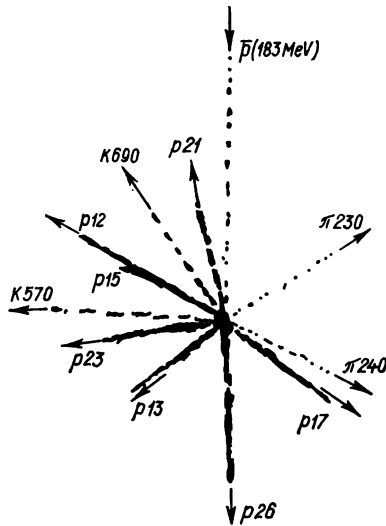


Fig. 1.10. A star in photoemulsion, produced in the capture of an antiproton by the nucleus. Traces of an antiproton, protons, and π^- and K -mesons are denoted by respective indices, with the energy given in megaelectronvolts

3. Antineutron. Antineutrons are also originated by bombarding atomic nuclei by protons of energy 6.2 GeV. However, antineutrons like neutrons have no electric charge and cannot be drawn out of the accelerator in a given direction by means of magnetic fields and be identified by their mass. Therefore, to observe antineutrons use was made of the phenomenon of antiproton recharging followed by the formation of antineutrons:



Antineutrons were detected by their annihilation in the sensitive volume of a large Cherenkov counter. The annihilation was accompanied by the appearance of fast mesons the luminescence of which was detected by photomultipliers. The condition of the detection of

an antineutron included a number of successive events detected by relevant counters: the appearance of the antiproton in the converter-body, its disappearance in the converter without annihilation, i.e., in recharging (1.93), and an annihilation light flash in the detector placed behind the converter in the line of movement of the original antiproton. An additional condition was that this flash be not preceded by the penetration of any charged particle into the detector. This was controlled by suitable scintillation counters [5]. When the enumerated events coincided, the light flash in the Cherenkov counter, corresponding by its intensity to the nucleon pair annihilation, could be caused by the antiproton alone. The process of the antineutron annihilation after the antineutron recharging was also observed in a photograph obtained in a propane bubble chamber. Antineutrons were first observed in Berkeley, USA, in 1956.

1.12. Antimatter

1. Probability of existence. The existence of elementary particles in the form of a particle-antiparticle pair is a conclusively established law of Nature. The physical properties of the pair members are absolutely similar, except for the charge signs, which makes them antipodes. And if ordinary protons, neutrons, and electrons form an ordinary substance, no physical reasons are known which would hinder the formation of an ant substance containing antiprotons, antineutrons and positrons. Such an ant substance would not differ at all from an ordinary substance only except that its contact with an ordinary substance would be accompanied by annihilation, i.e., by the transformation of the structural antibody particles which came close to each other into mesons and photons of high energy. After the antineutrons had been obtained and identified in accelerators, no doubt remained of the probability of the existence of antimatter. In 1965, at the Brookhaven National Laboratory, USA, in a proton accelerator to energy 30 GeV, an antideuteron, a bound antiproton-antineutron system, i.e., the simplest nucleus of the antimatter atom, was registered. In 1970, in a proton accelerator to 70 GeV at the Institute of High Energy Physics in Serpukhov, USSR, antihelium-3 nuclei were produced. Nuclei containing antinucleons originate during the collision of fast protons with the nuclei of the target atoms in the process of the multiple nucleon pair production. In the Serpukhov accelerator, the proton energy is high enough not to produce separate antideutons but their beams. In the experiments with antideutons it was possible to measure their bond energy which, as expected, was equal to the bond energy of the ordinary deuteron.

2. Antimatter in the Universe. The existence of a substance and an ant substance in the form of a single body is certainly impos-

sible. However the existence of individual cosmic antibodies is thought to be natural. It is possible to describe the processes which in their time led to the division of matter and antimatter and to their isolation in cosmic space [9]. Furthermore, from the point of view of the discoveries made in physical laboratories the absence of antimatter in the Universe would be considered to be strange. The preference given to the matter over the antimatter by Nature would not find any simple explanation. The appearance of the hypotheses of the existence of antibodies in the Universe is, therefore, understandable. The criterion of the verity of any hypotheses is experiment. However, there are great difficulties in the way of detecting cosmic antibodies. The overwhelming part of the observations of cosmic bodies was associated with the registration of electromagnetic signals sent by stars, i.e., light and radiowaves. The photon (see Table 1.2), however, has no antiparticle and, therefore, the electromagnetic signals from stars and antistars are absolutely similar. Only observations of cosmic radiation remain, the primary component of which consists of protons and, to a lesser degree, of atomic nuclei, and also the detection of neutrinos. As was already pointed out (Sec. 1.9-4), neutrinos are formed during the thermonuclear synthesis in stars, and antineutrinos, in thermonuclear synthesis in antistars. Neutrinos and antineutrinos are different particles which can be distinguished in experiment. However, calculations based on star luminescence show, that even the stars closest to the Earth send neutrino fluxes at least 10^{10} times less intense than those that can be registered nowadays. Nevertheless, neutrino astronomy is considered to have its future. The direct proof of the existence of antimatter in nature would be the detection of antinuclei in cosmic radiation. The existence of antiprotons among cosmic particles cannot be doubted since nucleon pairs naturally form in cosmic collisions of protons of high relative kinetic energy. However, the production (in space nuclear collisions) of antinuclei consisting of several antiprotons and several antineutrons is extremely inconceivable. The detection of the antinucleus in the cosmic radiation would directly imply the existence in space of stars made of antimatter since the antinucleus could only be the messenger of such a star. Attempts to find antinuclei in the composition of cosmic radiation with the help of an artificial satellite proved unsuccessful. As yet no observation can testify in favour of the existence of antimatter in the Universe and also no observations reliably deny such a probability. The nearest neighbours to the Sun system are formed by ordinary substances. The future will show whether there is any antimatter in cosmic depths.

CHAPTER TWO

PHYSICAL PROPERTIES OF ATOMIC NUCLEI

2.1. Nuclear Charge

1. **Moseley law.** Protons entering into the composition of the nucleus form the electric charge of the nucleus. The number of protons in the nucleus, Z , is called the nuclear charge, bearing in mind that the inherent nuclear charge is equal to Ze . The nuclear charge coincides with the atomic number Z of the element in the Mendeleyev periodic system. In 1913, the English physicist Moseley was the first to determine the charges on atomic nuclei. Measuring by means of a crystal the wavelength λ of the characteristic X -radiation, Moseley discovered that the wavelengths of the elements following one another in the periodic system (Fig. 2.1) vary regularly. Moseley explained this fact by the dependence of λ upon a certain atomic constant Z varying by unity from element to element and equal to 1 for hydrogen:

$$\sqrt{\frac{1}{\lambda}} = aZ - b \quad (2.1)$$

where a and b are constants. From experiments on the scattering of X -ray quanta by atomic electrons and of α -particles by atomic nuclei it was already known that the nuclear charge is approximately equal to half the atomic mass and therefore is close to the atomic number of the element. Since the characteristic X -ray emission is the consequence of electric processes taking place in the atom, Moseley drew a conclusion that the constant of the atom found in his experiments, which defines the wavelength of the characteristic X -radiation and coincides with the atomic number of an element, can only be the atomic nuclear charge (the Moseley law).

Measurements of the X -ray wavelengths are made to a high degree of accuracy so that on the basis of the Moseley law it is absolutely reliably established that the atom belongs to the chemical element. At the same time, the fact that the constant Z in (2.1) is the nuclear charge (although substantiated by indirect experiments) is in the long run based on a postulate, i.e., the Moseley law. Therefore, after Moseley's discovery nuclear charges were re-measured many times in experiments of α -particle scattering on the

basis of the Coulomb law (Sec. 1.2-3). In 1920, Chadwick improved the procedure of measuring a fraction of scattered α -particles and obtained the charges of copper, silver and platinum nuclei (Table 2.1). The data obtained by Chadwick fully confirmed the validity

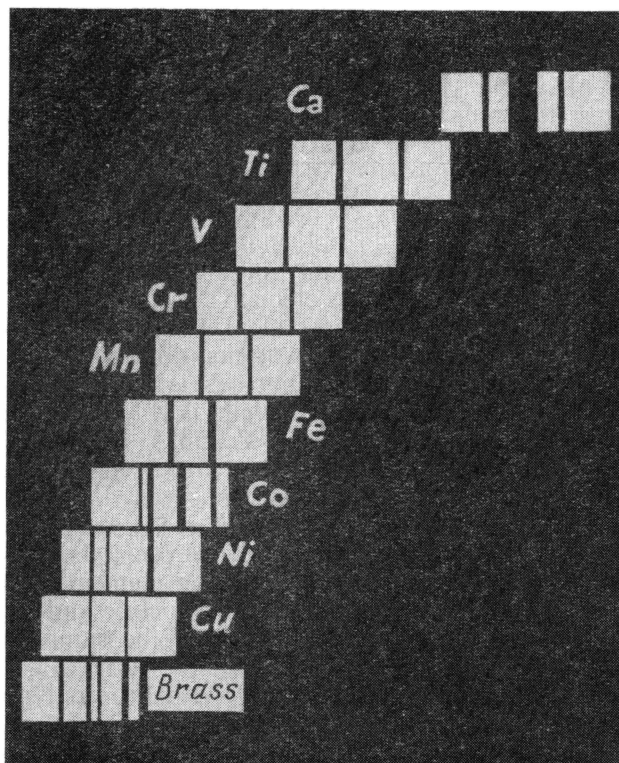


Fig. 2.1. X-ray spectra of the atoms of adjacent elements obtained by Moseley

of the Moseley law. In addition to the elements mentioned above, the nuclear charges of magnesium, aluminium, argon, and gold were determined by α -particle scattering [1].

2. Definitions. After Moseley's discovery it became clear that the principal characteristic of the atom is its nuclear charge, not its mass, as was supposed by 19th century chemists, since the nuclear charge determines the number of atomic electrons and consequently the chemical properties of the atom. The reason for the difference between the atoms of chemical elements is just in the fact that their nuclei contain different numbers of protons. On the con-

Table 2.1

Results of Chadwick's
Experiments

| Element | Atomic number Z | Charge Z (experiment) |
|---------|----------------------|----------------------------|
| Cu | 29 | 29.3 |
| Ag | 47 | 46.3 |
| Pt | 78 | 77.4 |

trary, the different numbers of neutrons in atomic nuclei having equal numbers of protons do not change the chemical properties of the atoms. Atoms differing only in the number of neutrons in nuclei are called *isotopes* of the element.

The atom with definite numbers of protons and neutrons in the nucleus is called a *nuclide*. As it was mentioned in Sec. 1.3-3, the total number of nucleons in the nucleus is the mass number A , and the number of neutrons in a nucleus is $A - Z$. Thus, the nuclear composition is given by numbers Z and A . If each of them has a definite value, for example, Z is represented by a chemical symbol of an element, and the mass number is given, then a nuclide is meant, for example, O^{16} , Na^{23} , Fe^{56} , U^{235} , etc. Every nuclide is an isotope of the corresponding element. However, it is reasonable to speak of an isotope only as belonging to an element. Thus, U^{235} is an isotope of uranium, but U^{235} is a fissile nuclide and not a fissile isotope. Atoms, the nuclei of which contain the same numbers of neutrons but different numbers of protons, are called *isotones*. Atoms with similar mass numbers but with different proton-neutron nuclear composition are called *isobars*.

2.2. Nucleus Size

One can judge of the geometrical size of the nucleus first of all by observing the violation of relation (1.5) during the scattering of heavy charged particles of sufficiently high energy. The greater the energy and the scattering angle of the particle, the smaller the minimum distance between the particle and the nucleus in the Coulomb scattering. The distance at which the scattering law $1/\sin^4 \theta/2$ becomes invalid should be taken for the radius of the nucleus. It may mean both the finite size of the electric charges of colliding bodies (Sec. 1.2-2), and the appearance of forces different from the Coulomb forces. Equally probable scattering at any angles when the particles reach the nuclear radii shows the effect of other, nuclear forces, which act within short distances.

A number of nuclear radii measurement techniques exist both direct, associated with the investigation of particle scattering by nuclei, and indirect [5]. The measurement results can be given by the following relation between radius R and mass number A :

$$R = r_0 A^{1/3} \quad (2.2)$$

where r_0 is a constant. Relation (2.2) means that the volume of the nucleus is proportional to the number of particles A . In this sense, the nucleus resembles a drop of liquid the volume of which is also proportional to the number of particles in the drop, and the density is independent of the volume. This analogy shows the similarity of the properties of nuclear forces ensuring the existence of nuclei with the properties of the forces responsible for the liquid drop stability. The constant r_0 is somewhat varied, depending on the measurement technique, and its averaged value is $r_0 = 1.23 \cdot 10^{-15}$ m. Therefore, the radius of the nucleus is

$$R = 1.23 \cdot 10^{-15} A^{1/3} \text{ m} \quad (2.3)$$

2.3. Nuclear Moments

1. **Spin.** Since nucleons have the intrinsic mechanical moment, or spin, equal to $\hbar/2$ (see Table 1.2), nuclei should also have their mechanical moments. In addition, in the nucleus, nucleons take part in relative motion which is characterized by a certain momentum of each nucleon. Mechanical moments in relative motion, or orbital moments, may take only integral values \hbar . All the mechanical moments of nucleons, both spins and orbital moments, are algebraically summed and constitute the mechanical moment, or nuclear spin I .

Although the number of nucleons in the nucleus may be very great, the nuclear spins are usually small and are not greater than several units of \hbar . This is determined by the peculiarities of the interaction of like nucleons. All the paired protons as well as all the paired neutrons interact only in such a way that their spins are mutually compensated, i.e., proton or neutron pairs always interact with antiparallel spins. Moreover, the total orbital moment of a proton or a neutron pair is always equal to zero. Consequently, nuclei composed of even numbers of both protons and neutrons [even-even (ee)] have no mechanical moment, and their spin $I = 0$. Only nuclei having unpaired protons or neutrons have spins different from zero. If there is one unpaired nucleon, its spin is summed with its orbital moment and the spin of the even-odd (eo) or the odd-even (oe) nucleus has a certain half-integral value $1/2, 3/2, 5/2$. Nuclei of odd-odd (oo) composition have spins of integral values 1, 2, 3, etc.

2. Magnetic moment. Nuclear spins are determined while observing the splitting of atomic spectrum lines under the influence of both the atomic magnetic fields (the superfine structure of atomic spectra) and the external magnetic fields, as well as while observing the splitting of atomic beams as they pass through nonuniform magnetic fields. All these measurements appear possible due to magnetic moments associated with nuclear spins. The magnetic moments of nuclei are measured in nuclear magnetons (1.62), and they are varied from -2 to $+5$ nuclear magnetons for different nuclei. Because of the relatively great mass of the nucleons the magnetic moments of the nuclei are very small compared to those of electrons. Therefore, observations of the effects due to the magnetic moments and measurements of the nucleus magnetic moments are more complicated than the corresponding determinations for electrons. The magnetic moments of nuclei, as also spins, are measured spectroscopically. The most precise method, however, is the nuclear resonance method [5]. The magnetic moments of even-even nuclei are, like the spin, equal to zero. The magnetic moments of nuclei with unpaired nucleons are produced by the intrinsic magnetic moments of these nucleons (1.63) and (1.64), and also by the magnetic moment associated with the orbital motion of the unpaired proton. At the same time, the majority of the observed moments may be explained if only one takes into account the contribution of the motion of many nucleons, which do not enter the filled nucleon shells [5], to the magnetic moment of the nucleus.

3. Electric quadrupole moment. Atomic nuclei, the spin of which $I \geq 1$, have quadrupole moments other than zero. It shows that their form is not strictly spherical. The quadrupole moment has a plus sign if the nucleus is extended along the spin axis (a spindle-shaped body) and a minus sign if the nucleus is extended in a plane perpendicular to the spin axis (a lenticular body). Nuclei with positive and negative quadrupole moments are known. A nucleus possessing a quadrupole moment produces a nonspherically symmetrical electric field. This results in the formation of additional energy levels of atomic electrons and gives rise to lines of superfine structure in the atomic spectra, the distances between which depend on the value of the quadrupole moment.

2.4. Nuclear Mass and Bond Energy

1. Mass of particles in bound state. Nucleon masses make up the atomic nuclear mass. The nuclear mass is, however, smaller than the total mass of the nucleons in the nucleus since the nucleons of the nucleus are in their bound state. The bound state is possible only under the influence of the forces of attraction which hold the particles in a limited volume. The work of forces of attrac-

tion brings about the transition of the particle system into a state with lower rest energy accompanied by the transfer of an equivalent quantity of free energy to some other bodies or its emission into the outer space. Subsequently the stability of the bound state is maintained by the relative minimum of the system energy or by its bond energy equal to the energy loss. The inverse process of splitting the bound system into constituents is possible only under some external influence if from the outer space there comes such a quantity of energy which makes up for the lack of it to do work against the forces of attraction holding particles in the bound state. Therefore, nuclei consisting of nucleons, atoms containing nuclei and electrons, and molecules comprising atoms have lower rest energies, i.e., masses lower than the total masses of their particles in the free state. It follows that the production of free energy is only possible when the particle bound states are formed in some or other processes, for instance, in nuclear or chemical reactions, at the cost of the particle mass reduction. In this case the quantity of the released energy ΔE is related to the lowering of the mass Δm by (1.26) as

$$\Delta E = \Delta mc^2 \quad (2.4)$$

Although this relation is universal, it was actually verified by experiment only for the atomic nuclear masses. The measurement of ΔE , i.e., the released energy, is usually not difficult. But to measure Δm is only possible when Δm is a noticeable fraction of the mass of free particles. This depends on both the particle mass values and the value of the binding forces. For example, electric forces at distances between elementary charges of the order of 10^{-10} m (which is characteristic of molecules) are relatively weak and their corresponding chemical bond energies ΔE are so low that no methods available can detect the change of molecular masses compared to the masses of their atoms in free states. On the contrary, nuclear forces are large and the corresponding bond energies per particle are at least 10^6 times the molecular bond energies. In this case, the mass decrease, when bound states are formed, appears somewhat less than 1% of the mass of particles in the free state and it can be reliably detected in experiment.

2. Nuclear mass and atomic mass. The masses of neutral atoms rather than the nuclear masses are usually used in nuclear physics. This is because it is impossible to measure the nuclear masses without their bonded electrons, which is true for practically all atoms except the lightest ones. Ionization of the atom is usually accompanied by the release of one or several electrons. In nuclear reactions, slightly ionized atoms, not their nuclei, are generated. Even in heavy nuclei fissioning when two newly born nuclei leave the volume of the original atom at high velocity, they carry away

more than half of the atomic electrons. Having been slowed down in the medium, all the products of nuclear transformations become neutral atoms, as were the original atoms. Moreover, it appears that the mass of the neutral atom, within the accuracy of the existing mass measurement techniques, is simply equal to the sum of the masses of the nucleus and electrons making up the atom, although in principle the atomic mass is

$$M_a = M_N + Zm - \frac{1}{c^2} \sum_i q_i \quad (2.5)$$

where M_N and m are the masses of the nucleus of electron, Z is the number of the atomic electrons and $\sum_i q_i$ is the bond energy of the nucleus and electrons known as the electron bond energy. The values of q_i are well known for all the electrons of any atoms [10] and

$$\sum_i q_i \approx 13.6Z^{1/5} \text{ eV} \quad (2.6)$$

However, the value of $(1/c^2) \sum_i q_i$ is negligibly small compared to M_a and practically has no influence on the value of the atomic mass. Thus, the mass of the nucleus is equal to the difference between the mass of the atom and the mass of the atomic electrons. The number of the atomic electrons is always known since it is equal to the charge on the nucleus, and, therefore, dealing with the masses of neutral atoms does not cause difficulties in nuclear physics.

3. **Mass unit.** The atomic masses are measured in relative units, or atomic units of mass. The atomic mass unit m_u is $1/12$ of the C^{12} nuclide mass. Its absolute value is

$$m_u = \frac{1}{N_A} = \frac{1 \text{ kg/kmole}}{6.023 \cdot 10^{26} \text{ 1/kmole}} = 1.66 \cdot 10^{-27} \text{ kg} \quad (2.7)$$

The energy equivalent of the atomic mass unit in megaelectronvolts, according to (1.26), is:

$$m_u c^2 = 931.4 \text{ MeV} \quad (2.8)$$

The adoption of the C^{12} scale for atomic masses put an end to the existence of the chemical and physical atomic mass units. Nevertheless, the chemical atomic masses differ from the physical atomic masses since the chemical masses refer to a natural mixture of the isotopes of an element whereas the physical masses are always the nuclide atomic masses. The chemical atomic masses can be obtained from the nuclide atomic mass, taking into account the isotopic abundance of elements.

4. Mass-spectroscopy. The method of measuring atomic masses, which is highly accurate and allows atomic masses to be directly compared with the reference mass, is based on the separation of beams of charged particles of different masses when they pass through electric and magnetic fields. Here, a quantitative analysis of samples by their atomic mass is also possible, for example, isotopic analysis of chemical elements.

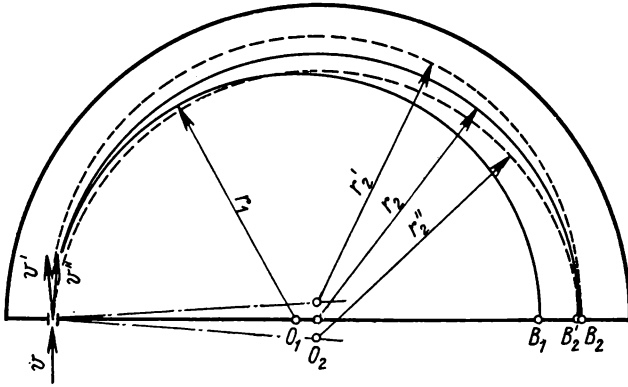


Fig. 2.2. Motion diagram of positively charged particles in a magnetic field. The magnetic induction vector is normal to the drawing plane and is directed towards the observer

A particle with charge e moving at velocity v in a constant magnetic field with induction \mathbf{B} is acted upon by the force

$$\mathbf{F}_1 = e [\mathbf{v}\mathbf{B}] \quad (2.9)$$

Since the force \mathbf{F}_1 is always normal to the velocity vector \mathbf{v} , it does not perform any work, i.e., it does not change the energy of the particle but only curves the trajectory of its motion. The force F_1 is maximum when the vectors \mathbf{v} and \mathbf{B} are mutually perpendicular. The particle describes a circular trajectory (Fig. 2.2), the radius of which is determined from the equality of F_1 to the centrifugal force Mv^2/r :

$$r = \frac{Mv}{eB} \quad (2.10)$$

It is evident that with different particle momenta Mv , the radii of their trajectories are different and such particles will be separated in space. The greatest separation is achieved after rotation of the momentum vector by the magnetic field through 180° (points B_1 and B_2 in Fig. 2.2). Analogous spatial separation of moving particles may be achieved by means of an electric field.

The particle with charge e in the electric field of strength E is acted upon by the force F_2 equal to

$$F_2 = eE \quad (2.11)$$

The force F_2 moves the charge in the direction of the vector E performing thereby the work $e\Delta\varphi$, where $\Delta\varphi$ is the potential difference. If the particle moves in a direction-constant field E with the velocity v normal to E , the particle is displaced in direction E , the displacement at its low value being inversely proportional to the particle energy [3]. This means that in the field E spatial separation

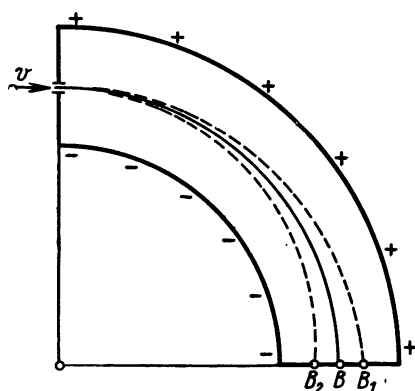


Fig. 2.3. Diagram of the motion of positively charged particles in a cylindrical electric field

of a beam of particles with different energies occurs. In the cylindrical field E (Fig. 2.3), a condition exists under which the force F_2 is always perpendicular to the velocity v . In this case the particle has a circular trajectory determined from the equality of the force F_2 to the centrifugal force Mv^2/r :

$$r = \frac{Mv^2}{eE} \quad (2.12)$$

Here too, spatial separation of particles with different energies takes place, since particles with energies higher or lower than those by condition (2.12) move along some other trajectories and arrive at points B_1 and B_2 , respectively, as shown in Fig. 2.3.

Thus, in both magnetic and electric fields it is possible to separate particles by their mass and to measure the mass values if, the narrow particle beams are directed for the analysis at velocities strictly the same in the value and direction. However, atomic and molecular ions being obtained in ionic sources have different velocities and the directions of their motion are arbitrary. The use of diaphragms with narrow slits and of electromagnetic velocity filters allows the desired ion beam to be formed but in this case the overwhelming majority of ions are lost, which worsens the sensitivity of the analyser. Therefore, in analysing the ion mass, methods of focusing divergent beams are used, i.e., focusing of directions, and focusing beams with ion velocity spread by absolute values, i.e., velocity focusing. In a constant magnetic field a slightly divergent beam of ions with similar velocities, i.e., with similar trajectory radii, is focused after the rotation of the inlet

slit through 180° with a slight blurring of the image ($B_2^1 B_2$ instead of B_2 in Fig. 2.2). Focusing is also possible at a smaller rotation angle but in this case the foci are beyond the limits of the magnetic field. The cylindrical electric field also focuses the divergent ion beam after the rotation of the velocity vector through the angle $(\pi/V\sqrt{2}) = 127^\circ 17'$. Passing ions through the electric and magnetic fields allows focusing ions of identical masses but with different velocity values. The use of the double direction and velocity focusing increases the ion transfer in the device and, hence, improves the device sensitivity and measurement accuracy, due to the reduction of the exposition time and, consequently, to the decrease of the external noise.

Ions are detected either by means of a photoplate, then the device is called a mass-spectrograph, or by the electric current carried by ions, this device being called a mass-spectrometer. Mass-spectrometers are used to perform quantitative measurements of the streams of ions of a given mass, necessary when analysing in a sample the percentage of particles of different masses, for instance, when analysing the isotopic composition of elements. The ion mass differences are determined either from the distance between the lines of blackening on the photoplate at constant parameters of the analyser or from the value of the change of the electrical parameters of the device, which ensure that the motion of particles of different masses be along the same trajectory. In the latter case, particularly accurate recording is achieved by oscillograph scanning of that portion of the spectrum where the lines of two compared masses are disposed, and by bringing into coincidence the two lines by changing the electric potential of the analyser. A photomultiplier detecting the ion flux is a vertical scanning generator of the oscillograph. A horizontal scanning generator of the oscillograph modulates the field of the magnetic analyser of the mass-spectrometer. As a result, mass spectrum scanning synchronous to the horizontal direction of the oscillograph ray takes place in front of the slit through which ions are transmitted onto the photomultiplier. This scanning is reproduced on the oscillograph screen. The subsequent field modulation of the electric analyser of the mass-spectrometer allows two lines on the oscillograph screen to be brought into coincidence and the mass difference to be found from the modulation value.

Most precise mass values are obtained in comparing ion masses with quite close values or in comparing mass-spectroscopic doublets. In such measurement, it is convenient to take the mass of the nuclide capable of forming many doublets as the reference mass. This circumstance played an important role in the choice of the C^{12} nuclide as such a reference. The C^{12} nuclide can be singly, doubly,

trebly and four times ionized, owing to which the light nuclide mass may be directly compared with the reference. For instance, the four times ionized C^{12} behaves in the mass-spectrometer as a particle with a single charge but with mass four times as low. Therefore, the mass of the He^3 nuclide and the mass of the H_3^{+1} three-atomic molecular ion, i.e., the mass of the H^1 nuclide may be directly compared with the reference mass. In addition, the C^{12} nuclide forms multiatomic molecules with the number of atoms 10 or more, which permits a direct comparison of the masses of the heaviest nuclides, for examples, the Pu^{240} mass with the reference mass, when measuring the mass difference of the doublet $(C^{12})_{10} - 1/2 Pu^{240}$ (Pu^{240} being doubly ionized). And finally, the C^{12} nuclide forms hydride and deuteride molecules of the type $C_m^{12}H_n^1$ or $C_m^{12}H_n^2$, the ion masses of which may be compared with the ion mass of all the nuclides with mass numbers from 1 to 210. The measurement error in this case comprises 10^{-5} of the mass being measured.

5. Method of nuclear reactions. The mass-spectroscopic method permits mass determination of only charged particles and, therefore, is inapplicable in measuring the mass of such most significant particle as the neutron. The method of nuclear reactions is based on the law of energy conservation (Sec. 3.5-7), and is not subject to the above limitation. If of all the particles, entering into reaction and forming after the reaction (or decay), the mass of only one particle is unknown, it may be found from the masses of the other particles and from the measured energy of the reaction (3.49). The mass in this case will not be less accurately determined than of all the other masses known. The energy of the threshold endoenergetic reaction is most simply determined because in this case only the energy of the bombarding particle corresponding to the threshold of the reaction E_{thr} is measured (1.44). In a general case, to obtain the reaction energy, it is necessary to measure both the kinetic energy of the bombarding particle and the kinetic energies of the reaction products, and in the spontaneous decay of the nucleus at rest, the energies of the conversion products. These measurements require the use of magnetic analyzers and spectrometers. That is, they are connected with charged particles or photons, the energies of which are determined by the energy of secondary electrons. The neutron mass is measured in such reactions where the neutron kinetic energy may be sufficiently accurately estimated or neglected at all, for instance, in reactions initiated by thermal neutrons [5].

6. Mass defect. Bond energy. The nuclide masses in the C^{12} scale are given in [11] where the nuclear bond energies are also given, and devices and mass measurement techniques are des-

cribed. The atomic masses (in m_u units) of the structural particles of atoms and the H^1 nuclide are given below:

| | |
|----------------------|------------|
| neutron . . . | 1.008665 |
| proton . . . | 1.007276 |
| electron . . . | 0.00054859 |
| H^1 nuclide . . | 1.007825 |
| C^{12} reference . | 12.000000 |

It is evident that the H^1 mass is equal to the total mass of the proton and electron. It is determined by that the bond energy of the proton and electron in the H^1 nuclide in m_u units is lower than $1.5 \cdot 10^{-8}$. On the other hand, the total mass of those particles which form part of the C^{12} nuclide is obviously greater than the mass of C^{12} . The C^{12} nuclide actually consists of 6 neutrons, 6 protons and 6 electrons, and the total mass of these particles in the unbound state is

$$6m_n + 6M(H^1) = 6 \cdot 1.008665 + 6 \cdot 1.007825 = 12.098922$$

whereas their total mass in the bound state, i.e., the mass of the C^{12} nuclide is 12.000000. The mass difference for C^{12} is 0.098922 and is known as the *mass defect* of the atomic nucleus. If the C^{12} mass defect is expressed in energy units according to (2.8), then one will have the value 92.1626 MeV. The energy equivalent to the mass defect is called the *bond energy* of the compound particle.

The bond energy of the atomic nucleus is the energy which is emitted into space with γ -rays when the nucleus is being formed from nucleons. On the contrary, the fission of the nucleus into nucleons is possible only when obtaining from the outer space the amount of energy not lower than the bond energy. The large value of the bond energy of nuclei renders them highly stable and is due to powerful nuclear forces of attraction between nucleons.

7. Peculiarities of bond energy value. The mass defects and bond energies of all the atomic nuclei have been determined from the exact values of the nuclide masses. The most significant feature of the nuclear bond energy appeared to be its proportionality to the number of nuclear particles. Otherwise stated, the mean bond energy per nucleon in the nucleus is constant for all nuclei (Fig. 2.4). True, the lightest nuclei, the mean bond energy of which changes strongly with the composition of the nucleus, make an exception. But, in all the other nuclei, it is about 8 MeV, has its maximum in the region of the Ni^{62} nuclide (8.8 MeV), and reduces to 7.6 MeV in the region of the U^{238} nuclide.

Such behavior of the specific bond energy reveals the capacity of nuclear forces to achieve saturation, i.e., the probability of nucleon interaction only with a small number of its partners. If nuclear forces did not possess the saturation property, then, in the

range of action of nuclear forces, each nucleon would interact with each of the rest and the interaction energy would be proportional to $A(A-1)$, i.e., A^2 , and the mean bond energy of one nucleon would not be constant in nuclei different in values and would increase with the increase of A , which is characteristic, for example, of the energy of the Coulomb repulsion of like charges. In this sense, nuclear forces appear to resemble electric forces holding particles in a liquid drop where, due to saturation, the mean bond

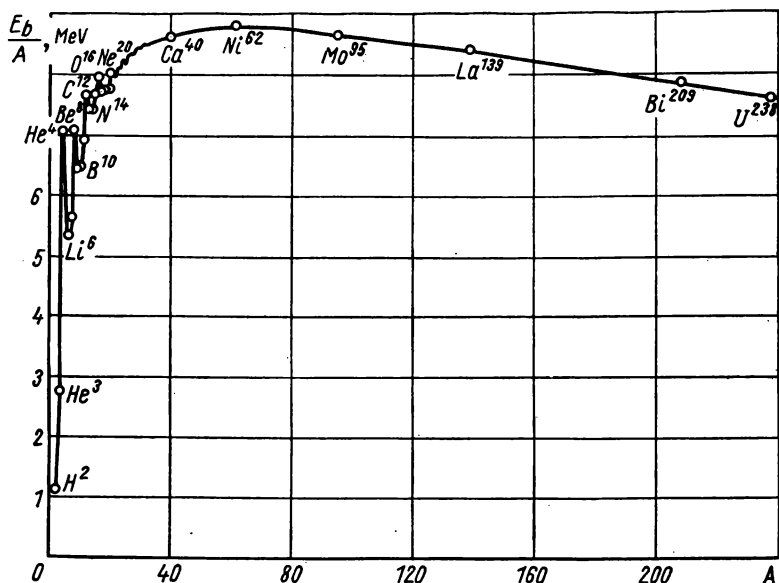


Fig. 2.4. The relation of the mean bond energy of a nucleon in the nucleus to mass number

energy of one particle is also constant. The saturation of nuclear forces prevents nuclei from constricting to very small sizes when the number of particles in the nucleus increases and the nuclear volumes appear to be proportional to the number of particles (2.2).

Figure 2.4 shows the relationship between A and the *mean* nucleon bond energy. But the bond energy of one nucleon, added to the nucleus or separated from it, may change over a great range and, first of all, depends on the parity of the number of protons and neutrons in the nucleus. If the nucleus contains an even number of protons and one more proton is added, the bond energy of the new odd proton proves to be much lower than the mean bond energy of the nucleon in the nucleus. On the contrary, the bond energy of the even proton added to the nucleus is higher than the

mean bond energy of the nucleon. The same is true of the bond energies of the even or odd neutron in the nucleus. It is significant that the parity-nonparity effect manifests itself separately for the numbers of protons and neutrons. The difference in the bond energy between the even proton or neutron and the odd proton or neutron preceding the former in the order of joining the nucleus, is 1-3 MeV. This difference is much higher for some light nuclei, the mean bond energy of two nucleons being 16 MeV. The result is that the mean bond energy of nuclei consisting only of proton and neutron pairs is higher than that of nuclei having one and, the more so, two odd nucleons.

The highest absolute value of the bond energy of one proton is observed in the He^4 nucleus (19.81 MeV), the highest value of the bond energy of the last neutron is also in the He^4 nucleus (20.58 MeV). These values, naturally, refer to the even proton and neutron. The lowest values of the bond energy of one proton and neutron among stable nuclides are for the deuterium and Be^9 which are the values of the bond energies of odd nucleons. Stable nuclides with low values of the bond energy of the last neutron are:

| | |
|---------------------------|----------|
| Be^9 | 1.67 MeV |
| H^2 | 2.22 MeV |
| O^{17} | 4.14 MeV |
| C^{13} | 4.95 MeV |
| Li^6 | 5.66 MeV |

If the bond energy of the proton or neutron added to the nucleus is negative, the absorption of the nucleon by the nucleus and the existence of the corresponding new nucleus are impossible. For example, the bond energy of the He^4 third proton joining the nucleus is 1.97 MeV, and that of the third neutron is 0.96 MeV. That is, the He^4 nucleus can absorb neither protons nor neutrons, and the corresponding nuclei with the number of nucleons five, i.e., Li^5 and He^5 , do not exist in nature. At the same time, the third proton or the third neutron acquires some positive bond energy, if the nucleons of some other kind are also three, i.e., the Li^6 nucleus exists and is stable. Thus, although the atomic nucleus resembles a liquid drop in its properties, the drop-nucleus cannot be produced with an arbitrary proton-neutron composition. This is where the specific features of nuclear forces show up. It turns out that nuclear forces achieve the best saturation in nuclei of any mass numbers if the number of protons equals the number of neutrons, $Z = A - Z$. Thus, the $Z = A - Z$ rule is fundamental. It determines the nuclear stability due to nuclear forces. The additional condition already discussed becomes valid for nuclei with close mass number values. It reads that nuclei with even numbers

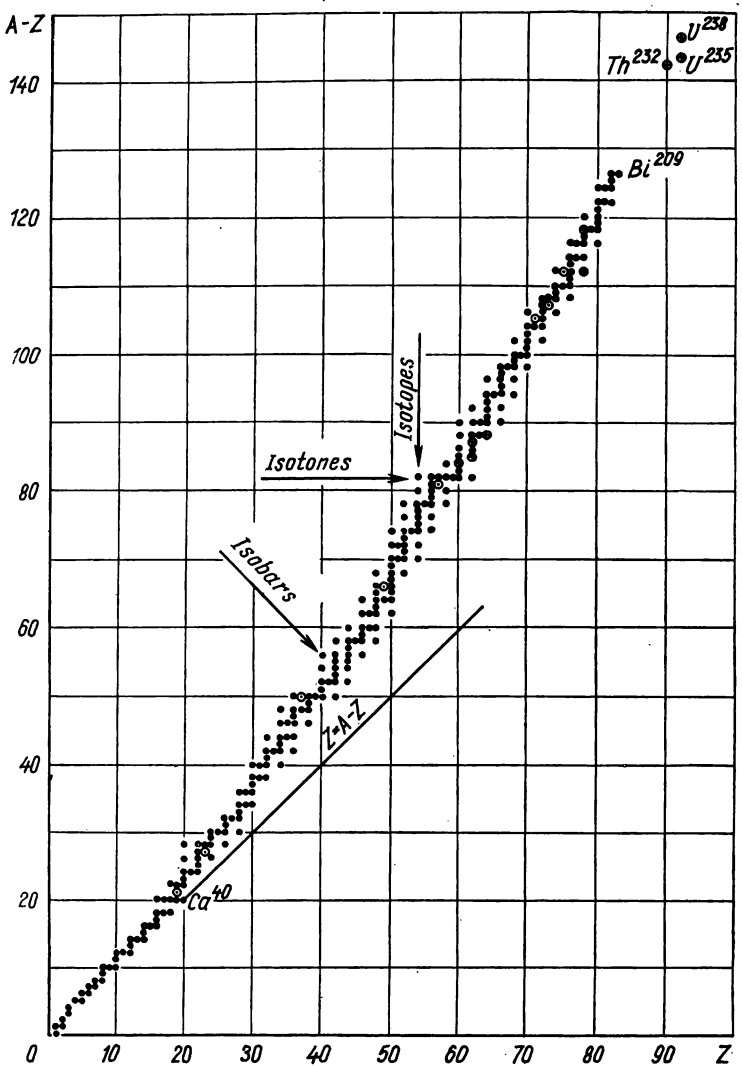


Fig. 2.5. Diagram of proton-neutron composition of stable and slow-decaying nuclides
 ● — stable; ⊕, ⊙ — unstable with respect to α - and β -decay

of protons and neutrons, even-even nuclei, are more stable than nuclei with one nucleon, i.e., nuclei of odd-even and even-odd compositions which, in turn, are more stable than odd-odd nuclei, i.e., nuclei having an odd number of both protons and neutrons,

8. Influence of Coulomb forces. Figure 2.5 shows the proton-neutron composition of naturally found stable or slowly decaying nuclides. It is interesting to note that in nuclear reactions no new stable nuclide has ever been obtained, i.e., all the stable nuclides are in their natural form. It means that nuclides of a proton-neutron composition, not shown in Fig. 2.5, are sure to be unstable to β - or α -decay. Stable nuclides are those which are most tightly bound. However, only a few light nuclides, the last of which is Ca^{40} , satisfy the fundamental rule responsible for nuclear stability, the $Z = A - Z$ rule. The remaining stable nuclides have more neutrons than protons, and the heaviest nuclei have one and a half times as many neutrons as protons. The deviation of the stable nuclide composition from the rule $Z = (A - Z)$ is explained by the influence of the Coulomb repulsion of many protons contained in a small volume. Even at the distance of the radius of action of nuclear forces, the Coulomb forces are weak compared to the nuclear forces, but they do not display their saturation features in nuclei. Therefore, the energy of the Coulomb repulsion increases in proportion to Z^2 , and as protons are accumulated in the nucleus it makes its own ever increasing contribution to the bond energy, thus decreasing its value. The destructive influence of the Coulomb forces leads to a lowering of the specific bond energy of nuclei with mass numbers $A > 60$ (see Fig. 2.4), and also to deviation from the rule $Z = A - Z$ (see Fig. 2.5).

The nuclear composition of stable nuclides with mass numbers $A > 40$ determines two tendencies: the best saturation of nuclear forces when $Z = A - Z$ and the destructive influence of the Coulomb repulsion, rapidly increasing with the increase of Z . The second factor proves to be so important that in nuclei with a very high number of nucleons the bond energy of two protons and two neutrons becomes lower than the bond energy of these four particles in the He^4 nucleus. It is energetically advantageous for such nuclei to lose superfluous nucleons by emitting the He^4 nucleus, i.e., an α -particle. As a result, the last stable nucleus appears to be the nucleus of the Bi^{209} nuclide ($Z = 83$), and all heavier nuclei are unstable to α -decay. The existence of 92 elements in nature is due to three α -active nuclides, namely, Th^{232} , U^{235} and U^{238} , having rather long half-lives compared to the geological age of the Earth ($4.5 \cdot 10^9$ years). Natural nuclides, with Z between 83 and 92, exist as intermediate decay products of the three elements in question.

9. Stability of isobars. The composition of most stable nuclei is defined by the rule $Z = A - Z$, with correction for the Coulomb repulsion of protons. Since there exists the β -decay mechanism, changing the proton-neutron composition of nuclei at constant A , one would expect that, with the given number of nucleons A , there

is only one stable isobar as the most stable, while all the other ones are β -radioactive. This is just the case with the odd A numbers corresponding to the even-odd or odd-even compositions. Fig. 2.6a presents the qualitative relationship between the nuclear mass of an odd A and the proton number Z . The deviation of the number of protons and neutrons (with a constant A) from some optimal value leads to a decrease in the bond energy, i.e., to the increase of the nuclear mass. If the mechanism of transition into the state with lower mass exists, such a transition always occurs spontaneously with the liberation of some free energy. The nucleus

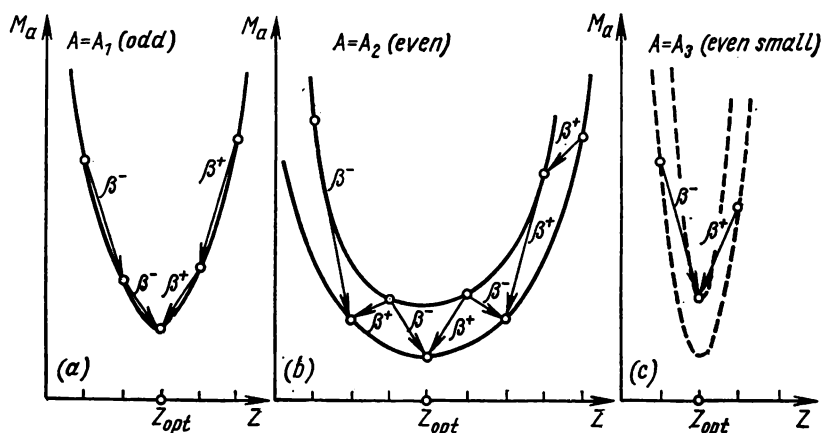


Fig. 2.6. Qualitative relation of the isobaric mass to Z

with excess neutrons undergoes β^- -decay, (3.25), until it acquires an optimal proton-neutron composition as a result of successive decays. A nucleus with excess protons undergoes β^+ -decay, (3.26) or (3.27).

For even mass numbers A , the existence of mainly isobaric pairs is observed, and several stable isobaric triplets are also known. In β -decay of nuclei with even numbers A , even-even nuclei are transformed into odd-odd nuclei, and vice versa. Since the bond energy of odd-odd nuclei is substantially lower than the bond energy of the neighbouring even-even nuclei due to two unpaired nucleons, the mass dependence of a nucleus with the given even A on Z is, in fact, described by two functions shifted along the coordinate axis by some interval (Fig. 2.6b). Therefore, the existence of two or even three stable isobars is possible, each of them having an even-even nuclear composition. The mass of one of the two or three isobars has the least value. However, because of a two unit difference in Z an isobar with the largest mass cannot transform

by decay into an isobar with the least mass. Such transformation could only take place during simultaneous emission of two β -particles by the nucleus (double β -decay). However, if the double β -decay does exist, its probability is so small that it cannot be detected experimentally, and both members of the isobaric pair should be considered stable.

The odd-odd nucleus situated along Z between two stable even-even isobars may undergo both β^- and β^+ decays, in each case of decay, transforming into an isobar with Z either one unit more or one unit less (see Fig. 2.6*b*). Among odd-odd nuclides four stable nuclides are known. All of them are light nuclides with an equal number of protons and neutrons. With a low number of nucleons in the nucleus, the bond energy very rapidly changes with changing the nuclear proton-neutron composition (Fig. 2.6*c*), which is the cause of greater stability of odd-odd nuclides compared to even-even nuclides, the composition of which does not satisfy the rule $Z = A - Z$ at all.

10. β -Decay regions. In the diagram of Fig. 2.5, each value of abscissa and coordinate corresponds to some nucleon composition of the nucleus. But only a small fraction of these values corresponds to stable nuclei which form a narrow path in the diagram. If nuclear reaction gives rise to a nucleus with a proton-neutron composition not coinciding with the composition of a stable nucleus, such a nucleus is unstable and undergoes β -decay in due course. If it is a heavy nucleus, it undergoes β^- or α -decay. Nuclei having an excess of neutrons, compared to stable nuclei, are β^- -radioactive, and those with an excess of protons are β^+ -radioactive. Therefore, it can be said that on the left of the path of stable nuclei in Fig. 2.5, there lies the region of β^- -radioactivity, and on the right, the region of β^+ -radioactivity.

11. Abundance of nuclides. A consequence of the enhanced stability of the nuclei of even-even nuclides is their predominance in nature. Almost two thirds of stable nuclides presented in Fig. 2.5 are of an even-even composition. The only four stable nuclides are H^2 , Li^6 , B^{10} and N^{14} . β -Decay transforms odd-odd nuclides into even-even nuclides, and as the latter are more tightly bound almost all the odd-odd nuclides prove to be unstable to β -decay. The nucleon number parity determines the variety of isotopes and isotones. All multi-isotope elements have even Z . On the contrary, elements with odd Z have no more than two isotopes, and elements with $Z = 43$ and 61 have no stable isotopes at all. The same is true of isotopes with even and odd $A - Z$, Fig. 2.5. Thus, the proton or neutron number parity provides some stability margin, which leads to the possible existence of several stable nuclides differing, respectively, by the number of neutrons for isotopes and by the number of protons for isotones. Finally, the number parity of

neutrons in the composition of heavy nuclei determines their ability of neutron-induced fissioning (Sec. 3.6-2).

Nuclide abundance according to composition parity is the following:

| | |
|---------------------|-----|
| even-even | 161 |
| even-odd | 54 |
| odd-even | 50 |
| odd-odd | 4 |
| <hr/> | |
| Total | 269 |

12. Possibilities of obtaining free energy. The relationship between the mean bond energy per nucleon and the mass number (see Fig. 2.4) points directly to possible ways of producing free energy in nuclear transformations. Energy must be released in the fission of heavy nuclei or in the fusion of light nuclei, as these processes result in the formation of nuclei with greater bond energy per nucleon. Fission of heavy nuclei (Sec. 3.6) is readily induced by neutrons which are in turn produced in fission. Fusion of light nuclei is hindered by the Coulomb repulsion between positively charged nuclei. Since the Coulomb repulsion is minimum with nuclei of the lowest Z (Sec. 2.5-2), it is most easy to realize fusion in the collision of the lightest nuclei such as the nuclei of the H^2 and H^3 heavy hydrogen isotopes. These nuclei are weakly bound and in nuclear reaction with their participation a strongly bound He^4 nucleus is produced and the reaction is accompanied by energy release (Sec. 3.5-14). However, to overcome the Coulomb repulsion high kinetic energy is required in this case as well. If the process is not a separate act of reaction but supposed to be realized in a large mass of the substance, then all nuclei must have such energy as the energy of thermal motion. Even for the hydrogen isotopes, the corresponding temperature exceeds 10^7 °K. The main difficulty of realizing the controlled thermonuclear fusion is to obtain such high temperatures in a gas not strongly rarified, even for a short period of time. Uncontrolled thermonuclear fusion has been realized in the hydrogen bomb, where high temperature results from the explosion of a bomb made of fissionable material.

2.5. Nuclear Forces

1. Basic properties. The nature of the forces holding nucleons in nuclei has not yet fully been clarified. At the same time, much data have been obtained on the physical properties of nuclei, as well as on interaction of free nucleons in collisions over a very wide range of kinetic energies, from 10^{-4} to 10^{10} eV. The analysis of the observed phenomena allows the following conclusions to be

drawn about the forces between nucleons. Nuclear forces are strong attractive forces, acting only at short distances. They possess the property of saturation, due to which nuclear forces are attributed exchange character (exchange forces), nuclear forces depend on spin, not on electric charge, and are not central forces.

2. Coulomb and nuclear potential of nucleus. Nuclear forces are said to be strong forces, in the sense that they are at least 100 times greater than very strong Coulomb forces taken at short nuclear distances of $\sim 10^{-15}$ m. The short range of nuclear forces leads to a strict demarcation of the regions where only long-range Coulomb forces, or only nuclear forces show up as the latter suppress the Coulomb forces at short distances. The dependence of the force on the space coordinates is described by means of the potential. The presence of one of the interacting bodies is expressed through the potential as a function of the distance from the body centre, and the force at the point r , directed from the first body to the second, is found as a potential derivative with respect to the space coordinates at this point. The electric potential φ of the charge Ze is

$$\varphi(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze}{r} \quad (2.13)$$

where ϵ_0 is the electric constant, and the potential energy of the interaction of the charges Ze and e equals

$$U(r) = \frac{1}{4\pi\epsilon_0} e \frac{Ze}{r} \quad (2.14)$$

i.e., it differs from the potential only by the constant and, therefore, the spatial dependences $U(r)$ and $\varphi(r)$ coincide. Hence, instead of the potential, the potential energy of a single or, in this case, elementary point charge is usually used. Then, nuclear forces are also introduced through the potential energy of the nucleon interaction. The positive potential creates repulsive forces, and the negative potential attractive forces. Therefore, the potential energy is positive if it corresponds to repulsive forces, and it is negative for attractive forces. As a result, the potential energy of the point proton interaction with the nucleus may be presented as in Fig. 2.7. The Coulomb repulsion changes abruptly to attraction at the distance of the radius of action of nuclear forces, i.e., at the boundary of the nucleus R . The transition from repulsion to attraction is likely to proceed, though rapidly but continuously, in the region of the space coordinate R . At the same time, the abrupt change of the interaction energy from U_c to $-U_0$ is nearly true, and, to a certain degree of accuracy, the nuclear potential is pictured in the form of a square potential well. The height of the Coulomb barrier U_c (see Fig. 2.7) can be calculated since the nuclear radius is

of a definite value. The value of U_C equals the potential value, Eq. (2.13), with $r = R$ from Eq. (2.3), multiplied by the elementary charge e :

$$U_C = e \frac{1.602 \cdot 10^{-19}}{4\pi \cdot 8.854 \cdot 10^{-12} \cdot 1.23 \cdot 10^{-15}} \times \times \frac{Z}{A^{1/3}} = e \left(1.17 \cdot 10^6 \frac{Z}{A^{1/3}} \right) \approx \frac{Z}{A^{1/3}} \text{ MeV} \quad (2.15)$$

i.e., U_C is approximately equal to 1 MeV in the lightest nucleus and is as high as 15 MeV in the uranium nucleus. The Coulomb

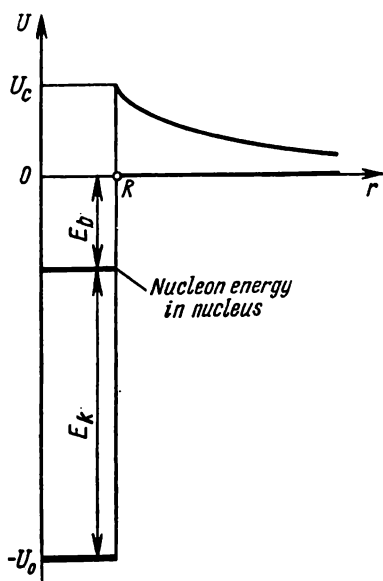


Fig. 2.7. Graphic representation of the nuclear and Coulomb potentials of the nucleus

barrier of the α -particle with the charge $2e$ is twice as high. It should be noted that the Coulomb barrier, Eq. (2.15), refers to the point particle with the proton charge. When calculating the barrier for real nuclei, one should bear in mind that each of the nuclei is characterized by finite radius R . Thus, the Coulomb barrier of the H^2 nuclei of deuterium and of the H^3 nuclei of tritium is about $1/3$ MeV. The Coulomb potential barrier prevents positive charged particles from approaching atomic nuclei and hinders nuclear reaction processes. If the kinetic energy is lower than the potential barrier, then in the nuclear collision, either the Coulomb scattering takes place or a reaction due to the barrier penetration mechanism (Sec. 3.5-13). Neutrons have no electric charge, they are free of the Coulomb inter-

action, and they freely approach nuclei at any energy. The nuclear potential of the neutron appears to be the same as that of the proton. Therefore, the interaction energy of the neutron with the nucleus is

$U = -U_0$ when $0 < r < R$ and $U = 0$ when $r > R$ (see Fig. 2.7)

3. Value and form of nuclear potential. The value of the nuclear potential, U_0 , in mega-electron-volts, i.e., in fact, the value of the potential energy of nuclear interaction, is determined by analysing the available experimental data, the same as the dependence of the

nuclear potential on the space coordinate, r . Nucleons possess wave properties, and for their bound states, there must be satisfied certain relations between the wavelength, λ , Eq. (1.14), and the spatial dimensions of the volume to which the stay of nucleons is limited. Since λ has the sense of the radius of the particle orbit in bound state, λ of the nucleon in the nucleus cannot be greater than the radius of the potential well, $\lambda < R$. The wavelength λ is expressed through the kinetic energy, and, hence, the nucleons of the nucleus must participate in the motion with the energy E defined by the relation:

$$\frac{\hbar}{\sqrt{2m_N E}} < R \quad (2.16)$$

where m_N is the nucleon mass. Relation (2.16) does not bear indications of how many times λ is less than R . It can only give the value of E below which there cannot be any kinetic energy of the nucleon in the nucleus. The absolute value of the potential well depth, U_0 , is higher than that of the kinetic energy, E , by the value of the nucleon bond energy in the nucleus, E_{bd} (Fig. 2.7).

The potential well depth is determined in calculating the nucleon bound states by quantum mechanics methods. In fact, such calculations can directly be performed for the simplest nucleus of the heavy hydrogen atom consisting of two nucleons, a proton and a neutron. Solving the Schrödinger equation with the known potential energy of particle interaction, one can find the energy values of their bound states. The value and character of the spatial dependence of the nuclear potential are not known, but the value of the bond energy of the deuteron is experimentally determined. It follows that in calculations one may select such a kind of potential which would provide an agreement of the calculated value of the bond energy with the value observed in experiment. In the case of the deuteron, the calculation gives $U_0 = 35$ MeV for the square potential well with $R = 2.1 \cdot 10^{-15}$ m. The same value of the depth of the potential well of more complicated nuclei allows one to correctly interpret experiments on neutron-nucleus scattering. Since the mean bond energy of nucleons is about 8 MeV, the kinetic energy of the motion of nucleons in nuclei must approximately be 25 MeV.

The independence of the nuclear potential of the space coordinate, i.e., the constant value of U_0 in the range of $0 \leq r \leq R$ is also substantiated by experimental facts. If the nuclear potential decreased when r approached zero, then there would exist some conditions for states with higher bond energy, since the energy of the bound state is, first and foremost, determined by the value of the interaction potential or by the depth of the potential well. The attenuation of the potential energy would, in addition, mean the

increase of the kinetic energy of the inner motion of the nucleon, i.e., the decrease of λ , and in this case the nucleon could be enclosed in a still smaller nuclear volume. Such a situation, for instance, occurs in atoms in which the radii of the Bohr orbits of electrons close to the nucleus decrease and the bond energy of electrons in these orbits increases with the increase of Z of the nucleus. Nothing of the kind is observed in atomic nuclei. The bond energy of certain nucleons is always near the mean bond energy and, therefore, all the nucleons of the nucleus have closely spaced energy levels. The accumulation of a great number of nucleons in the nucleus does not result in any change in the mean bond energy per nucleon, and the nuclear volumes are not constricted, and prove to be proportional to the number of particles in the nucleus. The analysis of the consequences of excitation of nuclei and nuclear reactions also indicates that the nucleus has neither too much strongly bound nor relatively weakly bound nucleons, which, with the increase of the nuclear volume proportional to A , directly points to the independence of the nuclear potential from the distance when $0 \leq r \leq R$. Some experiments show that there may happen to be some kernel inside the nucleon, and some strong repulsive forces between the nucleon kernels. However, no experiments unambiguously testify in favour of it, and, secondly, this peculiarity may be referred only to the potential of a separate nucleon, not of the nucleus.

4. Potential of a drop of liquid. Thus, the nuclear potential should be considered independent of the space coordinates in the limits of the nuclear volume. This is the potential of a drop of liquid. Just as in a drop of liquid, attractive forces have an effect on nucleons only when $r = R$, and in the range of $r < R$, nucleons may be considered to be free. (Force is the derivative of the potential with respect to the space coordinate, therefore, force is equal to zero if $U(r) = \text{const}$). At the same time the potential of a liquid drop cannot, of course, fully describe the nuclear interaction of nucleons. The explanation of the higher bond energy of nucleon pairs, as compared to the bond energy of the unpaired nucleon, is beyond the limits of the liquid drop model. The non-central character of nuclear forces contradicts the central potential, as shown in Fig. 2.7, which only depends on r . Therefore, the nuclear potential in the form of the square well is an approximation describing only those properties of the atomic nucleus which resemble the properties of a drop of incompressible liquid.

5. Exchange character of nuclear forces. Quantum mechanics explains saturation of forces in interaction of microbodies by the particle exchange mechanism. The knowledge of exchange interaction does not imply the existence of any special kind of forces. Exchange forces are the same electric or nuclear forces only ap-

pearing in a specific capacity resulting from the wave nature of microparticles. Interaction of nucleons in the limits of the meson theories is explained by the virtual meson exchange between them (Sec. 1.10-5); therefore the exchange character of nuclear forces directly results from such representation of their nature. Saturation of nuclear forces disclosed in the analysis of the physical properties of atomic nuclei also implies that nuclear forces must appear as the exchange forces. Finally, the mechanism of the nucleon exchange interaction was successfully observed experimentally. It became possible after such energies were obtained by particle accelerators that greatly exceed the absolute value of the nuclear interaction energy. The exchange interaction explains the unusual nature of neutron-proton and proton-neutron scattering compared to scattering without exchange. In this case, the exchange effect can be observed in experiment because the proton and the neutron are different particles.

Experiments were made with protons and neutrons as bombarding particles. It follows from the laws of energy and momentum conservation that particles of equal masses are scattered at the angle of 90° with respect to each other. On the other hand, if the energy of the neutron and proton interaction is limited, it means that the neutron moving with very high energy cannot in principle transfer a considerable portion of its energy to the target proton. The kinetic energy transferred to the proton will never be higher than the nuclear interaction energy because the limited work of forces corresponds to the finite interaction energy. The action of these forces, therefore, cannot significantly slow down a fast moving neutron. Thus, the neutron momentum must be much greater than the proton momentum after collision. It follows from the law of momentum conservation that in this case neutrons will scatter in the direction of their original flight, i.e., at small angles, and protons, at 90° or even greater angles to them.

Such a nature of fast neutron-proton scattering was proved experimentally. Neutrons scattered forward were actually observed. Along with the neutrons, many protons appeared in the forward flowing beam, and the corresponding low-energy neutrons were detected among protons scattered at great angles. Protons of very high energy were able to appear only as a result of exchange interaction in neutron-proton collision. These are, in fact, neutrons of the original beam which received their electric charges from the target protons. The same picture is observed in bombarding nuclei by protons of high energy. Protons, as well as neutrons formed of protons in exchange interaction, are forward scattered. As a result, angular distribution of scattered neutrons appears as in Fig. 2.8, which shows the dependence of the cross-section on the scattering angle in the laboratory coordinate system. Maxima at 0

and 90° are the characteristics of this distribution. The explanation of the increase of the scattering cross-section, when the scattering angle approaches 90° , may only be given on the basis of the exchange interaction mechanism. If one assumes that at high energies there appears stronger nuclear interaction due to possible approach of nucleons to each other at very short distances, then such a cause could only stop the decrease of the cross-section with the increase of the angle but in no way bring to the maximum at the angle of 90° . At the same time, the nature of the dependence in Fig. 2.8 implies that nuclear interaction is not only of an exchange

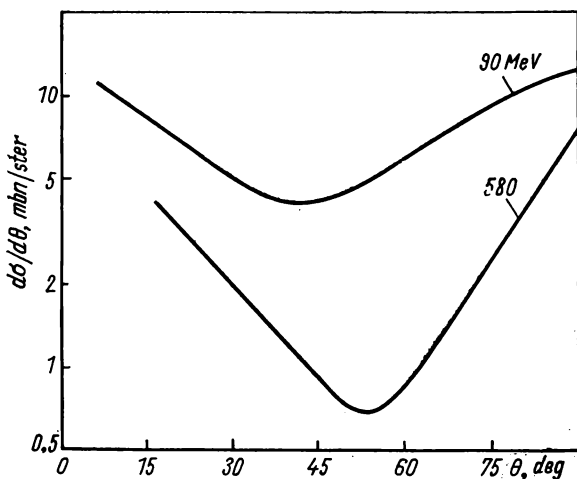


Fig. 2.8. Relation between the (np) -scattering cross-section and an angle at high energies

type. If it were so, the maximum of the scattering cross-section would only be at the angle of 90° and neutrons would not scatter through small angles at all because each of them in the collision would certainly change into a proton. Therefore, analysing experiments on fast neutron-proton scattering or proton-neutron scattering, one should draw a conclusion that nuclear forces may be both of the exchange and the ordinary (non-exchange) type.

6. Spin dependence. Nuclear forces depend on the spin orientation of interacting nucleons. This is, first of all, indicated by the absence in nature of the nuclei of deuterium with a zero spin. All deuterons consisting of one proton and one neutron have a spin equal to 1. It means that mutual nucleon attraction and, hence, the depth of the potential well, $U_0^{\uparrow\uparrow}$, in interaction of two opposite nucleons with parallel spins, are greater than the interaction and

depth of the potential well, $U_0^{\uparrow\downarrow}$, in the case of antiparallel spins. It follows from the deuteron theory and the analysis of the neutron-proton scattering cross-section that the potentials $U_0^{\uparrow\uparrow}$ and $U_0^{\uparrow\downarrow}$ differ at least by a factor of 2 [5].

The most direct proof of the dependence of nuclear forces on spin was experimentally obtained on molecular hydrogen scattering of very slow neutrons with the energy $\simeq 2 \cdot 10^{-4}$ eV. The wavelength of a neutron with such a low energy is longer than the internuclear distance in the hydrogen molecule. Therefore, such neutrons are scattered on two protons of the hydrogen molecule as on a single particle. Since the spins of the two protons of the hydrogen molecule may be oriented both parallel, the orthohydrogen molecule, and antiparallel, the parahydrogen molecule, (Sec. 1.4-11), then, in the first case, scattering occurs in the interaction with a system having spin 1, and in the second case, spin 0. The neutron-orthohydrogen scattering cross-section proved to be 30 times as great as the neutron-parahydrogen scattering cross-section. It appeared possible to separate the effects of the ortho- and parahydrogen molecule scattering when studying the scattering of neutrons on pure parahydrogen and on the mixture of ortho- and parahydrogen at a normal temperature.

The nuclear potential of the proton-neutron pair, $U_0^{\uparrow\downarrow}$, is so low in the absolute value that the formation of the bound state appears to be impracticable in this case. At the same time, the analysis of experiments on neutron-proton scattering shows that it was possible for the bound state to form if the potential well were a little deeper. This state would not be stable since the state with parallel spins has much lower energy. Such a state would be the excited state of the deuteron from which, in case it is formed during external excitation, the deuteron would spontaneously transform to its ground state with the bond energy $E_{bd} = 2.22$ MeV, emitting a γ -quantum. The energy of this state, however, turned out to be above the edge of the potential well by about 0.07 MeV. Such a state cannot be realized and the deuteron has no excited level. At the same time, attraction between the proton and the neutron in the state with antiparallel spins exists, and the energy value, which could correspond to the bound state at some other depth of the potential well, corresponds to the attraction potential $U_0^{\uparrow\downarrow}$. This energy level of the potential $U_0^{\uparrow\downarrow}$ is called the virtual excited level of the deuteron. Its proximity to the value of $U = 0$ leads to a very large scattering cross-section of low-energy neutrons on protons in collisions with antiparallel spins. According to estimates, $\sigma_{\uparrow\downarrow} = 78$ barns. Because of such a value of $\sigma_{\uparrow\downarrow}$ the total neutron-proton scattering cross-section, $\sigma_s = (3/4\sigma_{\uparrow\uparrow} + 1/4\sigma_{\uparrow\downarrow})$,

where the numerical coefficients are the statistical weights, Eq. (1.21), also appears to be abnormally large, and up to energies 10^4 eV, $\sigma_s = 20$ barns (see Fig. 4.23). Owing to the unusually high value of the neutron-proton scattering cross-section, the slowing-down power of hydrogenous substances with respect to neutrons of the energy lower than 1 MeV is great (see Table 5.2).

7. Charge independence. Protons and neutrons are presented quite equally in the atomic nuclei which is proved by the analysis of the bond energy of nuclei of different proton-neutron compositions. Coulomb proton repulsion surely brings about certain peculiarities in the nuclear structure and has an effect on the value of the bond energy. However, given the correction for the Coulomb forces, a pure nuclear interaction is the same for any nuclear pairs, i.e., (p, p) , (p, n) and (n, n) . In addition, the comparison of interaction potentials of these pairs in the same spin states, calculated when analysing the experiments on nucleon-nucleon scattering, also confirms the charge independence of nuclear forces [5].

8. Non-central forces. The field of nuclear forces is not spherically symmetric, which is indicated by electric quadrupole moments of nuclei (Sec. 2.3-3). The force does not only depend on the distance between particles but on the orientation of the radius vector of the particle with respect to the nuclear spin vector. The non-spherical nuclear potential cannot be described by the function depending only on the space coordinate r , as shown in Fig. 2.7, and is represented by tensor.

2.6. Nuclear Models

1. Drop model. The complete theory of nuclear forces presupposes that all the characteristics of any nucleus are obtained and the nuclear transformations are described as solutions of a certain general equation including the potential of nuclear forces strictly formulated from the mathematical point of view. In the absence of such a theory one has to use simplified nuclear models. Each of them is based on some characteristic feature of nucleon interaction, a feature most vividly displayed in the group of phenomena described by the model. The above physical characteristics of atomic nuclei point to some resemblance of the nucleus to a drop of incompressible liquid. How far this analogy may extend is seen from the deduction which can be obtained on the basis of the drop model. One of them is the semiempirical mass formula. It gives quite accurate values of atomic masses as a function of only two most significant parameters of the nucleus, namely, the mass number A and the atomic number Z . But to obtain accurate masses, one has to introduce corrections into the formula. The corrections

are born out by experimental data, not following from the drop model. Fissioning of heavy nuclei is also described in terms of the drop model. Consideration of competition between the forces of the Coulomb proton repulsion and the forces of surface tension of the drop-nucleus allows development of a quantitative theory of fission and prediction of the fission threshold and the stability limit of very heavy nuclei to spontaneous fission [5].

At the same time, non-symmetric nuclear fission due to thermal neutrons contradicts the drop model and is explained on the basis of the shell model.

2. Semiempirical mass formula. Since the atom consists of Z protons, Z electrons and $A - Z$ neutrons, the atomic mass is primarily determined by the mass of the particles it contains

$$M_0 = Zm_H + (A - Z)m_n \quad (2.17)$$

where M_0 is a rough approximation for the atomic mass, and m_H and m_n are the hydrogen atomic mass, i.e., the total proton and electron mass, and the neutron mass, respectively.

The first correction to Eq. (2.17) should allow for the fact that, as the drop nucleus forms from nucleons, the condensation energy is released or that the mass of bound nucleons is lower than the mass of unbound nucleons. As the bond energy of a single nucleon is approximately constant for all nuclei, the mass decrease ΔM_1 is simply proportional to the number of the nucleons of the nucleus:

$$\Delta M_1 = -a_1 A \quad (2.18)$$

where a_1 is the proportionality factor.

The stability of the drop is assured by the forces of surface tension which appear as the nucleons on the surface cannot use all their possible bonds in contrast to the nucleons inside the nucleus. This means that the nucleons on the surface are less strongly bound than the nucleons beyond the surface layer. Therefore, the correction (2.18), identical for all the nucleons, does not allow for the surface effect which gives rise to somewhat larger mass of the nucleons of the surface layer. The number of the surface nucleons is proportional to the surface of the nucleus which can be expressed through the mass number by means of formula (2.2). As a result, the second correction to the atomic mass is obtained in the form

$$\Delta M_2 = +a_2 A^{2/3} \quad (2.19)$$

where the factor a_2 includes a constant in the expression for the relation between the radius of the nucleus, its surface and the number of the surface nucleons, as well as the correction to the mass per one surface nucleon. With the increase of the number of particles in the nucleus, the portion of the surface nucleons all the

time decreases so that the mean bond energy per nucleon somewhat increases with the increase of A . It is this condition that determines some increase of the bond energy over the range from O^{16} to Ni^{62} , as shown in Fig. 2.4.

The Coulomb proton repulsion attenuates the bond energy of nucleons in the nucleus and increases the atomic mass. The third correction should be the energy of electrostatic interaction of Z protons in the nuclear volume, expressed in mass units. In the assumption of uniform distribution of the electric charge Ze over the volume of a sphere with the radius R , this energy is equal to

$$U = \frac{3}{20\pi\epsilon_0} \cdot \frac{(Ze)^2}{R} \quad (2.20)$$

where ϵ_0 is an electric constant. Substituting Eq. (2.3) and expressing U in mass units give

$$\Delta M_3 = + a_3 \frac{Z^2}{A^{1/3}} \quad (2.21)$$

where a_3 is a proportionality constant.

Nuclear forces attain the highest saturation when the number of protons is equal to that of neutrons. The deviation from the condition $Z = A - Z$ results in the attenuation of the bond energy and in the increase of the atomic mass. This correction cannot be explained by the nuclear drop model and is best presented in the form:

$$\Delta M_4 = + a_4 \frac{(A/2 - Z)^2}{A} \quad (2.22)$$

where a_4 is a constant, and ΔM_4 vanishes when $Z = A - Z$ and is equal to a positive value when deviating from this condition to any side.

One should also take into account the increase of the bond energy when proton or neutron pairs are formed in the nucleus. Nuclei of the even-even composition have the highest bond energy, nuclei of the odd-odd composition, the lowest bond energy, and the bond energy of nuclei of the even-odd and odd-even composition takes the intermediate position. The correction is introduced in such a way that it is equal to zero for the even-odd and odd-even nuclei and, hence, is negative for the even-even and positive for the odd-odd nuclei. Analysis of the bond energies with respect to the parity of the numbers of protons and neutrons results in the following dependence of the correction in question on the mass number with the proportionality constant (Table 2.2):

$$\Delta M_5 = a_5 A^{-3/4} \quad (2.23)$$

The correction allowances (2.18) through (2.23) to the total mass of atomic structural particles, (2.17), yield the following

Table 2.2
Constant a_5 in Formula (2.23)
for Nuclei of Different
Compositions

| A | Z | $A-Z$ | $(m_u \text{ units})$ |
|------|------|-------|-----------------------|
| Even | Even | Even | -0.036 |
| Odd | Even | Odd | 0 |
| Odd | Odd | Even | 0 |
| Even | Odd | Odd | +0.036 |

expression with indefinite coefficients for the atomic mass $M(A, Z)$:

$$M(A, Z) = Zm_H + (A - Z)m_n - a_1A + a_2A^{2/3} + a_3\frac{Z^2}{A^{1/3}} + \\ + a_4\frac{(A/2 - Z)^2}{A} + a_5A^{-1/4} \quad (2.24)$$

To define the constants a_i , one should apply formula (2.24) to five atoms of the known masses and find five unknowns from the five equations obtained. Since many precise atomic masses are known, equations for the definition of constants are more numerous than the constants themselves. It does not only allow one to define the constants but also to find their weighted values in the best way satisfying the atomic masses with any A and Z in the vicinity of stable atoms. The constant a_3 can be calculated directly. Upon substituting (2.3) into (2.30), the factor before $Z^2/A^{1/3}$ is expressed in joules, and if it is divided by the energy equivalent m_u in joules, then a_3 will be obtained in mass units. Since the mass formula, however, gives accurate results, a_3 is found along with other constants and is then used to calculate the nuclear radius with the given A and Z . Such a procedure is one of the techniques of finding nuclear radii. As a result, the mass formula acquires the form

$$M(A, Z) = 0.99176A - 0.00084Z + 0.019A^{2/3} + 0.00076\frac{Z^2}{A^{1/3}} + \\ + 0.10\frac{(A/2 - Z)^2}{A} \pm 0.036A^{-1/4} \quad (2.25)$$

where the atomic mass is expressed in mass units in the C^{12} scale. Because of the last term, formula (2.25) has different forms for nuclei with odd A , and for even-even and odd-odd nuclei, which is qualitatively reflected in Fig. 2.6. The mass formula gives the atomic mass values to approximately within 0.01% or higher, the

accuracy being the higher, the heavier the atoms, to whose nuclei the liquid drop model is applicable with best approximation. The accuracy of the mass formula can be improved if some other features of the atomic nuclear structure are considered [11].

3. Nuclear shell model. Like atomic electrons, nucleons of nuclei form proton and neutron shells. The shell formation is the consequence of the Pauli principle, according to which two particles with spin $\hbar/2$ cannot be in the same quantum states. Possible bound states of particles (for instance, possible electron orbits in the Rutherford-Bohr atom) are usually distinguished by their interaction energy, i.e., some definite energy level corresponds to each state. At the same time, groups of states exist within the limits of which energy difference is small or is absent at all, while the energy difference between the groups is relatively great. The bound particle tends to occupy the level of the lowest total energy, i.e., of the highest bond energy. But since in each state there may be only one particle with spin $\hbar/2$, then in the accumulation of particles in a compound system, gradual filling of the energy levels takes place according to the groups in the order of the increase of their total energy. The shell is formed by particles in states with close energy values. The shell is called closed or filled if all the shell levels are occupied by particles. A closed shell is of enhanced stability, i.e., it has a higher bond energy even in comparison with the bond energy of the same shell before its complete filling. On the contrary, the bond energy of a single particle which has made a transition to the next shell is always relatively low even in comparison with its own bond energy after filling up other levels in this shell. Thus, systems with the number of particles exactly corresponding to closed shells must possess some specific properties determined by the unusual stability of particle configurations in their composition. Closed inner shells always correspond to the filled outer shells.

The properties of atomic electron shells are well known. If the number of electrons in the atom equals 2, 10, 18, 36, 54 or 86, then the outer shell is closed and is of such a stability that the electron exchange with other atoms is never energetically advantageous. Therefore, neither chemical reactions nor the formation of homopolar molecules are feasible. The above numbers are the atomic numbers of inert gases. Analogous numbers of protons and neutrons, to which particular nuclear properties also correspond, were found in atomic nuclei as well. But these numbers are different:

$$2, 8, 20, 50, 82, 126 \quad (2.26)$$

They have been called *magic numbers*. The difference between the magic nuclear numbers and the corresponding numbers of electron shells is determined by the specific character of nucleon interac-

tion in the nucleus. The nuclear potential formed by all the nucleons of the nucleus has some other space dependence (see Fig. 2.7) than that of the atomic Coulomb potential (2.14). Because of that, a shift of the same quantum states along the total nucleon energy scale occurs. In addition, strong spin-orbital interaction is characteristic of nucleons. This interaction leads to a great energy difference of states with different mutual orientation of the nucleon spin s and of the mechanical moment l defining the orbital motion of the same nucleon in the nuclear potential well. The interaction energy is higher, i.e., the total energy is lower, with parallel orientation of s and l , or in the state with $l + 1/2$, than in antiparallel orientation, or in the state with the total mechanical moment $l - 1/2$. In electron shells, this effect is negligible and is of the opposite sign, i.e., it results in high interaction energy in the state with $l - 1/2$. On the contrary, the value of the level splitting, with the given l , is high for nucleons and rapidly increases with the increase of l . Therefore the states with $l + 1/2$ and $l - 1/2$ appear in different nucleon shells already with $l = 4$ or more. The account of the above-mentioned characteristics of nucleon interaction allows one to explain the magic nuclear numbers, (2.26).

4. Substantiation of shell model. Atomic nuclei containing the total number of protons and neutrons equal to one of the magic numbers, (2.26), are called *magic nuclei*. As these nuclei have closed proton or neutron shells, or both, they are of increased stability. This unusually high stability results in a number of properties which were initially detected experimentally and were the foundation for the nuclear shell model. The bond energy of such nuclei is especially high even in comparison with the neighbouring even-even but not magic nuclei. If the number of nucleons exceeds the magic number, a discontinuous decrease of the bond energy is observed, which, in particular, leads to an abnormally low bond energy of the neutron that is superfluous in comparison with the magic number. As a result, nuclei with the magic number of neutrons always have very small cross-sections of neutron radiative capture. On the other hand, if β -decay results in a nucleus with a neutron superfluous in comparison with the magic number, then the excitation energy left in the nucleus after β -decay often appears to be sufficient for the ejection of a neutron by the nucleus, i.e., the so-called delayed neutron (Sec. 3.3-9). Under other conditions, this excitation energy is not enough to emit a neutron, and then β -decay resulting in an excited product-nucleus gives rise to the emission of γ -quanta alone.

If a chain of α - or β -decays includes a nucleus with the magic number of protons and neutrons, then the decay resulting in the magic nucleus is always accompanied by an abnormally great release of energy, which directly points to the increased bond energy

of the magic nuclei. The stability range of heavy nuclides (see Fig. 2.5) is also determined by especially high stability of the magic nuclei. The lead isotopes with $Z = 82$ or the Bi^{209} nuclide with $A - Z = 126$ are the last stable nuclides and the final decay products of all heavier α -active nuclides. Apart from the heaviest nuclei, known is the α -activity of nuclides with the mass numbers about 150 whose decay is accompanied by the formation of nuclei with the magic number $A - Z = 82$. Finally, fissioning of nuclei into two unequal parts due to thermal neutrons can also be attributed to the fact that it is more energetically advantageous to have such fragments in the composition of which strongly bound closed neutron shells with neutron numbers 50 and 82 would appear. Meanwhile, from the point of view of the drop model, fissioning into equal parts is most advantageous. Nuclei with the magic number of protons or neutrons possess a certain stability margin, due to which nuclei preserve their stability with a great variety of the numbers of nucleons of some other kind. Thus, an element with $Z = 50$ (Sn) has the highest number of stable isotopes, 10, 3 of which are of the even-odd composition. That is also the only case among all the elements. The highest number of stable isotones corresponds to the magic number $A - Z = 82$. The last stable nucleus satisfying the rule $Z = A - Z$ is the twice magic nucleus of the Ca^{40} nuclide.

5. Applications of shell model. The nuclear shell model does not only explain the specific properties of the magic nuclei but, in some cases, can be used to calculate spins and magnetic moments of nuclei with unfilled nucleon shells. This model helps one to understand the law of the abundance of nuclear isomers among all the nuclides (Sec. 3.4-3) because it allows one to determine the mass number regions in which one should expect a great difference in the mechanical moment of the first excited and the ground states of the nucleus. At the same time, the nuclear shell model, like any other nuclear model, is not universal or useful in calculating the characteristics of nuclei whose form markedly deviates from the spherical one.

6. Model inconsistencies. The nuclear shell model and the drop model are contradictory, in some sense. It is anticipated with the shell model that the nucleons execute an ordered motion along their orbits with certain momenta l , while the motion of nucleons in the drop-nucleus must be accompanied by continuous collisions with other nucleons, which would seem to prevent nucleons from moving along their orbits. The nucleus, however, is a quantum-mechanical system with a definite *discrete* set of energy levels. All the lower energy levels are occupied by particles. Therefore, the transition to some other energy level means the transition only to an unoccupied level or a level with higher energy than that of any of

the occupied levels. But that is impossible without obtaining the corresponding energy from outside. Thus, nucleons of the nucleus in the ground energy state cannot exchange their energy, i.e., no collisions can occur in the ordinary sense. If one assumes two protons or neutrons to have exchanged their quantum states, then, on account of the principle of indistinguishability of identical micro-particles and in contradiction to classical physics, the new state of the two particles is the same initial state. Thus, the assumption that such a process is collision is of no sense by itself. Otherwise stated, in the nuclear potential well, nucleons are under some specific quantum conditions to which the concepts of the classical liquid drop where the energy of particles is continuously changing are not applicable. Therefore, the nuclear drop model does not exclude the existence of nucleon shells just as the existence of nucleon shells does not exclude the agreement of certain nuclear properties with the properties of a liquid drop.

7. Other models. Each model represents a certain group of nuclear properties, and beyond the limits of its applicability it cannot be conducive to understanding any phenomena. Therefore, various nuclear models exist which are intended to describe the ground or excited nuclear states, and the nuclear transformation processes. The shell and drop models refer to the ground nuclear states. Their development is embodied in the model of cooperative motion or a generalized nuclear model adapted to describe the properties of non-spherical nuclei and their low-excited states. The emission of nucleons by highly-excited nuclei is considered in the terms of the statistical model as a result of the random transfer of the excess energy of the nucleus to a single particle. Collisions of particles with nuclei are described in the model of the black nucleus or in the improved optical model where the nucleus is considered to be a body capable of absorbing, refracting and scattering a wave incident on it. Development of a universal model will only be feasible when the nature of nuclear forces is completely understood.

2.7. Excited Nuclear States

1. Excitation and decay. The atomic nucleus is usually in its ground energy state. It means that all the nucleons of the nucleus occupy the lowest energy levels. At the same time, the number of possible quantum states of nucleons is arbitrarily great. The transition to the state of a higher energy value is, however, possible only under some external influences when the required energy is transferred to the nucleus, for instance, when some particle collides with the nucleus or when the nucleus absorbs a γ -quantum. The nucleus having excess energy is called an *excited* nucleus. The excitation period is usually short, and in 10^{-14} s or less from the

moment of the energy absorption, the nucleus spontaneously transforms to its ground state (it decays).

If the excitation energy is so high that it exceeds the bond energy of the nucleon in the nucleus, then the transition to the ground state occurs mainly by emitting a nucleon which can carry away all the excitation energy, spending part of it, equal to the bond energy (about 8 MeV), on the work against nuclear attractive forces. This nucleon is most often a neutron because neutrons have no Coulomb potential barrier (see Fig. 2.7) that prevents protons

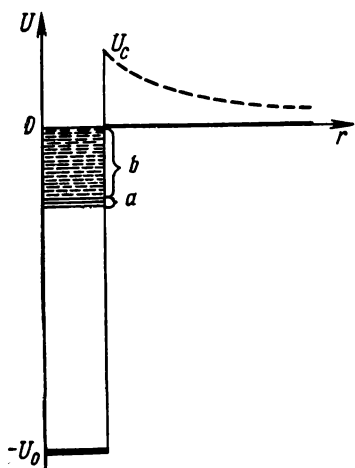


Fig. 2.9. Scheme of energy levels of nuclear nucleons (a) occupied levels; (b) unoccupied levels

both from penetrating into and escaping from the nucleus. Generally speaking, the transition to the ground state is performed by another nucleus having the number of nucleons less by unity. If the nuclear excitation energy is lower than the bond energy of the nucleon, then the transition to the ground state is performed by emitting one γ -quantum or, as is often the case, is a result of successive emission of several γ -quanta that carry away all the excitation energy. The γ -quantum participates only in electromagnetic interactions (Sec. 1.7-5). Therefore, γ -quantum emission is accompanied by an electric or magnetic transition in the atomic nucleus. As electric forces are much weaker than nuclear forces and the processes under their influence proceed slower than those due to

nuclear forces, then, if it is possible to transfer excitation energy to the nucleon, the nucleus usually emits a nucleon, not a γ -quantum though in this case γ -quanta emission is not only possible but under certain conditions even predominates over nucleon emission.

2. Energy spectrum of nucleus. Figure 2.9 schematically illustrates discrete energy levels of the nucleons in the nucleus, some of which having the lowest energy values are occupied by nucleons in the ground state of the nucleus. When excited externally, one or many of the nucleons of the nucleus may occupy higher levels. Since the nucleon levels are separated by finite energy intervals, the nucleus cannot receive any quantity of energy but only strictly definite portions precisely corresponding to the energies of nucleon transitions from the lower to the higher states. The values of these energy portions, referred to the nucleus as a whole, constitute a

system of nuclear excited levels, or the nuclear energy spectrum (Fig. 2.10).

The lowest quantity of energy that the nucleus can absorb corresponds to its first excited level. The first excited level is associated either with the transition of one nucleon into the nearest unoccupied state, which is characteristic of light and spherical nuclei, or with excitation of the lowest-frequency oscillations of a group of nucleons in an unfilled shell, which is sometimes observed in non-spherical intermediate and heavy nuclei. If the excitation energy is much higher than the energy of the first excited level, then a lot of nucleons are excited and the outer closed shells whose nucleons receive the excess energy, may be broken. The total excitation energy is distributed between many nucleons and may be several times greater than the bond energy of a single nucleon. Continuous energy exchange between nucleons does not allow the nucleus to rapidly release its excess energy, and the excited nucleus lives long enough (up to 10^{-14} s) in comparison with the time characteristic of nuclear interaction (10^{-23} s). Only the very weakly-bound nuclei of the lightest nuclides H^2 , H^3 , He^3 have no excited levels. Within the potential wells of these nuclei there is not a single unoccupied nucleon level, and the minimum energy which can be transferred to the nucleus is the bond energy of a single nucleon, i.e., the energy of nuclear destruction. The energy spectra of other nuclei are the more complex, the heavier the nucleus.

3. Features of nuclear spectra. Since there does not exist any comprehensive theory of nuclear forces, the prediction of nuclear spectra is impossible.

Experimental data show that there is no order in the arrangement of the nuclear energy levels, nor there practically exist spectra with a similar structure, which is characteristic of atomic spectra. Therefore, no empirical relations uniting nuclear levels into some groups have yet been obtained. Strong nucleon interaction and the absence of the central body in the nucleus determine the complex character of the spectra. The addition of every new nucleon to the nucleus leads to another mean nuclear potential and, hence, to another energy spectrum of the nucleus.

At the same time, experience reveals some common regularity of nuclear spectra. It may be characterized by the value of some energy interval-averaged distance between the levels. The higher the excitation energy and the more nucleons in the nucleus, the less the mean distance D between the levels and the closer the levels of the nucleus along the energy scale.

Most experimental data either refer to the lowest excited levels of the nucleus or to the levels disposed somewhat above the excitation energy equal to the neutron bond energy. The lowest levels are observed when γ -quanta are emitted by nuclei excited in

radioactive decay or in collision with particles, and the levels near the neutron bond energy are observed when neutrons are absorbed or scattered by nuclei. One-dimensional diagrams of Fig. 2.10, where the excitation energy is laid off along the y-axis, schematically present the spectra of the light and heavy nuclei. The nuclear ground state E_0 corresponds to the zero excitation energy, and each energy level of the nucleus is marked by the corresponding line. The first excited level E_1 of light nuclei ($A < 50$) is at the

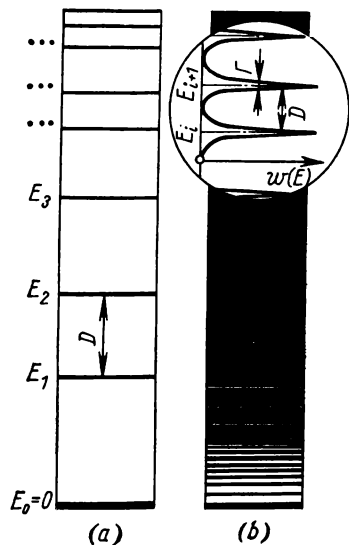


Fig. 2.10. Scheme of nuclear levels of light (a) and heavy (b) nuclei

energy ~ 1 MeV, while E_1 of heavy nuclei ($A > 200$) is ~ 0.1 MeV or somewhat lower. The distances between the first levels are of the order of the very first level because the value of the first level is its distance from the ground state E_0 . When the excitation energy increases, the mean distance between the levels is decreased. With light nuclei, however, even at the energy equal to the neutron bond energy (8 MeV), the distances between the levels still remain great (10^4 to 10^5 eV). At such excitation energy, heavy nuclei have the distance about 1 to 10 eV between the levels.

With a further increase of the excitation energy, the levels come even closer to each other, which finally results in the disappearance of the discrete structure of the nuclear energy levels, and the nuclear spectrum becomes continuous. It is observed in nuclei of different mass numbers, at

the excitation energies about 10 to 20 MeV. Merging of the discrete levels is also promoted by that the energy values of the nuclear levels are not strictly definite but have some width Γ . When the level widths become comparable with the distances between them, the nuclear energy spectrum ceases being discrete and the nucleus can absorb an arbitrary quantity of energy.

4. Level widths. In Fig. 2.10b, the excitation energy axis is added by the x-axis, along which the probability $w(E)$ of the formation of the excited state of the nucleus as a function of energy is laid off. This probability has its maximum and rapidly decreases when the energy deviates from its most probable value. The width of the level, Γ , is the distance, along the energy scale, between those values for which the probability of the formation of the excited state is half the maximum. The existence of the width of discrete energy

levels stems from the corpuscular-wave nature of microparticles (Sec. 1.4-8). The widths of energy levels are experimentally observed as resonance widths in the particle interaction cross-sections-to-energy ratios (Sec. 4.5-3). The width value depends on the individual properties of the level but it increases with the increase of the excitation energy. When nuclei absorb slow neutrons, levels of the width ~ 0.1 eV are observed, which is much less than the distances between them even in the excitation energy range of the order of the neutron bond energy (Sec. 2.7-3).

5. Spins of excited levels. Nuclear excited states appear when nucleons transfer to unoccupied levels, each of these nucleons having its own mechanical moment. The mechanical moments of excited nucleons are summed up and form the spin of the nuclear excited level J . Since the mechanical moments of relative motion are expressed only by integral units of \hbar , the excited level spins are half-integral in nuclei of the even-odd and odd-even nucleon compositions, and they are integral in nuclei of the even-even and odd-odd compositions (Sec. 2.3-1). In nuclei of the even-even composition, the spin must equal zero only in the ground energy state.

CHAPTER THREE

NUCLEAR TRANSFORMATIONS

3.1. Radioactivity

1. **Definition.** Radioactivity is spontaneous emission of nuclear radiation by a substance. This radiation occurs during α - or β -transformations of atomic nuclei as well as during other nuclear decays, i.e., in transitions of excited nuclei into their ground energy states, in spontaneous fission. All spontaneous nuclear transformations have some regularity in common: the character of the behaviour in time of great numbers of unstable or excited nuclei and, consequently, the amount of their decay products, i.e., radiation, is the same for any type of decay, as all the decays obey the probability laws. The constants of the corresponding time distributions always have the same physical meaning whether the decay takes place in a minute fraction of a second or lasts milliards of years.

2. **Decay law.** The main assumption from which follows the behaviour in time of a number of radioactive nuclei, consistent with experiment, is that the decay of any nucleus of a given kind during a certain time interval is equally probable. Hence, the mean fraction of nuclei which have undergone decay is constant for equal time intervals as the fraction of decays of a great number of nuclei is the probability of one nucleus decay. Stated otherwise, at any time the number of decays per unit time is proportional to the number N of available radioactive nuclei (decay law). As the number of decays per unit time is the decrease of the number of radioactive nuclei per unit time ($-dN/dt$), then

$$-\frac{dN}{dt} = \lambda N \quad (3.1)$$

where λ is the proportionality factor called a *decay constant*. The decay constant expresses the percentage of decays per unit time ($1/N$) ($-dN/dt$), i.e., it shows how fast the decay of nuclei of a given kind takes place.

Integration of relation (3.1) results in the expression

$$N(t) = N_0 e^{-\lambda t} \quad (3.2)$$

where N_0 is the number of radioactive nuclei, or atoms, at some arbitrary time taken to be the count onset, i.e., at time $t = 0$.

Thus, the number of radioactive atoms that have not yet decayed decreases with time, following an exponential law (Fig. 3.1). As the number of decays is always proportional to the number of radioactive atoms available, this value should also undergo change in time, following the same exponential law.

Designating the rate of decay, i.e., the number of decays per unit time $\mathcal{J}(t)$, and taking into consideration that $\mathcal{J}(t) = -dN/dt$, we obtain

$$\mathcal{J}(t) = \lambda N(t) \quad (3.3)$$

or substituting (3.2) into (3.3) we have

$$\mathcal{J}(t) = \lambda N_0 e^{-\lambda t} = \mathcal{J}_0 e^{-\lambda t} \quad (3.4)$$

where, in conformity with (3.3), \mathcal{J}_0 is the decay rate at the initial time instant, $t = 0$. Comparison of (3.4) and (3.2) shows that the dependence of the decay rate and the number of radioactive atoms on time is the same which is shown in Fig. 3.1. The decay constant λ is the only parameter of time distributions (3.2) and (3.4).

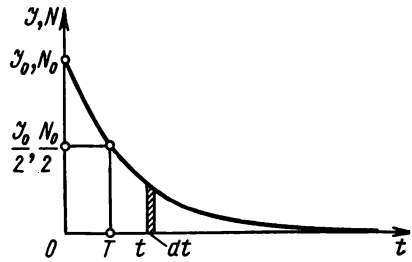


Fig. 3.1. Radioactive-decay curve

3. Decay characteristics. The mean life-time of a radioactive atom can be calculated on the basis of time distribution (3.2). According to the definition of the mean, the total time of the existence of all decayed atoms divided by the number of atoms which have undergone decay is the mean life-time. Only those atoms did live the time t which, at the time t within the interval dt , (see Fig. 3.1), underwent decay. The number of these atoms equals $(-dN/dt)dt$, their total time is $t(-dN/dt)dt$, and their mean life-time is

$$\tau = \frac{1}{N_0} \int_0^{\infty} t \left(-\frac{dN}{dt} \right) dt = \lambda \int_0^{\infty} t e^{-\lambda t} dt = \frac{1}{\lambda} \quad (3.5)$$

It is evident that the mean life-time is the time during which the number of radioactive atoms as well as their decay rate decrease by a factor of e . The same meaning has the mean life-time in all other decay processes.

In practice, the use of the mean life-time is inconvenient because the ratio of decay rates divided by the time τ is not an integer. Therefore, another time characteristic, *half-life period* T , is used. By

definition the half-life period is the time during which the number of radioactive atoms or the rate of decay decreases by a factor of 2 (see Fig. 3.1). The relation between T and the decay constant λ is obtained from the relationship

$$\frac{\mathcal{J}(T)}{\mathcal{J}_0} = e^{-\lambda T} = \frac{1}{2} \quad (3.6)$$

where

$$T = \frac{\ln 2}{\lambda} = \tau \ln 2 = 0.693\tau \quad (3.7)$$

Each constant, λ , τ or T , can characterize the radioactive substance. Experimental determination of any of them is the determination of the time distribution parameter, (3.2) or (3.4). If the life-time of the radioactive substance is very great, then it is impossible to find out a change in time of the number of decays. On the other hand, a slowly decaying substance can be obtained in great amounts, and by measuring N and $(-dN/dt)$, i.e., the number of radioactive atoms (from the mass of a sample) and the rate of decay (from the number of emitted particles, i.e., decay products), one can determine λ [see (3.1)]. If the radioactive substance is short-lived, so that the very fact of its formation can be found only by the onset of radioactivity, in this case it is easier to measure the half-life T by the decrease in time of radioactive radiation (see Fig. 3.1). Relation (3.1) allows obtaining any of the constants if one of them is found in experiment.

4. Statistic character of decay. The mean rate of decay changes with time, following (3.4). At the same time, the numbers of decays measured experimentally never precisely follow this relation. The decay per unit time of some number of atoms is an accidental event. That is why the numbers of decays are subject to statistical fluctuations. However, occasional deviations of the decay numbers from their mean value are within the limits defined by the laws of statistics [12], and their relative values are the lower, the greater the absolute value of the decay rate. The results of the experiment are in excellent agreement with expression (3.4) to the accuracy of the correction for the statistical spread of the decay rate values.

5. Radioactivity accumulation. If radioactive atoms result from a nuclear reaction, their number can be determined at any time on condition that the rate of their formation is known. The reaction cross-section σ and the particle flux density Φ being known, the number of radioactive atoms produced in unit volume of the substance in unit time is

$$Q = \Phi \sigma N' \quad (3.8)$$

where N' is the concentration of target nuclei (1.2). If one takes into consideration the production and decay of radioactive atoms, their balance in unit time in unit volume of a substance at any time can be expressed by the relation

$$\frac{dN}{dt} = -\lambda N + Q \quad (3.9)$$

where N is the number of radioactive atoms in unit volume. The term with the minus sign describes the number of decays in unit time, and that with the plus sign, the number of productions in unit

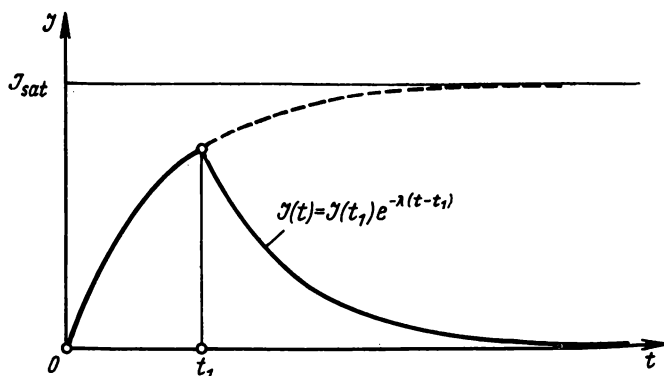


Fig. 3.2. Build-up and decay curve of radioactive substance

time. The competition between these two processes defines the sign of the derivative, that is, it shows whether the number of radioactive atoms decreases or increases in time unit.

The following expression is the solution of equation (3.9):

$$N(t) = \frac{Q}{\lambda} (1 - e^{-\lambda t}) = N_{sat} (1 - e^{-\lambda t}) \quad (3.10)$$

where N_{sat} is the ultimate number of radioactive atoms, with given Q and λ . The decay rate of produced atoms or the activity of the substance unit volume, by (3.3), equals

$$J(t) = Q (1 - e^{-\lambda t}) = J_{sat} (1 - e^{-\lambda t}) \quad (3.11)$$

where J_{sat} is the highest attainable activity, or saturation activity, just equal to the production rate of radioactive atoms Q . The saturation during activation means that the number of produced atoms equals the number of decaying atoms (dynamic equilibrium). In this case, $dN/dt = 0$ [see (3.9)], and the number of radioactive atoms as well as their decay rate do not change in time. The relation (3.10) or (3.11) is given in Fig. 3.2 which shows the activation curve during time t_1 . After the elapse of this time the activa-

tion discontinues and only the decay of a radioactive substance takes place, in conformity with (3.4).

The ultimate number of radioactive atoms, N_{sat} , in a sample is the greater, the more intense is the source of their formation, Q . When Q is the same, the ultimate number is greater if the half-life is greater, or if λ is lower. True, the activity limit of the sample, \mathcal{I}_{sat} , does not depend on λ and equals the production rate of radioactive atoms, Q . The saturation activity is achieved within a longer time with a lower λ . However, the induced substance activity with a lower λ is accordingly retained longer on account of a great number of accumulated radioactive atoms.

6. Transformation chain. As a result of radioactive substance decay, both a stable and a radioactive substance can be formed. In the latter case, decay chains appear. The balance of radioactive atoms is defined in this case by the following equations:

$$\left. \begin{aligned} \frac{dN_1}{dt} &= -\lambda_1 N_1 \\ \frac{dN_2}{dt} &= -\lambda_2 N_2 + \lambda_1 N_1 \\ \frac{dN_3}{dt} &= -\lambda_3 N_3 + \lambda_2 N_2 \\ &\dots \dots \dots \end{aligned} \right\} \quad (3.12)$$

where the subscript 1 refers to the original parent radioactive substance and the subscripts 2, 3, ..., to the daughter substances. The first equation in system (3.12) is the same as (3.1). The atom balance of each daughter substance is defined by the rate of inherent decay, proportional to the number of available atoms of the daughter substance and by the rate of production equal to the number of precursor decays per unit time. The solution of each equation (3.12) depends only on the kind of solution of the preceding equation. The solution of the first equation is Eq. (3.2). The general solution of the second equation obtained after substituting the function N_1 in its explicit form into (3.2) is as follows:

$$N_2(t) = N_{20}e^{-\lambda_2 t} + N_{10} \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \quad (3.13)$$

where N_{10} and N_{20} are the atomic numbers of the first and the second radioactive substance at the initial time $t = 0$. The addend in (3.13) is the change in time of the number of atoms of the second substance which existed before the time $t = 0$ and at the time $t = 0$, equal to N_{20} . The augend is the number of atoms of the second substance which appeared as a result of the first substance decay beginning from the time $t = 0$. It is defined by the number of atoms of the first substance at the time $t = 0$ and by the decay

tote when $\tau_2 = \tau_1$ and the dependences $N_2(t)$ and $\mathcal{J}_2(t)$ are

$$N_2(t) = N_{10}\lambda t e^{-\lambda t} \quad (3.17)$$

$$\mathcal{J}_2(t) = \mathcal{J}_{10}\lambda t e^{-\lambda t} \quad (3.18)$$

7. Units of activity. The activity of a radioactive sample is the number of decays in unit time over the entire sample volume. The

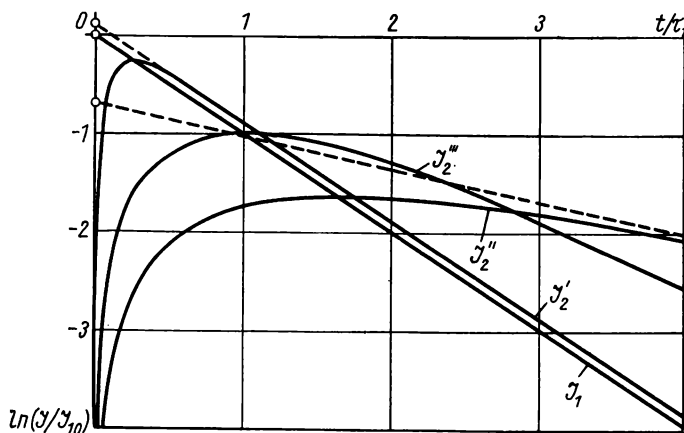


Fig. 3.3. Curves of build-up and decay of daughter radioactivity ($\tau'_2 = 0.1\tau_1$, $\tau''_2 = 3\tau_1$, $\tau'''_2 = \tau_1$)

unit of activity is 1 d/s. However, a more commonly used unit is a curie:

$$1 \text{ curie} = 3.7 \cdot 10^{10} \text{ d/s} \quad (3.19)$$

The activity of 1 curie is 1 g of Ra^{226} . The curie unit was first used to measure the quantity of radon which is in radioactive equilibrium with radium. Since 1947, expression (3.19) has become a definition of the curie as a unit of activity.

3.2. Alpha-Decay

1. Decay condition. α -Decay is characteristic of heavy nuclei for which, with the increase of A , the decrease of bond energy per nucleon is observed (see Fig. 2.4). In this region of mass numbers, the decrease of the nucleon numbers in the nucleus leads to the production of a more tightly bound nucleus. However, if A decreases by one, the energy gain is much lower than the bond energy of one nucleon in the nucleus. Therefore, the emission of a proton or a neutron having the bond energy beyond the nucleus equal to zero, is impossible. The emission of the He^4 nucleus is advanta-

geous from the viewpoint of energy since the specific bond energy of the nucleon in this nucleus is about 7.1 MeV. α -Decay is feasible if the total bond energy of nucleons in the product-nucleus and in the α -particle is higher than the bond energy of nucleons of the parent nucleus.

The increase of the bond energy of nucleons means the decrease of the rest energy just by the amount of energy E_α emitted in α -decay. Therefore, if the α -particle is presented as a whole in the composition of the product-nucleus, it should be in the positive E_α energy level (Fig. 3.4).

When the α -particle leaves the nucleus, this energy is emitted in a free form as the kinetic energy of the decay products, the α -particle and a new nucleus. The kinetic energy is distributed among the product-particles in inverse proportion to their mass [see (1.33)], and, as the mass of the α -particle is much lower than the mass of the newly formed nucleus, practically all decay energy is carried away with the α -particle. Thus, it may be said with great accuracy that E_α is the kinetic energy of the α -particle after its decay. The possibility of α -decay in terms of mass is expressed by the relation:

$$M(A, Z) > M(A - 4, Z - 2) + M_\alpha \quad (3.20)$$

Since in nuclear physics, the masses of neutral atoms are used as a rule, then (3.20) should be taken as the ratio of the atomic masses. But then M_α should be considered as the mass of the He^4 atom. The decrease of the mass in decay, expressed in energy units is the energy E_α of α -decay:

$$E_\alpha = [M(A, Z) - M(A - 4, Z - 2) - M_\alpha] c^2 \quad (3.21)$$

However, the Coulomb potential barrier U_c hinders the release of energy (see Fig. 3.4). The probability of the α -particle passage through the barrier is not great and quickly falls off as E_α decrea-

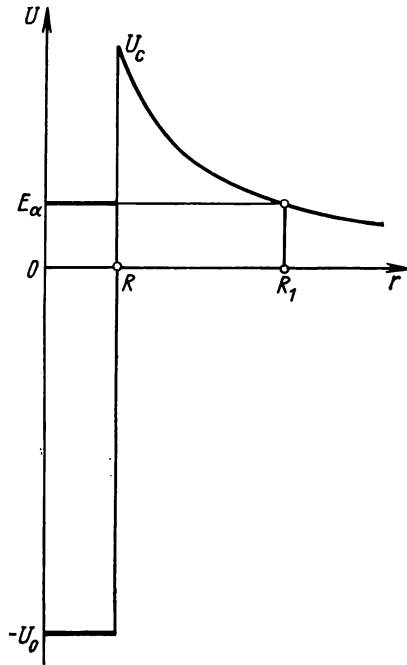


Fig. 3.4. Energy-level diagram of an α -particle in the heavy nucleus

ses. Therefore, (3.20) is not a sufficient condition for α -decay (Sec. 3.2-3).

The height of the Coulomb barrier for a charged particle penetrating into, or escaping out of, the nucleus increases proportionally to its charge. Therefore, the Coulomb barrier even stronger prevents other tightly bound light nuclei, such as C^{12} and O^{16} , from escaping out of the heavy nucleus. The mean bond energy of the nucleon in these nuclei is even higher than in the He^4 nucleus. Therefore, in some cases, the emission of the O^{16} nucleus would prove to be more advantageous from the standpoint of energy than the successive escape of four α -particles. However, the escape of nuclei heavier than the He^4 nucleus has not been observed.

2. Explanation of decay. Quantum mechanics explains the α -decay mechanism. From the point of view of classical physics, a body with energy E_α being in the region of $0 \leq r \leq R$ separated from the outer space by an energy barrier of the height U_C and width $R_1 - R$ (see Fig. 3.4) can never be beyond this region because, on reaching the coordinate $r = R$, the kinetic energy of the body becomes equal to zero and its further motion into the region $r > R$ ceases. The only possible way to leave the potential well is to get such a quantity of energy ΔE from outside that the total energy of the body, $E_\alpha + \Delta E$, becomes greater than the height of the Coulomb barrier U_C .

It should be noted that a particle leaves the potential well most readily on condition that $E_\alpha + \Delta E > U_C$. However, a particle possessing wave properties may be beyond the potential well even when its total energy $E_\alpha < U_C$. Moreover, it appears that only a potential barrier of an infinite width with the probability equal to unity limits the stay of the particle within the boundaries of the potential well. This would correspond in Fig. 3.4 to the negative values of the particle total energy. If the width of the barrier is finite, the probability of transition beyond the boundary of the potential barrier always differs in principle from zero, although this probability rapidly decreases with the increase of the width and height of the barrier. The quantum mechanics apparatus leads to the following expression for the barrier transparency, or the probability w for the particle to be found beyond the potential barrier when striking against its wall:

$$w = e^{-\frac{2}{h} \sqrt{2M_\alpha} \int_R^{R_1} \sqrt{U(r) - E_\alpha} dr} \quad (3.22)$$

where M_α is the α -particle mass. If the α -particle is imagined to be within the spherical potential well of the radius R , and to move at the velocity v_α , the frequency of impacts against the well walls

will be v_α/R . Then, the probability of the α -particle escape from the nucleus in unit time, or the decay constant [see (3.1)], will equal the product of the number of attempts per unit time and the probability of the passage through the potential barrier, with one impact against the wall:

$$\lambda = \frac{1}{\tau} = a_0 \frac{v_\alpha}{R} e^{-\frac{2}{\hbar} \sqrt{2M_\alpha} \int_R^{R_1} \sqrt{U(r) - E_\alpha} dr} \quad (3.23)$$

where a_0 is an unknown factor, since, on one hand, λ is the decay constant of the α -radioactive substance, and, on the other hand, the mechanism suggested for obtaining (3.23) is far from being valid. The α -particle does not move freely in the nucleus and, in general, there are no α -particles in the nucleus at all. The α -particle is made up of four nucleons at the instant of α -decay. The value a_0 has the sense of probability of the origination of an α -particle in the nucleus, the frequency of the α -particle impacts against the well wall equalling v_α/R .

3. Comparison with experiment. On the basis of relation (3.23), many phenomena observed during α -decay can be explained. The half-life of the α -particle nuclei is the greater, the less the energy E_α of α -particles emitted during the decay. However, if the half-lives vary from fractions of a microsecond to milliars of years, the range of the change of E_α is very small, approximately from 4 to 9 MeV for nuclei with the mass numbers $A > 200$. The regular dependence of the half-life on E_α was experimentally found among natural α -active atoms long ago and was interpreted by the following approximated relation:

$$\log \lambda = a + b \log E_\alpha \quad (3.24)$$

where a and b are constants somewhat different for different radioactive families. Expression (3.24) is called the Geiger-Nuttal rule and is the power dependence of decay constant λ on E_α with a very large index b . This strong dependence of λ on E_α immediately follows from the mechanism of the α -particle passage through the potential barrier. The barrier transparency and, consequently, the decay constant λ depend exponentially on the integral value over the region ($R_1 - R$) and rapidly increase with the increase of E_α . When E_α approaches 9 MeV, the life-time with respect to α -decay is minute fractions of a second, that is, with the energy 9 MeV or higher, α -decay occurs almost instantaneously. It is typical that such a value of E_α is even much lower than the height of the Coulomb barrier, U_c . With heavy nuclei, the height of the Coulomb barrier (2.15) for a two-charge point particle is about 30 MeV. The barrier for the α -particle of a finite size is somewhat

lower and can be estimated to be 20 to 25 MeV. Thus, the passage of the α -particle through the Coulomb potential barrier proceeds rather efficiently if its energy is not lower than a half, or one third of the barrier height.

The transparency of the Coulomb barrier depends also on the nuclear charge Z since U_C depends on Z (2.15). α -Decay is observed among nuclei with the mass numbers $A > 200$ and in the region $A \approx 150$. It is clear that with the same E_α , α -decay proceeds easier in nuclei with $A \approx 150$ than in the heaviest nuclei. At the same time, the emission of α -particles with arbitrarily low energy has not actually been observed, although, in principle, the transparency (3.22) differs from zero at any E_α . (It is impossible to detect α -decay in experiment if the half-life exceeds 10^{17} to 10^{18} years). The corresponding minimum value of E_α is higher in heavier nuclei and comprises 4 MeV in nuclei with $A > 200$ and about 2 MeV in nuclei with $A \approx 150$. Hence, the fulfilment of relation (3.20) does not necessarily testify to the instability of the nucleus to α -decay. It appears that (3.20) holds for all nuclei with the mass numbers higher than 140. However, about one third of all natural stable nuclides are found in the region $A > 140$.

Relation (3.23) relates the life-time of α -active nuclei, τ , to their decay energy E_α . The values $U(r)$ and R_1 in (3.23) can be calculated by the known Z and E_α . In this case the only unknown value is the nuclear radius R . Therefore, relation (3.23) was used to analyse the values of the atomic nuclei radii of natural α -active substances for which half-lives and decay energies were experimentally measured and it was found to what chemical elements they belong. Relation (2.2) was first obtained just in this way. It turned out later on that (2.2) is valid for the radii of nuclei with any mass numbers A .

4. Stability limits. Radioactive families. The limits of stability of heavy nuclei to α -decay may be explained using the nuclear shell model (Sec. 2.6-4). Nuclei having only closed proton or neutron shells are extremely tightly bound. Therefore, although the bond energy per nucleon of the intermediate and heavy nuclei decreases with increasing A (see Fig. 2.4), this decrease is always slowed down when A approaches the magic composition, and is accelerated after A has passed the magic number of protons and neutrons. As a result, the energy E_α (3.21) appears to be much lower than its minimum value at which α -decay is observed if the nucleus (A, Z) is magic, or the mass number of the nucleus is lower than the mass number of the magic nucleus. On the contrary, the energy E_α increases discretely in nuclei with mass numbers exceeding the values A of magic nuclei and surpasses the minimum of practical stability to α -decay.

In the region of the mass numbers $A \approx 150$, α -active nuclides are nuclides whose nuclei contain by two or several neutrons more than the magic number 82. Some of these nuclides have their half-lives much longer than the geological age of the Earth and are, therefore, present in a natural mixture of the isotopes of elements (for instance, Nd^{144} , Sm^{147} , Sm^{149} , Gd^{152}). Others were obtained as a result of nuclear reactions. The latter are short of neutrons as compared to stable nuclides of the corresponding mass numbers. In these nuclides, β^+ -decay usually competes with α -decay. The heaviest stable nuclide is Bi^{209} , its nucleus containing the magic number of neutrons 126. Lead which precedes bismuth has the atomic number $Z = 82$, and the nuclei of all the lead isotopes contain the magic number of protons. Pb^{208} is a twice magic nuclide. All heavier nuclei are radioactive. Since the product-nucleus is enriched with neutrons as a result of α -decay, then several α -decays are followed by a β -decay. The latter does not vary the number of nucleons in the nucleus, and any nucleus with the mass number $A > 209$ may become stable only after several α -decays. As the number of nucleons changes by four during α -decay, the existence of four independent decay chains is possible, each having a final product of its own. Three of them exist in nature and are called natural radioactive families. The decay of natural families terminates in the formation of one of the lead isotopes, the final product of the fourth family being the Bi^{209} nuclide (Table 3.1). The existence of natural radioactive families is due to three long-lived α -radioactive nuclides, Th^{232} , U^{235} and U^{238} having half-lives comparable to the geological age of the Earth, i.e., $4.5 \cdot 10^9$ years.

Table 3.1

Radioactive Families

| Family | Formula for the mass number of the family (c is an integer) | The most long-lived representative | Its half-life, years | Final decay product |
|------------------|--|------------------------------------|----------------------|---------------------|
| Thorium | $4c$ | Th^{232} | $1.39 \cdot 10^{10}$ | Pb^{208} |
| Uranium-actinium | $4c - 1$ | U^{235} | $7.1 \cdot 10^8$ | Pb^{207} |
| Uranium-radium | $4c - 2$ | U^{238} | $4.51 \cdot 10^9$ | Pb^{206} |
| Neptunium | $4c - 3$ | Np^{237} | $2.2 \cdot 10^6$ | Bi^{209} |

The Np^{237} nuclide is an isotope of the transuranium element neptunium. It is the most long-lived representative of the extinct fourth family. At present, a great deal of nuclides have been obtained by

bombarding heavy nuclei with neutrons and light nuclei, which are isotopes of transuranium elements ($Z > 92$). All of them are unstable and belong to one of the four radioactive families.

A sequence of decays in natural families is shown in Fig. 3.5. In case when the probabilities of α - and β -decay are comparable,

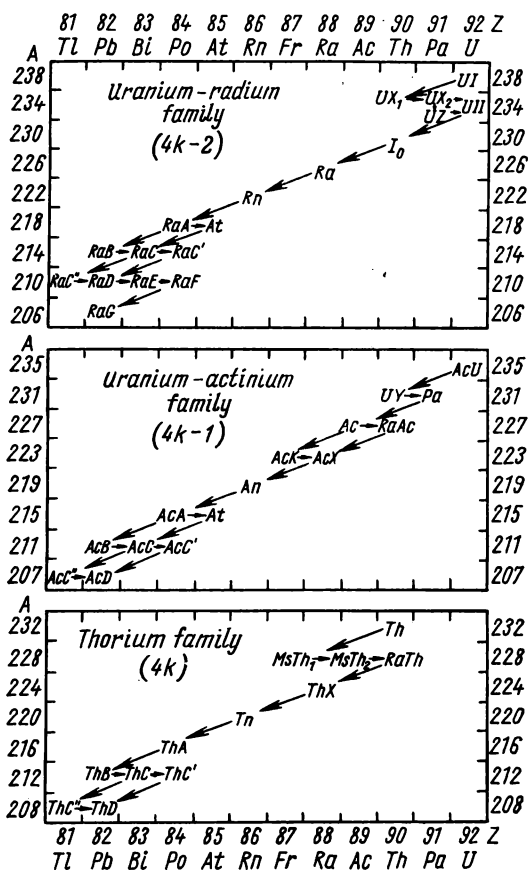


Fig. 3.5. Natural-family decays. The given designations are assigned to radioactive nuclides in the early studies of natural radioactivity

branchings are formed that correspond to nuclear decays with either α - or β -particle emission. Within the branching, α -particle emission is followed by β -decay and β -particle emission is followed by α -decay so that decay branching does not result in any different final products. In addition to α -active nuclei belonging to four families, heavy nuclides obtained artificially are known. They decay with the emission of α -particles, but they do not belong to

the above families. As a rule these are nuclides with neutron-deficient nuclei and mass numbers about 200. They have an excess of several protons compared to the magic number 82. Like other nuclides with neutron deficiency in nuclei, in addition to α -decay, they undergo β^+ -decay.

5. Alpha-spectra. Since α -decay results in one nucleus and one particle, the energy of each α -particle is always the same. At the same time, when α -particles are analysed in an alpha-spectrometer, in addition to the main energy group which is of the greatest intensity, one often observes groups of α -particles having lower energies, each belonging to its energy value E . Such an energy spectrum is called a line spectrum (Fig. 3.6). α -Particles with the energy lower than the energy of the main group have a shorter particle range in the substance

and are called short-ranged particles. The emission of short-ranged particles during α -decay is followed by γ -radiation. The energy measurement of the γ -quanta showed that any group of short-ranged particles has its corresponding γ -quanta with such energy that the total energy of the γ -quanta and of the short-ranged particle just makes up the energy of the main α -particle group which is defined by the mass ratio (3.21), (E_0 in Fig. 3.6). Thus, the short-ranged particles are emitted when the product-nucleus is produced in its excited state. In this case the excitation energy is a part of the α -decay energy and, hence, the emitted α -particle has a lower energy value. The nuclear energy levels are discrete. Therefore, the groups of short-ranged particles are also energy-discrete. The transition of the excited nucleus into the ground state occurs almost instantaneously, and γ -quanta may be considered to be emitted together with α -particles, although, in fact, the emission of γ -quanta follows α -decay. α -Decay is not followed by γ -radiation at all if the daughter nucleus is immediately originated in its ground state.

The diagram of α -transition with possible formation of excited states of the product-nucleus is given in Fig. 3.7. The energy of nuclear states is plotted without scale along the conventional y -axis. Each nuclear state is marked by a corresponding line. Z nuclei are qualitatively presented by the conventional x -axis. The arrow-lines indicate transitions with Z and energy changes. The indices on the arrow-lines indicate the energy carrier particle. Such diagrams are very convenient and widely used to display

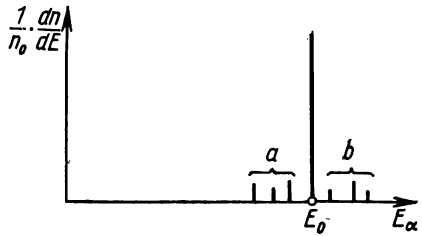
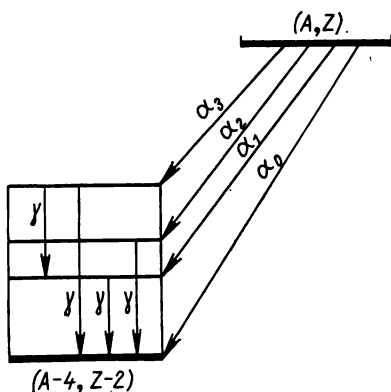
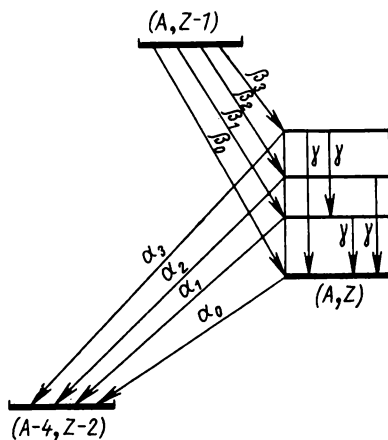


Fig. 3.6. α -Particle line-spectrum
(a) short-ranged α -particles; (b) long-ranged α -particles; n_0 is the total number of the α -particles analysed

nuclear transformations. The energy values of nuclear states are denoted, if necessary, by numerical values in the diagram. It follows from the diagram that it is possible to determine the energy of the excited levels of the daughter nucleus directly from the energy spectrum of short-ranged α -particles. As a rule, the intensities of short-ranged α -particles are usually lower than the intensity of the main group, as the transparency of the Coulomb barrier is lower for α -particles with a lower value of E_α .

The production of long-ranged particles during α -decay (see Fig. 3.6) is possible in some specific cases. Among natural α -ra-

Fig. 3.7. α -TransitionsFig. 3.8. Production of long-range α -particles

dioactive substances, two are known, RaC' and ThC' (see Fig. 3.5) which have very short half-lives ($1.6 \cdot 10^{-4}$ and $2 \cdot 10^{-7}$ s, respectively) and the nuclei of which result from the preceding β -decay, mainly in their excited states. The transition into the ground state, with γ -quantum emission is usually the first to take place, and then α -decay follows. However, because of their very short half-life compared to α -decay, a small but measurable portion of nuclei in their excited states undergoes α -decay faster than the transition into the ground state. The excitation energy of the parent nucleus is added to the α -decay energy (3.21) and the emitted α -particle carries away more energy than do α -particles emitted by nuclei in their ground state. Fig. 3.8 shows a transition diagram for the above case. It is evident that the energy spectrum of long-ranged α -particles bears information on the system of nuclear levels of the parent nucleus.

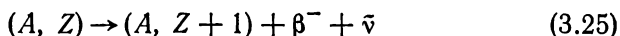
6. Alpha-spectrometer. In its operational principle the magnetic alpha-spectrometer is analogous to a mass-spectrometer. When

charged particles move in a constant magnetic field, an analysis of particles by their momenta Mv (2.10) is possible because the particles with different momenta move along circular paths with different radii. If the velocities of all particles are the same, the momenta analysis is the analysis of the particle masses. If all the particle masses are the same, as in the case of α -particles, the momenta analysis is the analysis of the velocities or energies of α -particles. A very thin radioactive sample should be the source of α -particles in the alpha-spectrometer so that slowing down of α -particles in the substance be negligible.

3.3. Beta-Decay

1. **The condition of decay.** The mass quantity of a nucleus with a given number of nucleons depends on the nuclear proton-neutron composition. A nucleus with the least mass can correspond to only one combination of the numbers of protons and neutrons (see Fig. 2.6). From the standpoint of energy, transformation of a nucleus with any other nucleonic composition into a nucleus with an optimum number of protons and neutrons is advantageous. In fact, spontaneous changes in nuclei take place because there exists a mechanism of nuclear interconversion, i.e., β -decay, (1.65)—(1.67). During β -decay a proton is transformed into a neutron or a neutron is transformed into a proton. The direction of the process only depends on the state of the nucleon — whether a proton or a neutron — when the nucleon together with its associated particles has its least total energy. The mass of a free neutron in energy units is by 1.3 MeV greater than the mass of a free proton. Therefore, in free state, spontaneous transformation of a neutron into a proton is only possible. The total energy of a single nucleon in the atomic nucleus is on the average by 8 MeV (i.e., the amount of the bond energy) lower than the total energy of a free nucleon and changes rapidly when the nuclear composition varies. Since the bond energy is much higher than the energy differences of the rest energies of the free neutron and proton, the transformation of the proton into a neutron inside the nucleus is also possible, as such a process may appear advantageous from the energy point of view.

2. **Types of β -decay.** β -Decay is called β^- -decay or β^+ -decay, depending on which state is the initial and which is the final state of the nucleon. If the parent nucleus has excess neutrons, compared to the optimum nuclear composition of the same mass number A , it undergoes β^- -decay:



during which the number of protons in a nucleus increases by unity on account of the decrease of the number of neutrons by one. In

this case, the nucleus ejects an electron and an antineutrino. Electrons emitted by nuclei are called β -particles. If the nucleus has excess protons, the decrease of their number by unity and the increase of the number of neutrons by unity take place in β^+ -decay:

$$(A, Z) \rightarrow (A, Z - 1) + \beta^+ + \nu \quad (3.26)$$

In this case, the nucleus emits a positron and a neutrino. Process (3.26) often occurs without emitting a positron but then the nucleus absorbs an atomic electron:

$$(AZ) + e^- \rightarrow (A, Z - 1) + \nu \quad (3.27)$$

Here, e^- is an atomic electron, in distinction to the electron emitted from the nucleus. Capture of the electron from the K -shell of the atom is most probable in transformation (3.27). Therefore, the process is called K -capture. K -capture is identical to β^+ -decay, as far as the change in the nuclear composition is concerned, and is its modification. Hence, both processes can take place simultaneously during the decay of nuclei of a given kind. The competition between them is determined by the properties of the radioactive atom and the product-atom, and the percentage of decays of each of these processes may be in the range from 0 to 1 in different cases. In the case of K -capture, the nucleus always emits a neutrino.

It is very easy to detect experimentally β^- - and β^+ -decays by the emitted electrons having high energy. The neutrino very weakly interacts with the substance and its appearance cannot be detected by conventional laboratory methods. During β -decay, it is also very difficult to detect recoil nuclei because of their very low kinetic energy (1.34). Therefore, K -capture is identified by its secondary effect. After the electron has been captured by the nucleus, electron transition takes place in a newly formed atom with a vacancy in the K -shell. It is accompanied by the emission of a characteristic X -ray quantum. The wavelength of the characteristic X -radiation is defined by the nuclear charge [Moseley law (2.1)]. From this it follows that the spontaneous emission by a substance of the characteristic X -radiation belonging to the element having Z less by a unity than Z of the parent substance is indicative of the K -capture process. To observe this phenomenon is also quite difficult because of the low quanta energy. Nevertheless, this is the easiest way of detecting K -capture.

3. Decay energy. Spontaneous processes take place only with the formation of particles of lower mass. Since in atomic β -decay nucleons are bound with other nucleons and electrons, the mass ratios must include the atomic mass M_a . The condition for β^- -decay or K -capture is as follows:

$$M_a(A, Z) > M_a(A, Z \pm 1) \quad (3.28)$$

and that for β^+ -decay is:

$$M_a(A, Z) > M_a(A, Z - 1) + 2m \quad (3.29)$$

In the latter relation, the correction $2m$ is added because during β^+ -decay an atom with $Z - 1$ electrons is formed and not only a positron but an atomic electron appears to be beyond the atom. In β^- -decay, the product-atom lacks one electron due to the increase of Z by unity. Therefore, the mass of a neutral atom is obtained if the electron emitted by the nucleus is taken into account. In the case of K -capture, the balance of atomic electrons is automatically equalized.

It follows from relations (3.28) and (3.29) that K -capture may also take place when β^+ -decay proper is energetically impossible since the mass of the product-atom during β^+ -transformation with electron absorption (i.e., K -capture) may be by $2m$ larger than the ultimate mass of the product-atom allowing β^+ -decay with positron ejection. This peculiarity of β^+ -decay follows from the fact that the emission of a positron from an ordinary substance means an additional release of energy, $2mc^2$, which is evolved during the annihilation of a positron and an electron. β^- -Decay would possess this peculiarity if it took place in antimatter. If β^+ -decay is possible, then K -capture is even more probable. However, K -capture does not always accompany β^+ -decay and is most probable with heavy atoms that have small radii of electron K -shells.

The energy of β^- -decay and of K -capture equals

$$E_\beta = [M_a(A, Z) - M_a(A, Z \pm 1)]c^2 \quad (3.30)$$

and that of β^+ -transformation is

$$E_\beta = [M_a(A, Z) - M_a(A, Z - 1) + 2m]c^2 \quad (3.31)$$

If the energy released in positron annihilation is taken into account, then (3.31) will be transformed into (3.30), strictly following (2.4), because the released energy is proportional to the decrease of the atomic mass. Since during β -decay Z of the nucleus is changed simultaneously with the change of the bond energy of nucleons the electron bond energy also changes (2.6). The latter component may be of significance if E_β is small. For instance, in the decay of a natural radioactive substance RaD (Pb^{210} nuclide) with the production of a daughter nucleus in the excited state, $E_\beta = 0.014$ — 0.018 MeV, according to different measurements whereas the increase of the electron bond energy in the transformation of the lead atom into the bismuth atom equals 0.015 MeV, that is, practically all the energy of the β -transformation is released on account of the change of the bond energy of the atom, not of the nucleus.

Part of β -decay energy may be spent to excite the product-nucleus or the electron shells of the product-atom. This energy imme-

diately after β -decay is released as radiation. The atom emits X -ray quanta after K -capture, and the excited nucleus emits γ -quanta. The remaining energy, or the whole energy E_β in the absence of excitation, is carried away by three particles produced during β -decay (the product-atom, the β -particle and the neutrino) or by two particles (the atom and the neutrino) in K -capture. The energy is distributed among the particles, following the law of energy and momentum conservation. If there are two particles, any of them receives the same energy in each case of decay. If they are three, the energy of each particle can take on a value from zero to some maximum, depending on the mutual orientation of the momenta in different cases of decay. The maximum energy value of the atom recoil (1.34) is very low as the atomic mass is comparatively great, and practically the whole energy of β -decay is carried away by the β -particle and the neutrino, being distributed between them. If the energy of the β -particle is at its maximum, the neutrino energy equals zero, and vice versa. Therefore, in the absence of excitation of the product-nucleus, the maximum energy value of any of these particles is, with high accuracy, the energy of β -decay, (3.30) or (3.31). However, more probable is the emission of the β -particle and the neutrino with arbitrary intermediate energy values. The neutrino does not practically interact with the substance, and it is impossible to observe the neutrino energy distribution. In fact, it is possible to detect only the energy spectrum of β -particle, or the β -spectrum.

The observed absolute values of E_β lie in a very wide energy range of from 0.019 MeV with H^3 to 16.4 MeV with N^{12} . There is no limit as to the energy release in β -decay. The most essential restrictions concern the case of a considerable difference between the spin values of the parent and finite nuclei. Therefore, β -decay practically always takes place when conditions (3.28) and (3.29) are fulfilled.

4. **Beta-spectrometer.** The device for obtaining β -spectra is called a beta-spectrometer. It is a magnetic electron momentum analyser, and in its operating principles is analogous to the mass-spectrometer. Electrons emitted during β -decay pass through a constant magnetic field where they move along circular paths with radii (2.10) proportional to the particle momenta (1.25). The number of particles of a given momentum is detected by a beta-counter at the output of the device. The total energy (1.24) and the kinetic energy (1.29) of β -particles are determined by their measured momentum. Distribution of the numbers of β -particles by the kinetic energy values is β -spectrum.

5. **Beta-spectrum.** Beta-spectra are continuous. In the energy distribution of β -particles, there are particles with any energy from zero to the maximum E_β (Fig. 3.9). Figure (3.9) shows the fraction

of β -particles of their whole number, n_0 , per one energy interval, as a function of the β -particle energy. It is characteristic that the greatest part of β -particles has the energy much lower than the maximum value of E_β so that the mean energy of the β -spectrum approximately equals $1/3$ of E_β . Thus, the greatest part of the β -decay energy is carried away by the lightest particle formed during β -decay, i.e., a neutrino. The neutrino does not transfer its energy to the substance and this energy is not transformed into heat evolved in the slowing down of other particles. The seeming disappearance of a large portion of the β -decay energy led Pauli to the assumption that during β -decay a particle with a very low mass and no electric charge is emitted. Fermi called this particle a neutrino, i.e., a small neutral particle. The β -decay theory developed by him in the assumption of the neutrino emission, showed fair agreement with experiment. However, it took over 20 years to closely study the properties of this subtle particle (Sec. 1.9).

6. Theory. β -Decay takes place due to weak forces (Sec. 1.7—5). Weak interactions develop in time not faster than for 10^{-10} s, and the life-time in relation to β -decay is much greater and has the values from some frac-

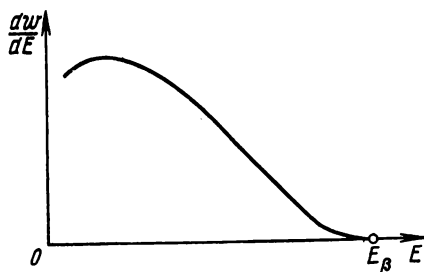


Fig. 3.9. Beta-spectrum

tions of a second to millions of years. There are several reasons for such deceleration of β -transformations. First of all, the β -decay rate strongly depends on the decay energy E_β . The lower E_β , the slower the decay proceeds because the range of the values of momenta which can be gained by the electron and the neutrino produced in the decay process becomes narrower. In addition, the process of β -decay is the transformation of the proton into the neutron or vice versa, which, except for the β -decay of a free neutron, takes place inside the atomic nuclei. In the nuclei, protons and neutrons occupy definite levels in nucleonic shells. The final state of the nucleon which undergoes transformation can greatly differ from the initial state in β -transformation of the nuclei with the number of protons not being substantially equal to the number of neutrons, which is characteristic of nuclei with intermediate and high mass numbers, whereas the probability of transition from one state to another is determined by the overlap of the wave functions of final and initial states in space. For different states of particles in the nucleus this overlap is very small. Finally, the probability of β -transformation greatly depends on the diffe-

rence in the spin values of the initial and final nucleus. If this difference exceeds unity, then the β -decay rate abruptly decreases. The decay rate also decreases if the wave functions which describe the nucleonic states of the initial and final nuclei have different parity, which, in the case of the nucleus, is defined by the parity of the orbital moment of the nucleon l .

The theory gives the following expression for the probability of the atomic nuclear β -decay per unit time ω with the emission of an electron having the momentum p , which is referred to a single interval of the electron momentum scale:

$$\frac{d\omega}{dp} = \frac{g^2 |\mathfrak{M}|^2}{2\pi^3 \hbar^7 c^3} (E_\beta - E)^2 p^2 \quad (3.32)$$

where g is a constant of weak interaction responsible for β -decay; $|\mathfrak{M}|^2$ is a square modulus of the matrix element of the nucleonic transition computed by quantum mechanics methods and describing the degree of complexity of the transition from the initial state of the nucleon into its final state after its β -transformation, taking into consideration the change in the nuclear spin and the parity of the wave function; E is the kinetic energy of the β -particle with the momentum p . The integral of the right-hand side of (3.32) over all the electron momentum values from zero to maximum, or, otherwise stated, the integral over all the electron energies from zero to E_β , gives the probability of the decay per unit time ω , with the emission of an electron of an arbitrary momentum, i.e., the probability of any case of β -decay. The probability of decay per unit time is the decay constant λ [see (3.1)] equal to $1/\tau$, where τ is the mean life-time with respect to β -decay. Therefore,

$$\frac{1}{\tau} = \frac{g^2 |\mathfrak{M}|^2 (mc^2)^5}{2\pi^3 \hbar^7 c^6} f(E_\beta^0) \quad (3.33)$$

where $f(E_\beta^0)$ is a dimensionless part of the integral over the β -particle energy, depending on the upper limit of integration presented in the form of (1.30). The function $f(E_\beta^0)$ is directly calculated and, with high energy values of β -decay, most strongly depends on E_β^0 [in proportion to $(E_\beta^0)^5$]. It should, however, be noted that this dependence is essentially weaker than the dependence on the α -decay rate energy (3.23).

Since the energy dependence of β -decay rate is allowed for by the function $f(E_\beta^0)$, the expression $f\tau$ does not depend on the energy:

$$f\tau = \frac{2\pi^3 \hbar^7}{m^5 c^4 g^2 |\mathfrak{M}|^2} \quad (3.34)$$

Thus, τ and E_β measured in experiment allow one to determine $f\tau$ for any β -active substance, and this value only depends on the transition matrix element $|\mathcal{M}|^2$. In general, f also depends on the nucleus charge Z and the β -particle sign because electrically charged electrons emitted by nuclei interact with the Coulomb nuclear field. The greater Z of the radioactive nucleus and the lower E_β , this effect is the more pronounced and can properly be allowed for.

7. Comparison with experiment. The deductions from the β -decay theory as regards the forms of β -spectra and the distribution of radioactive atoms according to the values of $f\tau$ are in fair agreement with experiment [16]. In fact, β -spectra (see Fig. 3.9) are described by expression (3.32), and their life-times depend on the β -transition energy, E_β , and the degree of forbiddenness of the transition, defined by $|\mathcal{M}|^2$ in accordance with (3.33). The square modulus of the matrix element $|\mathcal{M}|^2$ equals unity if the state of a nucleon before and after the decay is the same (for instance, the state of a free neutron decaying and transforming into a free proton). The value of $|\mathcal{M}|^2$ rapidly decreases with the increase of the difference between the original and final states of the nucleon, and this value cannot, in general, be accurately calculated. It drops discontinuously at least by a factor of 100 with the increase of the spin differences of the original and final nuclei by each unity or when the nuclear wave function parity changes. β -Transitions with value of $f\tau \approx 10^5$ s are called allowed transitions and correspond to the transitions with a change of the nuclear spin $\Delta I = 0$ or $\Delta I = 1$ and without the parity change of the wave function. The rest transitions are called forbidden transitions. Very high values of $f\tau$ being classified according to the groups of forbiddenness correspond to these transitions, and the decay of the corresponding β -radioactive atoms proceeds very slowly. For instance, the natural K^{40} has a half-life of $1.3 \cdot 10^9$ years. Its decay occurs with a spin change $\Delta I = 4$, and the value of $f\tau$ for K^{40} equals 10^{18} s.

The cases of β -decay where the matrix element of transition $|\mathcal{M}|^2$ can with adequate accuracy be taken as a unity are used to find the most important constant of the nuclear physics, i.e., the weak interaction constant g , by the measured τ and E_β [see (3.34)]. The constant g is the perturbation energy which is created by weak forces and which brings the nucleon to β -transformation if there are proper conditions for weak forces to exist, i.e., a transition advantageous from the viewpoint of energy is possible. Since the theory of β -decay is in good agreement with experiment, the determination of the weak interaction constant by the values measured in β -decay is reliable:

$$g \approx 10^{-62} \text{ j} \cdot \text{m}^3 \quad (3.35)$$

Weak forces should correspond to a very small absolute value of the constant, characterizing the interaction energy in β -decay. The assumption of weak forces existing in nature forms the basis for the understanding of the causes of the slow transformation processes of elementary particles. Not only β -decay of atomic nuclei but other processes with the neutrino participation as well as the decays of some heavy elementary particles take place under the action of weak forces.

From the experimentally measured constant g , one can determine the characteristic time of weak interactions (Sec. 1.7-5). The characteristic interaction time means the time interval during which the process will actually take place with great probability if the conditions for the realization of this process are most favourable during this time period.

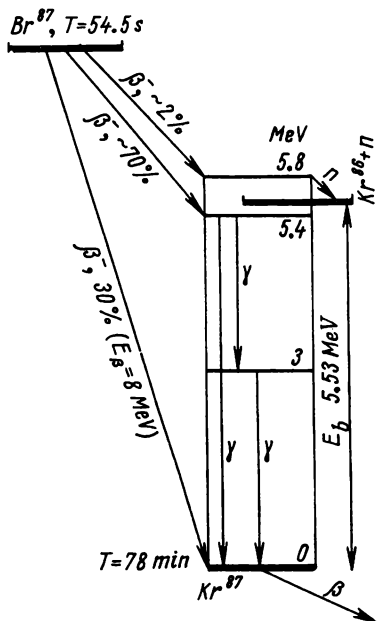
8. γ -Emission and delayed neutrons. β -Decay followed by the formation of the daughter nucleus in the ground energy state in each case of decay is a rare exception rather than a rule. Usually, along with β -transition into the ground state, one can observe transitions with the formation of several excited states of the product-nucleus (Fig. 3.10). In cases when the spins of the ground states of the parent and daughter nuclei differ by some units of \hbar , and E_β is

Fig. 3.10. Emission of delayed neutrons in Br^{87} decay

high enough for the daughter nucleus to be produced in excited states, the mechanical moments of which are rather similar to the spin of the decaying nucleus, the daughter nucleus is not produced in its ground state at all. Immediately after β -decay, the excited product-nuclei are transferred into their ground state, emitting predominantly γ -quanta. Therefore, β -decay is usually accompanied by γ -emission which presents the main radiation danger when dealing with radioactive substances.

The nuclear excitation occurs on account of the β -decay energy E_β . Only some part of the energy (E_β) is carried away by the β -particle and the neutrino. If the excitation energy of the daughter nucleus is E_d^* then

$$E_\beta = E_\beta - E_d^* \quad (3.36)$$



In the transition of the excited nucleus into its ground state, either one γ -quantum with the energy $E_\gamma = E_d^*$ or several γ -quanta with the total energy E_d^* are emitted. The energy E'_β is distributed among the product-atom, the β -particle and the neutrino, just as in the ground state transition. β -Particles also have continuous energy distribution, but with the maximum energy level E'_β . If in β -transition, the formation of a product-nucleus is possible in several excited states, the full β -spectrum is an overlap of several spectra with maximum energy values E_β , E'_β , E''_β , etc. Each spectrum component in this case is characterized by its output, i.e., by a fraction of decays resulting in its formation.

Thus, dependence (3.32) for the momentum or energy distribution of β -particles is valid for the energy values $E > E'_\beta$ because, with lower energies, two or more functions (3.32) with different proportionality factors are summed up.

The absolute values of the excitation energies are defined by the system of the energy levels of the daughter nuclei (Sec. 2.7-2), and by the value of E_β and are usually in the range of 0.1 to 3 MeV. However, the excitation energy may be much higher than 3 MeV, in rare cases reaching 8 to 11 MeV, which is comparable to, or higher than, the bond energy of the nucleon in the nucleus. If the nuclear excitation energy exceeds the nucleon bond energy, the nucleus gets rid of its excess energy by emitting a nucleon, not a γ -quantum. Thus, following β -decay, protons, neutrons or α -particles can be emitted, apart from γ -quanta. Cases of the nucleon or α -particle emission in β -decay are rare and all of them refer to short-lived artificial radioactive substances which have high values of E_β , at least exceeding the bond energy values of the above particles in the product-nucleus. The process itself is called the emission of *delayed* nucleons or α -particles. The matter is that the separation of nucleons from stable nuclei is only possible due to nuclear reactions (Sec. 3.5). Nuclear reactions result in short-lived radioactive substances. The production of nucleons as reaction products immediately ceases after the collision with the nuclei of the particles initiating nuclear reactions. However, the production of nucleons goes on with the delay by the life-time of the β -radioactive substance if β -decay is accompanied by nucleon emission. The change in time of the number of emerging delayed nucleons, after the initiation of the nuclear reaction has been stopped, is described by the law of radioactive decay (3.4) with the decay constant of the β -radioactive substance, i.e., the delayed-nucleon precursor.

9. Scheme of delayed-neutron emission. The emission of delayed neutrons by radioactive products of the fission of heavy nuclei is used to control the self-sustained reaction in nuclear reactors. Fig. 3.10 is an energy scheme which explains the production of

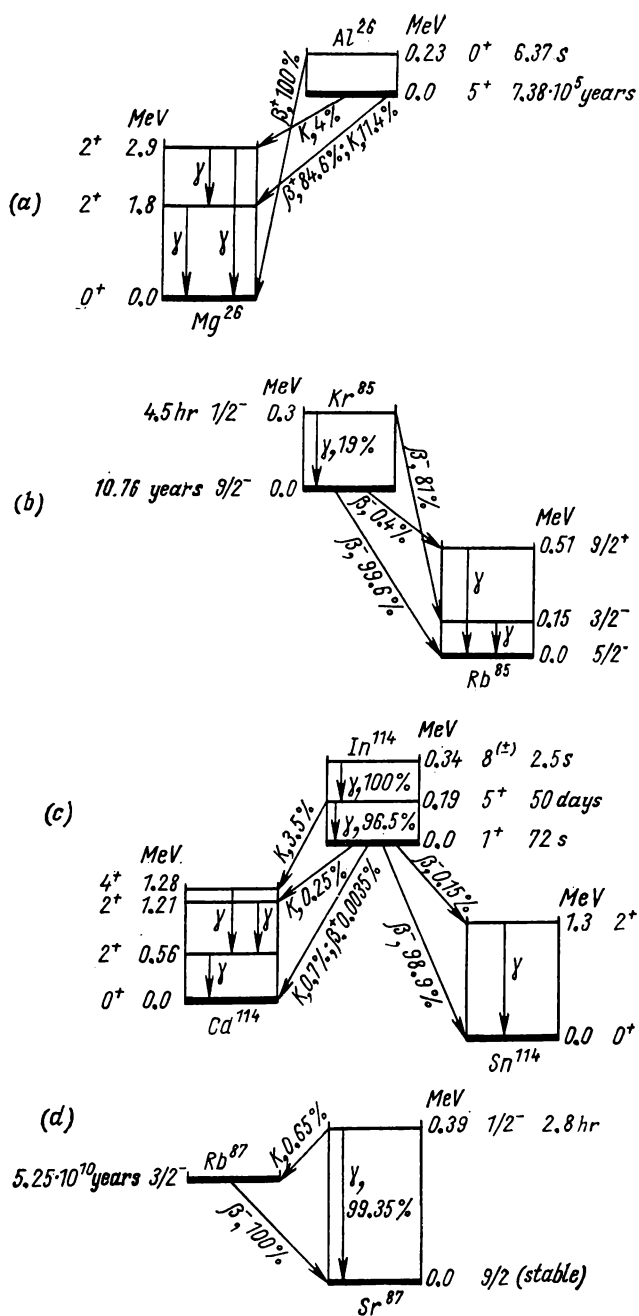


Fig. 3.11. Isomer decay diagrams

delayed neutrons during the β -decay of the Br^{87} nuclide produced in the U^{235} fission. Approximately in two cases out of one hundred, β -transition of Br^{87} is accompanied by the formation of a strongly excited state of the Kr^{87} nucleus, $E^* = 5.8$ MeV. The nuclei of all radioactive fission products are oversaturated with neutrons, and therefore, the bond energy of the last neutrons in their composition is relatively low. In this case, the Kr^{86} nuclide is magic as regards neutrons so that one neutron above the closed neutron shell of 50 neutrons in the nucleus of the Kr^{87} nuclide has particularly low bond energy which equals $E_b = 5.53$ MeV. Hence, the emission of neutrons is possible from the level $E^* = 5.8$ MeV, which is the case in the production of the stable Kr^{86} nuclide. In β -transitions to a lower excitation state of Kr^{87} only γ -quanta are emitted. In addition to Br^{87} , about ten more radioactive nuclides are known among fission products. These nuclides emit delay neutrons during β -decay.

10. Decay from excited states. β -Decay usually proceeds from the ground states of atomic nuclei. The decay of radioactive isomers is an exception (Sec. 3.4-2). In the β -decay of an isomer (Fig. 3.11) the energy of the parent nucleus excitation, E_m^* , is added to the β -transformation energy. The β -particle spectrum, as with the neutrino spectrum, remains continuous in this case. Only the maximum β -spectrum energy, E_β' , appears to be higher than the β -decay energy, E_β , by the excitation energy value, E_m^*

$$E_\beta' = E_\beta + E_m^* \quad (3.37)$$

if the transition into the ground state of the product-nucleus takes place. The transition into the excited state of the daughter nucleus in its turn attenuates the maximum β -spectrum energy by the value of the excitation energy of the daughter nucleus [see (3.36)]. Complete schemes of nuclear radioactive decays are given in [13, 14].

3.4. Nuclear Gamma-Radiation

1. Time of γ -quantum emission. In distinction from X -ray quanta and from visible light quanta emitted in atomic electron transitions, photons emitted by nuclei are called γ -quanta. γ -Quantum emission is the fundamental process of releasing the nucleus from its excess energy if this energy does not exceed the nucleon bond energy in the nucleus. The photon formation is only due to electromagnetic forces and is accompanied by the nuclear redistribution either of electric charge (electric transitions) or of magnetic moments, both nucleon and orbital (magnetic transitions). In this case, the nuclear spin and its components are sure to change (Sec. 2.3-1) because the γ -quantum carries away the mechanical moment l at least equal to unity. Emission with $l = 1$ is called a

dipole emission, and the life-time of the nucleus emitting a dipole quantum is approximately 10^{-14} s, i.e., the emission of a γ -quantum follows the formation of the nuclear excited state almost instantaneously.

At the same time, quantum electrodynamics shows that the photon corresponding to the multipole transition of the multiplicity factor 2^l , i.e., with the change of the mechanical moment of the emitter by l units, carries away the angular momentum $l\hbar$ with respect to the source of emission. But the emission of quadrupole ($l = 2$), octopole ($l = 3$) quanta, etc., with $l > 1$ is the more difficult, the greater the mechanical moment of the quantum. The life-time in the emission of such quanta is, therefore, longer than the time of the dipole emission, and it abruptly increases with the increase of l . In addition, the time of the emission of a γ -quantum by the excited nucleus depends on the value of the transition energy and increases with the decrease of the energy difference between the initial and final states.

Thus, transitions between nuclear levels with small energy difference and with great difference in the values of the mechanical moment take relatively long times. These transitions cannot compete with the transitions corresponding to the small change of the mechanical moment. Therefore, when the mechanical moment difference is great, several successive transitions take place between the excited and the ground states of the nucleus. But if there are no intermediate levels between the ground and the excited states with great spin difference, which implies a low absolute value of the excited level energy, then the corresponding level appears longlived, metastable. The transition time between such states is measured in seconds, hours and even years. The nucleus in its metastable excited state and a similar nucleus in its ground energy state are called isomers, and the metastable levels, isomeric levels.

2. Nuclear isomerism. Nuclear isomers are found among stable, and predominantly among β -radioactive nuclides. In the case of the stable nuclide, the transition into the ground state from the excited isomeric level formed as a result of a nuclear reaction or the preceding decay is performed by emitting a γ -quantum. This time, the substance is only γ -radioactive obeying the decay law (3.4). The isomeric excited level of the β -radioactive nuclide does not necessarily transform into the ground state with the emission of a γ -quantum but may undergo independent β -decay. Half-lives, both with respect to the γ -quantum emission and to β -decay, depend upon the transition energy and especially upon the spin difference of the initial and final states. The half-lives of the radioactive isomers of the same nuclide are, therefore, always different. The presence of γ -radioactivity in a substance stable to α - and

β -decays and the presence of two, or in some cases of three, half-lives of the radioactive substance implies the formation of the metastable nuclear levels in some preceding process, i.e., the phenomenon of nuclear isomerism.

The first pair of isomers was detected among the radioactive uranium decay products just by the two half-lives of the same Pa^{234} nuclide. These are β^- -radioactive UX_2 and UZ (see Fig. 3.5). At present, more than 250 twos and even threes of isomers with the half-life of the metastable level of more than a millisecond are known [13, 14]. Fig. 3.11 presents isomer decay schemes, each of their nuclear levels having its designations for energy, spin and parity of the wave function (+ or -), and unstable isomers for the half-life as well. The Al^{26} isomers (Fig. 3.11a) undergo only independent β -decay because the life-time, with respect to the γ -quantum emission with the nuclear spin change by $\Delta I = 5$, proves so long that a β -decay due to weak forces but with no change in spin or parity of the wave function of the initial and final nuclear states takes place, rather than the electromagnetic transition. The metastable level of the Kr^{85} nuclide (Fig. 3.11b) undergoes a mixed decay both with the γ -quantum emission and formation of the Kr^{85} isomer of the lowest energy, and directly a β -decay. One of the three In^{114} isomers (Fig. 3.11c) disintegrates only with the γ -quantum emission, transforming into the second isomer which undergoes mixed decay. β -Transitions from a metastable level may compete with γ -quantum emission only in case the difference in the spin levels of the parent and daughter nuclei is much less than the difference in the isomer spins. All the cases of the independent decay of isomers result in the formation of different levels of the daughter nuclei the spins of which are close to the spins of one or another of the decaying isomers (see Fig. 3.11a, b, c).

Among the isomers of the nuclide stable in the ground state, one is unstable and transforms into the stable isomer during the emission of a γ -quantum. Sr^{87} (Fig. 3.11d) may be an example. The case of Sr^{87} is, however, unique in the sense that the ground state of the radioactive Rb^{87} nuclide which itself, due to β -decay, transforms into Sr^{87} happens to have a lower energy than the energy of the Sr^{87} isomer which is of the highest energy. Since the spin difference of this isomer and of the ground energy state of Rb^{87} is not great, K -capture, i.e., the process of β^+ -transformation successfully competes with γ -quantum emission. Thus, instead of emitting a γ -quantum, the Sr^{87} nucleus in the metastable state may first absorb the atomic electron with the formation of the Rb^{87} nucleus and then eject an electron, and then again transform into the Sr^{87} nucleus. Such a way of decaying is accompanied by the emission of a neutrino in every transition [see (3.27), (3.25)]. This

case shows once more that the direction of the process of the nucleon β -transformation is determined only by the values of the total energy of the initial and final nuclear states.

3. Isomeric abundance. Nuclear isomers are nonuniformly abundant among nuclides of different mass numbers. The highest number of nuclear isomers is observed in the following ranges of the numbers of protons and neutrons in the nuclear composition: from 39 to 49, from 69 to 81 and from 111 to 125, i.e., with the proton and neutron numbers preceding the magic numbers 50, 82 and 126 (2.26). Such a distribution of isomers at least with odd mass numbers is in agreement with the nuclear shell model (Sec. 2.6-3) which predicts that, with great l 's the states with different mutual orientation of l and s are found in different nucleon shells and are, in the energy scale, in the vicinity of states of very low values of the mechanical moment. States with great $I = l + s$ often appear to be the last in the shell. Thus, in an under-filled shell, one may really expect such a situation where the last filled state corresponds to the low value of the nuclear spin and the first unfilled state, which is the first excited level in this nucleus, corresponds to a very high value of the mechanical moment. Transitions between such states are most difficult in both directions, and metastable levels are not at all excited from the ground levels. However, as a result of the preceding decay or the cascade de-excitation in high excitation, the nucleus may happen to be in the state with the spin rather different from the spin of the ground level or from the spins of several lower-lying levels. In this case, further de-excitation of the nucleus is extremely slow and this nucleus has more than one isomer.

4. Electrons of internal conversion. The emission of γ -quantum is not the only process resulting in the release of the energy excess from the atomic nucleus. The Coulomb field of the nucleus can transfer all the excitation energy directly to the atomic electron. The nucleus transforms into its ground state without emitting γ -quantum, and the atom ejects an electron of internal conversion. Most probable is the transfer of energy to the K -electron nearest to the nucleus. However, the emission of electrons of internal conversion, with L , M , etc., atomic shells is possible. Since the excitation energy of the nucleus, E^* , is a strictly definite value, the kinetic energy of the electrons of internal conversion, E_e , is also the same in all cases of the electron emission from the given shell:

$$\begin{aligned} E_e^{(K)} &= E^* - E_K \\ E_e^{(L)} &= E^* - E_L \end{aligned} \quad (3.38)$$

where E_K, E_L, \dots , are the bond energies of electrons in the corresponding shells of the atom. Thus, the energy spectrum of electrons

of internal conversion is a line spectrum in distinction from the continuous spectrum of electrons emitted during β -decay. If the excitation energy of the nucleus $E^* < E_K$, then the separation of the K -electron from the atom does not appear feasible and electrons of internal conversion can be ejected only from the subsequent electron shells.

Apart from the possibility of emitting γ -quanta, there is a probability of emitting electrons of internal conversion by the nucleus. Otherwise stated, part of the excited nuclei transfer their energy to γ -quanta, and others, to the atomic electrons. The fraction of both decays of excited nuclei is described by the internal conversion coefficient α . The emission of electrons of internal conversion is an additional mechanism of releasing excess energy by nuclei, but not a conversion of γ -quanta pre-emitted by nuclei, although in principle such a process is also possible. Therefore, the number of decays with the emission of conversion electrons is usually determined with respect to the number of ordinary decays with the emission of γ -quanta, N_γ , but not to the total number of decays, $N_e + N_\gamma$, and the internal conversion coefficient in this case has the form of

$$\alpha = \frac{N_e}{N_\gamma} \quad (3.39)$$

The absolute value of the coefficient α is the higher, the longer the life-time with respect to the emission of a γ -quantum, and the higher Z of the nucleus, i.e., the closer the electron shells of the atom to the nucleus.

5. X-radiation and Auger-electrons. As a result of the emission of the electron of internal conversion, the atomic nucleus passes to its ground state. The atom, however, remains excited because of the lack of an electron in one of its shells. The vacancy is quite rapidly, during $\sim 10^{-15}$ s, filled with the electron from the outer electron shell. Therefore, the emission of the internal conversion electrons is accompanied by the radiation of the characteristic X -ray quanta or by the emission of the Auger-electrons. The characteristic X -radiation belongs to the same chemical element from the atoms of which the conversion electrons were emitted.

The emission of the electron of internal conversion is most probable from the K -shell. In this case, the excitation energy of the atom is equal to the bond energy of the lost electron, E_K . Filling the vacancy in the K -shell occurs mainly in the transition of the electron from the nearest L -shell and the atom emits a K_α X -ray quantum. However, the transition of the electron from the L - into the K -shell may take place without emitting an X -ray quantum but with ejecting one more electron, and again predominantly from the L -shell, and, hence, with the kinetic energy equal to $E_K - 2E_L$.

where $2E_L$ is the bond energy of the two electrons in the L -shell. These electrons are called Auger-electrons after the French physicist who discovered this effect. Having emitted the Auger-electron, the atom appears to be doubly-ionized in the L -shell (much rarer in other shells) and before the atom has completely released its excitation energy, low-energy electron transitions should occur in the atom and the addition of electrons which the atom lacks. In the formation of a vacancy in the K -shell, the lightest atoms emit in the main the Auger-electrons, not the characteristic X -ray quanta. Whereas, atoms with $Z > 32$ mainly emit X -ray quanta and with less probability the Auger-electrons [10].

6. Conversion with pair production. One more mechanism of releasing the excess energy from the nucleus is called conversion with pair production. If the excitation energy of the nucleus $E^* > 2mc^2 = 1.02$ MeV, then in the Coulomb field of the nucleus, an electron-positron pair may be produced (Sec. 1.8), which carries off all the excitation energy of the atomic nucleus. As the emission of conversion electrons, conversion with pair production is not the conversion proper, i.e., is not the transformation of a γ -quantum, previously emitted from the nucleus, into an electron-positron pair, but is an additional method of giving off the nuclear energy into the outer space. The probability of this process is always low in comparison with the probability of emitting a γ -quantum. Near the threshold, the fraction of the decays of the nuclear excited levels with pair production is $\sim 10^{-4}$ of the decays with γ -quantum emission, and with the excitation energy close to the bond energy of the nucleon in the nucleus, it makes some tenths of per cent. In contrast to the internal conversion coefficient (3.39), the probability of conversion with pair production is slightly lower with the increase of Z of the nucleus as well as with the increase of the nuclear transition multipolarity [16].

The kinetic energy released in the process of pair production, $E_p = E^* - 2mc^2$, is distributed between the electron, the positron and the remaining atom. Since the atomic mass is large, all the released energy is practically carried off by the electron and the positron. Their energy distributions are continuous, are in the range from zero to the maximum value of E_p and are symmetric with respect to $E_p/2$, in neglecting the interaction of these particles with the Coulomb field of the nucleus.

3.5. Nuclear Reactions

1. Definitions. The process of the production of new nuclei and elementary particles in collisions of particles and nuclei is called nuclear reaction. If upon collision original nuclei and particles remain and no new nuclei or particles are produced, then the pro-

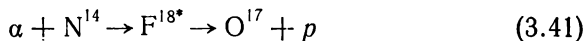
cess is called scattering. Rutherford was the first to observe nuclear reaction in 1919, bombarding nitrogen atomic nuclei by α -particles. Nuclear reaction was detected by the appearance of secondary ionizing particles having the path in gas longer than the α -particle path and identified as protons. Subsequently, photographs of the process were taken with the help of the cloud chamber (see Fig. 1.5). This reaction can be presented by the expression



There is an abbreviated expression of nuclear reaction: $\text{N}^{14}(\alpha, p)\text{O}^{17}$, where before the parenthesis and after the parenthesis are the target nucleus and the final nucleus, respectively, and within the parentheses, the bombarding particle and the product-particle. If some class of nuclear reactions irrespective of the nuclei on which they take place is meant, we speak of the (α, p) , (p, n) , (n, γ) , etc., reactions, where the bombarding particle and the product-particle are put in parentheses.

2. Mechanism of compound nucleus. Nuclear reactions with not very high kinetic energy of colliding particles, at least up to 10 MeV, proceed in two stages through the mechanism of a compound nucleus. The first stage of the reaction is the absorption of the bombarding particle by the target-nucleus and the production of an intermediate, or compound, nucleus. The compound nucleus is always highly excited because the absorbed particle brings both its kinetic energy and the bond energy of the absorbed nucleons into the produced nucleus. The last component of the excitation energy is equal to the bond energy difference of the new nucleons in the compound nucleus and in the original particle if this particle is compound, as, for example, the α -particle. The second stage of the reaction is the decay of the compound nucleus with the emission of this or that particle. The original particle may always be such a particle, and here again the original nucleus is formed, and scattering is observed instead of nuclear reaction. Scattering with the production of a compound nucleus is called *resonance* scattering in distinction from *potential* scattering without production of a compound nucleus. Possible ways of the decay of the compound nucleus are called *channels* of nuclear reaction.

In view of the above, reaction (3.40) should be written in the form



the index (*) meaning that the compound nucleus has some energy excess.

3. Excitation energy. The excitation energy, E^* , of the compound nucleus produced in absorbing a free nucleon is equal to the sum

of the bond energy E_b of the nucleon and the portion E' of its kinetic energy (3.54):

$$E^* = E_b + E' \quad (3.42)$$

Usually, due to the great difference between the masses of the nucleus and the nucleon, E' is approximately equal to the kinetic energy E of the nucleon bombarding the nucleus and, with sufficient accuracy, the excitation energy of the compound nucleus is

$$E^* = E_b + E \quad (3.43)$$

The bond energy is, on the average, 8 MeV, slightly changing depending on the individual characteristics of the compound nucleus being produced. However, for the given target nucleus and the nucleon this value is a constant. The kinetic energy of the bombarding nucleon may be of any value. In initiation of nuclear reactions by neutrons the potential of which has no Coulomb barrier, the value of E may be close to zero. Thus, E_b is the minimum value of the excitation energy of the compound nucleus, and since it changes in megaelectron-volts, the compound nucleus is, in fact, always highly excited. If nuclear reaction is initiated by a charged particle, then the contribution of the kinetic energy to the excitation energy is also large since only with a sufficiently high kinetic energy charged particles can overcome the Coulomb barrier (2.15) and initiate nuclear reaction.

The bond energy of the particle in the nucleus is the energy required for its separation from the nucleus. As $E^* > E_b$, the condition is present of the reverse escape out of the compound nucleus of the particle, in the capture of which the compound nucleus has been originated. Thus, resonance scattering can always accompany nuclear reactions. It proves negligible if the compound nucleus, with prevailing probability, decays in other channels leading to some nuclear reactions.

4. Energy and spin limitations. The compound nucleus formed in the intermediate stage of the reaction does not differ in its properties from any other nuclei. It is called compound only because it has absorbed a particle, is highly excited and, hence, unstable, but the latter is, in general, characteristic of any excited nucleus. As with other nuclei, the produced nucleus possesses the inherent system of excited levels (Sec. 2.7-2). It means that not any amount of energy (3.43) can be transferred as excitation energy to the nucleus, but only a strictly definite set of energy portions, each of which must coincide with the energy of some nuclear level, with an accuracy determined by the level width (Sec. 2.7-4). Since E_b is a constant for the given nucleus-particle pair, the coincidence (3.43) with the energy values of the levels of the intermediate nucleus can be provided only on account of

the kinetic energy of the bombarding particle. Thus, nuclear reaction is possible only at some selected values of the kinetic energy of the bombarding particle. With any other of its values, the compound nucleus is not formed, and in the collision of the particle with the target nucleus only scattering called potential takes place.

The second limitation is associated with the spin of the nucleus. Each excited level is characterized by its own mechanical moment J (Sec. 2.7-5). The target nucleus in its turn has the spin I in its ground state, and the particle possesses the spin s . The relative motion of the particle and the nucleus is also characterized by its angular momentum l . However, at moderate energies of bombarding nucleons, $l = 0$ more often, and with $E < (10^4 - 10^5)$ eV, $l = 0$ always, due to a very high value of λ (1.14) exceeding the nuclear radius. With $l = 0$, the total angular momentum of colliding particles may be in the range from $|I + s|$ to $|I - s|$ through unity (Sec. 1.4-9), and if the bombarding particle is a nucleon, then the mechanical moment is either $I + 1/2$ or $|I - 1/2|$. If the spin J of the excited level of the compound nucleus is not equal to one of the possible values of the total mechanical moment of colliding particles, then the formation of the compound nucleus is impossible, and in collision, one observes only potential scattering of the bombarding particles. If J falls within the range from $I + s$ to $|I - s|$, the formation of the compound nucleus is possible, but only in cases when the equality of the total momentum of the colliding particles with the excited level spin of the compound nucleus, J , is realized. The fraction of such collisions is determined by the statistical factor g (1.21), all the other collisions being accompanied by potential scattering of particles.

5. Substantiation of compound nucleus mechanism. Experimental data confirm the mechanism of the compound nucleus. First of all, in detecting nuclear reactions, one may observe resonances in the dependence of the reaction cross-section on the energy of the bombarding particle or the abrupt increase of the cross-section at some values of the kinetic energy (see Fig. 4.12). It is in the formation of the compound nucleus, i.e., of the bound target nucleus-particle system, and with the participation of the captured nucleon in orbital motion in the new nucleus that the interaction cross-section may be as high as $\pi\lambda^2$, (Sec. 1.4-4). On the contrary, beyond the resonance range, the cross-section is small because potential particle scattering takes place there without any compound nucleus formation.

The measurement of the resonance widths allows one to estimate the mean life-time of the compound nucleus (1.20). This life-time proves very long (10^{-14} s) in some cases, in comparison with the characteristic time of nuclear interaction (10^{-23} s) (Sec. 1.7-5).

Such estimates are also in favour of the compound nucleus formation in nuclear reactions since the direct collision of the bombarding particle with one or several nucleons in the nucleus accompanied by knocking the secondary particle out of the nucleus must take time comparable with 10^{-23} s. In fact, a particle colliding with the nucleus is captured by it if the energy and spin relations are met. It means that, because of the strong interaction between nucleons, the energy (3.43) introduced by the particle for a time less than the time of the particle flight over the diameter of the nucleus is distributed among all or many nucleons. Although the compound nucleus has a great excess of energy, at least sufficient for the reverse ejection of the captured particle, the nuclear decay process is delayed because each excited nucleon has the energy too low to escape out of the nucleus, and the probability of the transfer of all the excitation energy distributed among many nucleons to a single nucleon is negligible. Thus, the great value of nuclear forces ensures efficient energy exchange between nucleons in the excited nucleus and prevents the compound system of particles, the compound nucleus, from fast decay due to the same nuclear forces.

Under some circumstances, the decay of the compound nucleus with the emission of a nucleon may appear to be so prolonged that decay with γ -quantum emission due to weaker electromagnetic forces begins to successfully compete with it. Continuous energy exchange between the nucleons of the compound nucleus hinders the γ -quantum emission too. However, with the sufficiently great life-time of the compound nucleus, this process may turn out to be predominant. After the γ -quantum emission, the nucleus transforms into its ground energy state, and the process as a whole is called *radiative capture* of a particle. Radiative capture resonances are usually of very narrow widths because electromagnetic forces determining γ -quantum emission can compete with nuclear forces, due to which nuclei eject nucleons, only at very long life-times of compound nuclei (1.20).

Finally, the spherical symmetric distribution of the directions of the escape of nuclear reaction products and of resonance scattering bears witness to the compound nucleus production. The direction of the motion of the particle which has initiated nuclear reaction can readily be detected in experiment because it coincides with the axis of the particle beam bombarding the target. The absence of any correlation of the directions of secondary particles with respect to the direction of the bombarding particles is in agreement with the notion of the formation of the compound nucleus, a bound nucleus-particle system. If a particle is captured by the nucleus, its location in space is limited by the size of the nucleus, and in this case the particle momentum has no definite

value (Sec. 1.4-5). Hence, the connection between the momentum of the captured particle and the momentum before the capture has no sense. The momenta of the particle before the emission and after it, during the decay of the compound nucleus, are in the same relation. Consequently, no relation can exist between the momenta of the primary, bombarding, and the secondary particle, resulting from the reaction. The emission of product-particles proceeds equally probably in all directions. True, in the laboratory coordinate system, the asymmetry for the directions of the forward and backward escape with respect to the direction of the motion of the primary particle is possible. This asymmetry is not of the nuclear origin but is connected with the motion of the compound nucleus itself in the laboratory coordinate system in the direction of the bombarding particle momentum (Sec. 3.5-11). In the inertia centre coordinate system, where the compound nucleus rests, the distribution of the directions of the secondary particle momenta is spherically symmetrical. On the contrary, the asymmetry of the distribution of the directions in the inertia centre system always becomes apparent if the bombarding particle initiates nuclear reaction in the direct interaction with one or several nucleons of the nucleus.

6. Direct interaction. It is possible for nuclear reactions to proceed through the mechanism of direct interaction. However, such a mechanism shows up mostly at very high energies of the bombarding particles when the nucleons of the nucleus may be considered free. At the same time, already at energies near 10 MeV, this second mechanism begins to manifest itself. The direct interaction differs from the mechanism of the compound nucleus, first of all, in the distribution of the vectors of the product-particles momenta with regard to the momentum of the bombarding particles. As distinct from the spheric symmetry of the compound nucleus mechanism, the direct interaction is characterized by the predominant direction of the flight of the reaction products forward with respect to the direction of the primary particles motion. The energy distributions of the product-particles are also different in these two cases. An excess of high-energy particles is characteristic of the direct interaction. In collisions with the nuclei of compound particles, i.e., also nuclei, processes of transferring nucleons from one nucleus to another, or the process of nucleon exchange, are possible. These reactions take place without producing compound nucleus, and all the characteristics of direct interaction are inherent in them.

7. Law of energy conservation. All the conservation laws of classical physics are fulfilled in nuclear reactions. Due to the work of the strongest forces of nature, nuclear forces, the free energy quantities are so high in nuclear reactions that they appear to be noticeable in comparison with the rest energies of the particles

participating in reactions. For nuclear reactions, the law of energy conservation should, therefore, be formulated in the most general form: the total energy of the particles before the reaction is equal to the total energy of the new particles after the reaction, where the total energy means the energy (1.23) or (1.24), including the rest energy of the particles. If \mathcal{E}_1 , \mathcal{E}_2 , \mathcal{E}_3 and \mathcal{E}_4 are the total energies of two particles before the reaction, and of two particles after the reaction, then, on the basis of the law of energy conservation

$$\mathcal{E}_1 + \mathcal{E}_2 = \mathcal{E}_3 + \mathcal{E}_4 \quad (3.44)$$

In the production of more than two particles, the number of components on the right-hand side of (3.44) should be correspondingly higher. The total energy of the particle is the sum of its rest energy Mc^2 , M being the rest mass, and of the kinetic energy E [see (1.29)]. Therefore, (3.44) is reduced to

$$M_1c^2 + M_2c^2 + E_1 + E_2 = M_3c^2 + M_4c^2 + E_3 + E_4 \quad (3.45)$$

The quantity

$$Q = (E_3 + E_4) - (E_1 + E_2) \quad (3.46)$$

is known as the reaction energy. As the target nucleus is usually at rest, $E_2 = 0$ and the reaction energy equals

$$Q = (E_3 + E_4) - E_1 \quad (3.47)$$

where E_1 is the kinetic energy of the particle bombarding the nucleus, and E_3 and E_4 are the kinetic energies of the reaction products. Taking into account (3.46),

$$M_1 + M_2 = M_3 + M_4 + Q/c^2 \quad (3.48)$$

The factor $1/c^2$ is usually omitted and it can be written

$$M_1 + M_2 = M_3 + M_4 + Q \quad (3.49)$$

bearing in mind that in the balance computation, it is necessary to express either the particle masses through the energy units or the energy in the mass units (2.8). If $Q > 0$, then the reaction proceeds with the release of free energy and is called *exoenergetic*. If $Q < 0$, then the reaction is accompanied by the absorption of free energy and is termed *endoenergetic*. It is easy to see that $Q > 0$ when the mass sum of the product particles is less than the mass sum of the original particles, i.e., the free energy release is feasible only on account of the decrease of the mass of the reacting particles. On the contrary, if the mass sum of the secondary particles exceeds the mass sum of the primaries, then such reaction is possible only on condition that some quantity of the kinetic energy is

spent on the increase of the rest energy, i.e., of the mass of the new particles. This spent energy is usually a part of the kinetic energy of the bombarding particles (Sec. 3.5-11), and to initiate the endoenergetic reaction, E_1 should be at least higher than the energy of the reaction, Q . The minimum value of E_1 at which the endoenergetic reaction is feasible, is called the threshold energy (1.44).

The law of conservation of energy in the form of (3.49) may be directly confirmed in experiment. The masses of all the participating particles are measured by the mass-spectrometer (Sec. 2.4-4). The energy of the reaction, Q , can also be independently measured by controlling the kinetic energy E_1 of the bombarding particle and by determining the kinetic energies E_3 and E_4 of the reaction products. If the masses and the reaction energy are expressed in the same units, it turns out that in nuclear reactions, the value of the reaction energy is much higher than the experimental error in the measurement of the value $(M_1 + M_2) - (M_3 + M_4)$. This means that the comparison of the measured values $[(M_1 + M_2) - (M_3 + M_4)]c^2$ and Q is possible. Experience proves with high accuracy the validity of expression (3.49). Following the law of energy conservation (3.49), one can, therefore, determine both the unknown masses of particles and the reaction energies. In fact, if the mass of one of the four particles participating in the process is unknown, then by the masses of the three particles and by the measured reaction energy the mass of the fourth particle can be found. The precision of this method will not be worse than the precision of mass measurement in the mass-spectrometer, and this method is widely used for measuring the atomic masses. In particular, it is just on the basis of the law of energy conservation that the neutron mass was measured, which cannot be measured in the mass-spectrometer because the neutron has no electric charge. If the masses of all the four particles are known, the reaction energy is directly computed.

Expression (3.49) is universal. It can, however, be checked experimentally only in the case of nuclear reactions. In chemical reactions, i.e., in molecular transformations, the reaction energies Q are so low that their absolute values in corresponding units are much lower than the errors in the molecular mass measurement performed by means of most precise up-to-date techniques.

8. Law of momentum conservation. The total momentum of particles before the reaction equals the total momentum of the product-particles of the reaction. If \mathbf{p}_1 , \mathbf{p}_2 , \mathbf{p}_3 and \mathbf{p}_4 are the vectors of two particles momenta before the reaction, and of two particles after the reaction, then, on the basis of the momentum conservation law,

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_3 + \mathbf{p}_4 \quad (3.50)$$

If the target-nucleus is at rest, then $\mathbf{p}_2 = 0$ and

$$\mathbf{p}_1 = \mathbf{p}_3 + \mathbf{p}_4 \quad (3.51)$$

The absolute values of the particle momenta can be measured with the help of magnetic spectrometers (2.10) or by measuring the energies of the particles since the energy and the momentum modulus there are directly related as: $E = p^2/2M$ or (1.24), (1.29) at relativistic velocities. Angles between the momentum vectors are measured by observing reactions in tracking devices or with the help of other particle detectors so oriented with respect to the target, where nuclear reactions are initiated, as to simultaneously detect both product-particles. Thus, each vector can be measured independently in experiment, and relations (3.50) or (3.51) be subject to verification. The experimental data, in total, confirms the validity of the momentum conservation law both in nuclear reactions or transformations and in the processes of microparticle scattering.

9. Law of mechanical moment conservation. Angular momentum is conserved in nuclear reactions. Collisions of microparticles result in the production of only such compound nuclei the mechanical moment of which is equal to one of the possible values of the mechanical moment obtained when summing up the mechanical moments of particles and the mechanical moment of their relative motion (Sec. 3.5-4). The ways of the compound nucleus decaying can also be only such in which the angular momentum would be conserved. No exceptions to these rules are experimentally observed.

10. Other conservation laws. Electric charge is conserved in nuclear reactions: the algebraic sum of elementary charges before the reaction is equal to the algebraic sum of charges after the reaction. The number of nucleons is conserved in nuclear reactions, which, in the most general case, is interpreted as the baryon charge conservation. If the kinetic energies of colliding nucleons are very high, then reactions of producing nucleon pairs are possible (1.91). Since opposite signs of the baryon charges are assigned to nucleons and antinucleons, the algebraic sum of the baryon charges always remains unchanged in any processes. In nuclear reactions which proceed under the action of nuclear or electromagnetic forces, the wave function parity describing the state of the particles before and after the reaction is conserved. The wave function parity is conserved in the transformation due to weak forces. All the conservation laws set some limit on the possibility of attaining chain reactions. A process advantageous from the energy point of view always appears impossible if its realization leads to the violation of some conservation law.

11. Contribution of kinetic energy to excitation energy. The target nucleus is usually at rest. The total momentum of the particle-nucleus system is the momentum of the particle, and, on the basis of the conservation law, the momentum of the compound nucleus that has captured the particle is equal to the momentum of the particle. As energy is always associated with momentum, it means that some part of the kinetic energy of the particle, E , is conserved as the kinetic energy of the compound nucleus, E_n . The remaining energy, $E' = E - E_n$, is transferred to the compound nucleus as an excitation energy component. Denoting the momentum of the particle, $M_1 v_1$, the mass of the target nucleus, M_2 , and the mass of the compound nucleus, neglecting its increment due to the absorbed energy, $M_1 + M_2$, we obtain

$$M_1 v_1 = (M_1 + M_2) v_n \quad (3.52)$$

where v_n is the velocity of the compound nucleus motion, which is also the velocity of the inertia centre of the particle and of the target nucleus, because, having been absorbed, the particle does not move relative to the nucleus. Hence, the kinetic energy of the compound nucleus is

$$E_n = \frac{1}{2} (M_1 + M_2) v_n^2 = E \frac{M_1}{M_1 + M_2} \quad (3.53)$$

and

$$E' = E - E_n = E \frac{M_2}{M_1 + M_2} \quad (3.54)$$

transforms into the excitation energy. Usually $M_1 \ll M_2$ and $E' \approx E$. However, $E' < E$, in principle, and with the low mass of the target nucleus, E' may greatly differ from E .

The energy E' is the total kinetic energy of the particle and of the target nucleus in the coordinate system of the inertia centre. Only this kinetic energy transforms into the excitation energy of the compound nucleus and may turn to the rest energy. For this reason, the threshold energy of the endoenergetic reaction is higher than the reaction energy Q (Sec. 3.5-7). Only in the particular case of the coincidence of the coordinate system of the inertia centre and of the laboratory coordinate system, which corresponds to the motion of the particle and the nucleus towards each other with momenta equal in modulus, $E' = E$, E being the sum of the kinetic energies of the particle and the nucleus. All the kinetic energy transforms into the excitation energy of the compound nucleus, and the threshold energy of the endoenergetic reaction equals Q .

12. Reaction yield. The number of reaction events referred to the number of the particles which have bombarded the target, or $\Delta n^F/n_0^F$ in (1.48), is called *nuclear reaction yield*. Reaction

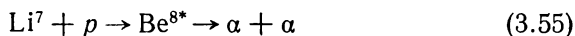
yield is usually a value directly determined in experiment during qualitative measurements. Since the yield is simply related to the reaction cross-section, measuring the yield is measuring the cross-section which is the most significant characteristic of each nuclear process.

13. Reaction under the action of charged particles. The electric charge of atomic nuclei hinders the initiation of reactions due to charged particles, α -particles, protons, deutons, etc. For such a particle to reach the range of action of nuclear forces, it should possess some kinetic energy sufficient to overcome the Coulomb potential barrier (2.15) which is of the order of 1 MeV even in the interaction of singly charged particles with the lightest particles. True, nuclear reactions are also feasible at energies E below the potential barrier. These are so-called under-barrier reactions. Just as an α -particle in α -decay passes through the potential barrier, with the probability (3.22), beyond the boundaries of the nucleus, so in colliding with the nucleus it can with the same probability appear within the boundaries of the nucleus. The penetrability of Coulomb barrier increases very rapidly as E approaches U_c (2.15). Therefore, if E does not greatly differ from U_c , the under-barrier reactions take place with remarkable probability. The yield of the reactions at the under-barrier kinetic energy is, of course, low and rapidly decreases with the decrease of E . Some strongly exoenergetic reactions on the lightest nuclei for which the yield remains essential even with $E \approx 0.01$ MeV are the only exception.

Under the action of protons and α -particles the most probable are reactions with the neutron emission, (p, n) and (α, n) . The probability of reactions (p, α) or (α, p) is much less because the same Coulomb barrier prevents the charged particle from escaping out of the nucleus. Therefore, if, as a result of the capture of a proton or an α -particle, the formed compound nucleus has its excitation energy exceeding the bond energy of the neutron, a neutron is emitted during the decay of the compound nucleus. Charged particles as well as neutrons are ejected by the compound nuclei at high excitation energies. In cases when the excitation energy of the compound nucleus is lower than the bond energy of the neutron, either scattering of charged particles $[(p, p), (\alpha, \alpha)]$ or radiative capture $[(p, \gamma), (\alpha, \gamma)]$ take place. Narrow resonances due to the long life-time of the compound nucleus emitting a γ -ray quantum are characteristic of the reactions of the type (p, γ) .

Reactions due to protons and α -particles are most interesting on light nuclei where the Coulomb barriers are not very high. The (α, p) -reaction (which is not the most probable one) on the N^{14} nucleus (3.41) appeared to be just the first of the detected nuclear reactions. Its yield is $2 \cdot 10^{-5}$, and $Q = 1.06$ MeV. The reaction (α, p) may be initiated on many other light nuclei by α -particles

of spontaneous emitters. The discovery of the neutron is also associated with the bombardment of light nuclei by α -particles. The $\text{Be}^9(\alpha, n)\text{C}^{12}$ reaction has the highest yield, $2.5 \cdot 10^{-4}$. Therefore, it is used in laboratory practice for the production of neutrons. The energy of this reaction is $Q = 5.44$ MeV. An example of the (p, α) -reaction can be the reaction on Li which is the first nuclear reaction performed with artificially accelerated particles (Cockroft and Walton, Great Britain, 1932):



The reaction is strongly exoenergetic ($Q = +17.3$ MeV) and has a noticeable yield even with $E_p = 0.01$ MeV. The Be^{8*} compound nucleus may decay in another way which leads to the $\text{Li}^7(p, \gamma)\text{Be}^8$ reaction with the production of Be^8 in its ground state and of a γ -quantum of the energy 17.2 MeV. Since the mass of the Be^8 nucleus is slightly greater than the sum mass of two α -particles, then Be^8 immediately decays into α -particles of the energy 0.1 MeV. Of the (p, n) -reactions, the $\text{Li}^7(p, n)\text{Be}^7$ and the $\text{B}^{11}(p, n)\text{C}^{11}$ reactions are of interest for the production of neutrons.

14. Specific character of reactions with deutons participation.

Reactions due to deutons possess some specific features of their own. The deuteron consists of one proton and one neutron, its bond energy is 2.22 MeV, or about 1 MeV per nucleon, which is much less than 8 MeV, the mean bond energy of the nucleon in most nuclei. In addition, the mean distance at which nucleons are spaced from each other in the deuteron composition is relatively long ($\sim 4 \cdot 10^{-15}$ m). The particle possessing such properties proves to be able to interact with nuclei not only with the production of a compound nucleus but by direct interaction. If in the deuteron nucleus collision, a compound nucleus is produced with the capture of both nucleons, then its excitation energy (3.43) appears to be very high due to the great difference in the bond energies of two nucleons in the nucleus and in the deuteron, i.e., about 14 MeV. Therefore, all the reactions due to deutons, (d, p) , (d, n) , (d, α) , are exoenergetic and have high yields.

Apart from the production of the compound nucleus, the reactions (d, p) and (d, n) can proceed in some other way. Because of the weak binding of nucleons in the deuteron and of the relatively great distance between them, the deuteron-nucleus interaction may result in the absorption of only one nucleon, while the other nucleon will remain beyond the boundaries of the nucleus and continue its motion predominantly in the direction of the initial flight. If the kinetic energy of the deuteron is lower than the height of the potential barrier of the nucleus, then the yield of the (d, p) -reaction turns out to be comparable with the yield of the (d, n) -reaction for light and intermediate nuclei, and for heavy nuclei it is

even several times higher than the latter. Such a behaviour of the yields of the (d, p) -, (d, n) -reactions contradicts the compound nucleus mechanism because in the decay of the compound nucleus the emission of protons is always more difficult than that of neutrons, and especially in the case of heavy nuclei. The predominant capture of neutrons by heavy nuclei in the interaction with deuterons of moderate energy takes place because, as a result of the strong electrostatic repulsion, the deuteron in coming close to the heavy nucleus is oriented by its proton in the direction of the nucleus. The capture of one of the nucleons of the deuteron by the nucleus at the under-barrier kinetic energy of the deuteron is called the Oppenheimer-Phillips process (after the physicists who assumed the existence of the above mechanism and described it theoretically).

At the very high kinetic energy of the deuteron, the probability for both the proton and the neutron to penetrate into the nucleus is about the same for all the nuclei. Nucleons, remaining free after reaction, move in a slightly diverging beam in the direction of the deuterons bombarding the target. The mean energy of these nucleons is approximately equal to half the initial energy of the deuterons. The process of the absorption of one of the nucleons of the deuteron by the nucleus at the under-barrier energy is called stripping reaction. The stripping reaction is used for the production of neutron beams of the energy of tens of megaelectron-volts. The contribution of the compound nucleus mechanism or of the direct interaction to the reactions of deuterons with nuclei of a given kind can be estimated by the relation of the yields of the (d, p) - and (d, n) -reactions, by the change of this relation with the increase of the deuteron energy and by the angular distributions of the reaction products. If the compound nucleus mechanism predominates, the yield of the (d, p) -reaction is lower than the yield of the (d, n) -reaction, the ratio of the former to the latter increases with energy, and the angular distribution of the reaction products is spherically symmetrical.

High yields of the reactions due to deuterons make these reactions attractive for the production of free neutrons, for example, the $\text{Be}^9(d, n)\text{B}^{10}$ -reaction. The deuteron-deuteron collision results in the (d, p) - and (d, n) -reactions with approximately equal probability: $d + d \rightarrow \text{H}^3 + p$ and $d + d \rightarrow \text{He}^3 + n$. The energy of the former reaction is $Q = 4$ MeV, that of the latter being $Q = 3.25$ MeV. The heavy hydrogen isotope, tritium H^3 , and the light He^3 isotope were discovered just in studying these reactions. Subsequently, He^3 was found in natural helium as a small admixture to He^4 . H^3 is a β -radioactive nuclide and, with $T = 12$ years, it transforms into He^3 : $\text{H}^3 \rightarrow \text{He}^3 + \beta^- + \bar{\nu}$. The collision of the deuteron with the triton, the tritium nucleus, is accompanied only by the (d, n) -reaction: $d + \text{H}^3 \rightarrow \text{He}^4 + n$. The reaction results in the tightly bound

He^4 nucleus, therefore the reaction is strongly exoenergetic, $Q = 17.6$ MeV. The neutron, the lightest of the product-particles, carries off the greater part of the released energy (14.1 MeV). This reaction is characterized by the lowest Coulomb barrier and by the high yield. It is, therefore, of particular interest for thermonuclear synthesis (Sec. 2.4-12). Its initiation requires a lower environmental temperature than that for any other possible nuclear reaction, of the order of tens of millions of degrees Kelvin. The deuteron reactions produce artificial radioactive nuclides, used in scientific research, for instance, C^{11} and P^{32} in the $\text{B}^{10}(d, n) \text{C}^{11}$, $\text{P}^{31}(d, p) \text{P}^{32}$ reactions.

15. Neutron reactions. The most common is the reaction of the neutron radiative capture (n, γ) , the cross-section of which is the higher, the lower the neutron energy (Sec. 4.5-5). The (n, p) - and (n, α) -reactions are unlikely because the compound nucleus originated in the neutron absorption ejects a neutron rather than a charged particle. Only in some cases of highly exoenergetic reactions on light nuclei, the (n, p) - and (n, α) -reactions proceed with prevailing probability. In heavy nuclei, neutrons readily initiate the fission process (n, f) (Sec. 3.6). The behaviour of the neutron reaction cross-sections as a function of energy is discussed in Sec. 4.5.

Neutron reactions have found wide practical application. The (n, f) -reaction serves as energy source in nuclear reactors. New fissile materials are produced as a result of the (n, γ) -reaction [(3.56) and (3.57)]. For thermonuclear synthesis, tritium is produced in the $\text{Li}^6(n, \alpha)\text{H}^3$ -reaction in radiating lithium by the neutrons of the nuclear reactor. In addition, artificial radioactive nuclides are produced by means of neutron reactions, and reactions accompanied by the emergence of charged particles $[(n, \alpha), (n, p), (n, f)]$ are used to detect neutrons. The most significant of them is the $\text{B}^{10}(n, \alpha)\text{Li}^7$ -reaction.

16. Neutron activation. Neutron reactions and first of all the (n, γ) -reaction are often the cause of induced radioactivity. The radiative capture of the neutron by the nucleus results in the production of an isotope of the same element with the mass number one unit higher. The newly formed isotope is stable if it is in the natural isotopic mixture of the element. Consequently, the emergence of the radioactive atom does not accompany every neutron absorption, and the total activity of the chemical element depends on its isotopic composition.

Sodium used as coolant in nuclear reactors may provide an example of a strongly activated substance. Sodium is a monoisotopic element with a sufficiently high neutron absorption cross-section. Each act of the neutron capture leads to the production of the radioactive Na^{24} , with $T = 15$ h, i.e., to sufficiently long-lived radioactivity. In the Na^{24} decay, γ -ray quanta of the energy up to

4 MeV are emitted. These are most dangerous in handling radioactive materials. Light water serving as moderator and coolant in thermal reactors is an example of a slightly activated substance. The isotopic composition of hydrogen and oxygen is such that activation of water, as a result of the (n, γ) -reaction, takes place but to a very low degree. Hydrogen consists of the H^1 and H^2 isotopes, and oxygen, of the O^{16} , O^{17} and O^{18} isotopes, the isotopes of the lowest mass numbers practically being 100% (99.99% H^1 and 99.76% O^{16}), i.e., the neutron capture in the bulk quantities of hydrogen and oxygen does not give rise to radioactivity. The radiative capture cross-sections of H^2 and O^{18} are very small, and one of the radioactive products of the capture reaction, tritium, does not give γ -radiation. As a result, water radioactivity arising from the (n, γ) -reaction appears to be negligible. More substantially water is activated due to the $O^{16}(n, p)$ -reaction. The cross-section of this reaction is, however, very small and the reaction is initiated only by neutrons of the energy ~ 10 MeV and higher, but such neutrons are too scarce in the nuclear reactor. Therefore, the total intrinsic radioactivity of water is very low.

17. Nuclear photoeffect. Since atomic nuclei emit γ -ray quanta, they must absorb them as well. In absorbing a γ -ray quantum, the nucleus acquires some excess energy without changing its nucleon composition, and the nucleus with excess energy is a compound nucleus. It is a matter of fact that absorption of a γ -ray quantum by the nucleus is feasible only if the required energy and spin relations are fulfilled. If the energy imparted to the nucleus exceeds the bond energy of the nucleon in the nucleus, the decay of the produced compound nucleus proceeds mainly with the emission of nucleons and, first of all, of neutrons. Such a decay results in nuclear (γ, n) - and (γ, p) -reactions which are called photonuclear reactions, and the phenomenon of emitting nucleons from nuclei due to γ -rays is called a nuclear photoeffect.

Photonuclear reactions proceed with the production of a compound nucleus. However, in initiating (γ, p) -reactions on nuclei with $A > 100$, an yield too high in comparison with the yield predicted by the compound nucleus mechanism is observed. In addition, the angular distribution of protons of the highest energy value appears nonisotropic. These facts point to an additional mechanism of direct interaction which is essential only in the case of the (γ, p) -reaction on heavy and intermediate nuclei. The (γ, n) -reaction always proceeds with the production of a compound nucleus.

The first photonuclear reaction observed was the reaction of photospitting of the deuteron: $\gamma + H^2 \rightarrow p + n$. It proceeds without producing the compound nucleus since the deuterium nucleus has no excited states. This reaction may be initiated by γ -ray quanta of

moderate energy. Nevertheless, there are only a few nuclides with a low nucleon bond energy (Sec. 2.4-7) and photons of the energy as high as 8 MeV are necessary to initiate photonuclear reactions. Photons of this energy emerge in some nuclear reactions or are produced in slowing down very fast electrons in the substance. In radioactive decay such γ -ray quanta are not produced, as a rule. Therefore, γ -ray quanta of β -decay cannot initiate photonuclear reactions or give rise to new induced radioactivity in other substances. From this it follows that no matter how great is the radioactivity of the coolant-substance circulating in the primary circuit of the nuclear reactor, the contact with the coolant of the secondary circuit in the heat exchanger cannot give rise to induced radioactivity in the secondary circuit although γ -quanta readily penetrate through the walls of the heat exchanger. The danger of such radioactivity can be associated only with the delayed neutrons, accompanying β -decay in rare cases. In coolant-substances of nuclear reactors containing oxygen, the emitter of delayed neutrons emerges in the $O^{17}(n, p)N^{17}$ -reaction. This reaction is, however, threshold, ($E_{thr} \approx 9$ MeV), its average cross-section for the fission spectrum is very small ($\sigma = 7 \cdot 10^{-6}$ barns), and the O^{17} content in natural oxygen is 0.037%. Therefore, the absolute value of radioactivity connected with N^{17} is low. The N^{17} nuclide decays rapidly, ($T = 4.14$ s), emitting delayed neutrons with the energy up to 1.8 MeV.

If the moderator in the reactor is berillium or heavy water, then because of the extremely low bond energy of the neutron in Be^9 and H^2 , the photonuclear (γ, n)-reactions efficiently proceed on the nuclei of these nuclides under the action of the γ -ray quanta of radioactive decay. Radioactive products of uranium fission give off quite a lot of γ -ray quanta. Some other neutron-activated materials of the nuclear reactor also emit γ -ray quanta. Thus, an additional neutron source due to the photonuclear (γ, n)-reaction is available in heavy-water and berillium reactors.

18. High-energy particles. Nuclear reactions due to very fast nucleons of the energy many times higher than the mean bond energy of the nucleon in the nucleus are accompanied by the escape of many nucleons out of nuclei. If such a nuclear reaction is detected in nuclear photoemulsion, then the tracks left by the charged particles emerging from the same point make a characteristic star. The analysis shows that there are usually a few very long tracks in the stars corresponding to high-energy protons emerging as a result of the direct interaction mechanism. However, shorter tracks corresponding to protons emitted in the decay of the compound nucleus are also observed. Also emitted, of course, are neutrons which leave no tracks in the emulsion, neutrons the escape of which can, no doubt, be also determined by the mecha-

nisms of both direct interaction and the compound nucleus. From this it can be inferred that a very fast nucleon penetrating into the nucleus, colliding with one or several nucleons, transfers the greater part of its energy to them. The remaining energy of the nucleons is distributed among many nucleons of the nucleus, which results in the production of a highly excited compound nucleus in whose decay some more nucleons are emitted. The estimations show that, with the nucleon energy about 100 MeV, only about half the energy is spent on direct interaction, while the second half is spent on the excitation of the remaining nucleus. Consequently, the compound nucleus mechanism is neither excluded in reactions due to very fast particles.

19. Reactions with meson and nucleon pair production. If the kinetic energy of colliding nucleons in the system of their inertia centre exceeds the rest energy of the π -meson, of several π -mesons, or of two nucleons, then the reactions of π -meson production (1.78), (1.79), (1.88), of the multiple production of these particles, and, finally, of nucleon pairs (1.91), respectively, are feasible, along with some other reactions. E_p 's, in the laboratory coordinate system, are determined by the mass of the colliding and emerging particles (1.39).

3.6. Heavy Nuclei Fission

1. Possibility of fission. In the heaviest nuclei, the mean bond energy of the nucleon is about 1 MeV lower than that in most stable nuclei (see Fig. 2.4). Since the bond energy is the attenuation of the rest energy of the particle in its bound state, the transformation of the heavy nucleus into two lighter ones is accompanied by the release of the free energy. If the process advantageous from the energy point of view is possible but does not proceed slowly, it means that the energy barrier hinders it. The separation of one part of the nucleus from the other is first accompanied by the increase of the potential energy which decreases only upon reaching some value, the height of the energy barrier, U_b (see Fig. 3.12). In fission, the barrier is determined by the forces of surface tension (Sec. 2.6-2), which tend to retain the spherical form of the nucleus, corresponding to the minimum of the surface potential energy. Consequently, the initial change of the shape of the nucleus which can result in fission is possible only by acquiring some amount of energy from outside, i.e., by exciting the nucleus.

The fission process is energetically advantageous for nuclei with mass numbers higher than 80. The energy gain is, however, very low at the beginning, and the barrier height, U_b , is so great that in the nuclear excitation, reactions with nucleon emission, not fission, take place. Only with the heaviest nuclei, the energy barrier

appears approximately equal to the value of the bond energy of the nucleon, so that the decay of compound nuclei along the channel of fission becomes substantial in comparison with other channels of decay, and in some cases is predominant. Experimental thresholds of fission due to γ -ray quanta give the picture of the absolute values of the barriers, U_b :

| | Nuclide | Threshold energy, MeV |
|-------------------|---------|-----------------------|
| Th ²³² | | 5.9 |
| U ²³³ | | 5.5 |
| U ²³⁵ | | 5.75 |
| U ²³⁸ | | 5.85 |
| Pu ²³⁹ | | 5.5 |

These data show that the potential barrier is 5.5 to 6 MeV with respect to the fission of the heaviest nuclei and depends but slightly on the composition of the nucleus. The relatively low values of the barriers make the probabilities of spontaneous fission of these nuclei experimentally measurable (Table 3.2).

Table 3.2

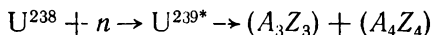
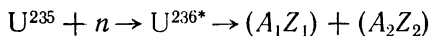
Rate of Spontaneous Fission

| Nuclide | Half-life in spontaneous fission, years | Number of fissions in 1 s per kg of substance |
|-------------------|---|---|
| Th ²³² | $1.4 \cdot 10^{18}$ | 0.04 |
| U ²³³ | $3 \cdot 10^{17}$ | 0.2 |
| U ²³⁵ | $1.9 \cdot 10^{17}$ | 0.3 |
| U ²³⁸ | $8 \cdot 10^{15}$ | 7 |
| Pu ²³⁹ | $5.5 \cdot 10^{15}$ | 10 |

By analogy with α -decay (Sec. 3.2-2) fission product-nuclei possessing wave properties have the probability of being beyond the energy barrier of a finite width different from zero. Otherwise stated, if fission is energetically advantageous then it is as well possible, with some low probability without any pre-excitation of the original nucleus.

2. Fissile and fissionable nuclides. Of greatest interest is neutron-induced fission of heavy nuclei as each fission act gives rise to new free neutrons capable of initiating subsequent fission acts, i.e., a possibility arises for producing a self-sustaining chain reaction. In distinction from fission due to γ -ray quanta where the

target nucleus is fissioned, when the process is initiated by neutrons a nucleus with the mass number a unit higher is fissioned. For example, when neutrons are absorbed by the U^{235} or U^{238} nuclei,



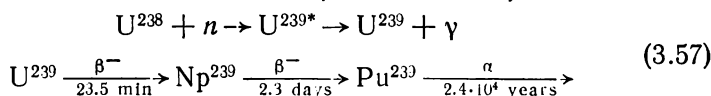
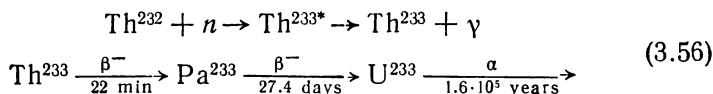
the compound nuclei of U^{236} and U^{239} are, in fact, fissioned. To ascertain the possibility of fissioning of the original nuclei, one should compare the excitation energies of compound nuclei produced in neutron capture with the values of the energy barriers. The minimum excitation energy of the compound nucleus is the bond energy of the neutron joining the nucleus (Sec. 3.5-3). If this bond energy is higher than the energy barrier, the original nucleus can fission in absorbing neutrons of any kinetic energy. If the bond energy is lower than the value of the barrier, then fission is possible only on condition that the kinetic energy of the neutron is high enough to exceed, when summed with the bond energy, the value of the barrier. The bond energies of neutrons in nuclei which are compound in fissioning of most significant heavy nuclides are listed below:

| Nuclide | Bond energy, MeV |
|----------------------|---------------------|
| Th^{233} | 5.07 |
| U^{234} | 6.77 |
| U^{236} | 6.40 |
| U^{239} | 4.76 |
| Pu^{240} | 6.38 |

The bond energy of the paired neutron is always higher than that of the unpaired neutron (Sec. 2.4-7). For this reason, the bond energy of the neutron in the nuclei of U^{234} , U^{236} and Pu^{240} appears to be higher than the value of the energy barrier of fission, and in the nuclei of Th^{233} and U^{239} it is lower because the value of the barrier differs but slightly in nuclei similar in composition (Sec. 3.6-1). This circumstance is responsible for the possibility of fissioning U^{233} , U^{235} and Pu^{239} by neutrons of any energies. Such nuclides are called *fissile*. On the contrary, Th^{232} and U^{238} can be fissioned by neutrons only of sufficiently high kinetic energy. Fission thresholds are 1.2 MeV in Th^{232} and about 1 MeV in U^{238} . These nuclides appear to be unable of sustaining a chain reaction (Sec. 6.1-8) and are called *threshold*. Among heavy nuclides, any other nuclides with an odd number of neutrons in the composition of nuclei are fissile, and those with an even number of neutrons are threshold. For example, Pu^{240} , like U^{238} , is threshold, while Pu^{241} is fissile. Nuclides formed in radiative capture of neutrons

in U^{233} and U^{235} , and in U^{234} and U^{236} , respectively, are threshold nuclides.

Th^{232} and U^{238} can be reprocessed into fissile nuclides, and as they exist in their natural form, they are considered to be suitable for reprocessing. The idea of reprocessing of fissionable nuclides is in joining one more neutron to the nucleus which is even in the number of its neutrons:



Th^{233} and U^{239} are produced as a result of radiative capture of neutrons in Th^{232} and U^{238} , respectively. The former rapidly decay producing short-lived products. After two successive β -decays, U^{233} and Pu^{239} are produced, which are odd in the number of their neutrons and, therefore, are fissile nuclides. They are α -active but have sufficiently long half-lives to be considered stable from the point of view of accumulation and storage. Accumulation of the fissile U^{233} and Pu^{239} nuclides is usually achieved in nuclear reactors, which always have a great excess of free neutrons.

The five heavy nuclides listed in Table 3.2 are most significant for nuclear engineering. Three of them, Th^{232} , U^{235} and U^{238} , are found in nature, and two, U^{233} and Pu^{239} , are produced from fissionable nuclides. Of fissile nuclides, U^{235} alone is presented in its natural form. Natural uranium contains 0.7% of U^{235} .

3. Fissioning mechanism. The fissioning process is explained on the basis of the drop model (Sec. 2.6-1). If the nucleus is imparted the activation energy, oscillations accompanied by the deviation from the original form appear in it (see Fig. 3.12). In an underformed state, forces of attraction are counteracted by forces of Coulomb repulsion which prevent nucleons from their most tight binding in the nucleus. The energy of nuclear attraction is proportional to the number of particles, and the energy of Coulomb repulsion is proportional to the square number of charged particles. Therefore, when the drop-nucleus is deformed and the neutrons and protons are dispersed, the efficiency of Coulomb repulsion in each half of the drop falls. If the activation energy is so high that $E_a > U_b$ (Fig. 3.12), critical deformation, $r = r_{crit}$, becomes feasible at which electric forces do not already prevent nuclear forces from binding nucleons more efficiently. It is, however, possible only in two new nuclei each of which has fewer protons. Increasing the bond energy of the nucleons participating in the process means that

the work of nuclear forces has resulted in the attenuation of the rest energy of all nucleons from the initial value U_{in} , taken to be zero in Fig. 3.12, to the final U'_{fin} , which is 180 MeV in absolute units.

4. Fission energy. On account of the work of nuclear forces two new nuclei, *fission fragments*, appear to be under very high electric potential. Electrostatic repulsion scatters the fragments, and the

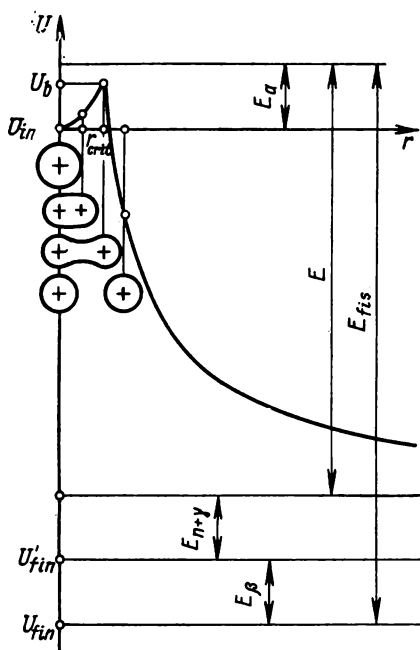


Fig. 3.12. Energy diagram and nuclear-fission deformation scheme (r is the distance between the effective charge centres of an oscillating nucleus and the centres of the fragments produced)

potential energy of the Coulomb field, U_C , transforms into the kinetic energy of the fission fragments equal to E as $r \rightarrow \infty$ (see Fig. 3.12). Mutual acceleration of the fragments is practically completed as they reach the boundaries of the original atom, $r \approx 10^{-10}$ m. Subsequently, moving in the substance, the fragments ionize other atoms (Sec. 4.2-2), and their kinetic energy transforms into the energy of the thermal motion of the environmental particles.

Part of the energy released in fission turns into the excitation energy of the fission fragments. The excitation energy of each fragment is much higher than the bond energy of the neutron so that in the transition of the new nuclei to the ground energy states, the first to be emitted are neutrons, one or two by each fragment, and then

γ -ray quanta (Sec. 2.7-1). Their energy in Fig. 3.12 is $E_{n+\gamma}$. Neutrons and γ -ray quanta emitted by the excited fragments are called *prompt*. If neutrons penetrate into the moderator-substance, they rapidly (see Table 5.5) lose their kinetic energy that transforms into the thermal motion energy. Finally, neutrons are absorbed by nuclei, usually in the (n, γ) -reaction, and the γ -ray quantum energy is also transformed into heat. The time before the absorption of neutrons is as short as 10^{-3} s. Therefore, the energy of neutrons and γ -ray quanta of radiative capture turns into heat, in fact, immediately after fission.

Upon slowing down in the medium, fission fragments are transformed into neutral atoms with nuclei in their ground energy states and are called *fission products*. Since stable heavy nuclei have an excess of neutrons in their composition as compared to stable nuclei of moderate mass numbers (Sec. 2.4-9), the fission products are supersaturated with neutrons and are β -radioactive (Sec. 2.4-11). Very many product-atoms are produced (Sec. 3.6-6), and each of them undergoes, on the average, three β -decays before it becomes stable. As a result of β -decays, the nucleons of the fission products become even more tightly bound in nuclei, their rest energy being reduced to the value of U_{fin} (see Fig. 3.12). The energy of radioactive decays, E_β , is distributed among β -particles and neutrinos, and its greater part is carried off by γ -ray quanta accompanying β -decay (Sec. 3.3-8). In rare cases of β -decay of fission products, *delayed* neutrons are emitted (Sec. 3.3-9). The energy of β -particles and of γ -ray quanta turns into heat while the energy of neutrinos is carried off beyond the boundaries of the medium as neutrinos do not practically interact with the substance (Sec. 1.9). The total fission energy, E_{fis} , includes both the instantaneously released energy and the β -decay energy, as well as the energy released in neutron absorption, which is shown in Fig. 3.12 as E_a , for one of the fission neutrons.

In a nuclear reactor, only the energy transformed into heat is of interest. In addition, a matter of principle is whether the energy carrier belongs to penetrating or non-penetrating nuclear radiation as well as the time of transforming the released energy into heat. Thus, fission fragments and β -particles transfer their energy to the substance immediately in the vicinity of the point of the heavy nucleus fissioning while neutrons and γ -ray quanta transfer the energy over great distances (Chapter Four). The energy of fission fragments, neutrons and prompt γ -ray quanta is transformed into heat immediately after the fission process. The energy of β -decay appears with a great time shift determined by accumulation and decay of many radioactive atoms (Sec. 3.1-6). Table 3.3 gives an approximate energy balance in U^{235} fission. The value of each constituent depends on the way of fissioning the compound nucleus. The energy of the γ -ray quanta of capture depends on the properties of nuclei absorbing neutrons. In nuclear reactors, three fourths of secondary neutrons remaining upon the subtraction of one neutron initiating subsequent fission are absorbed by uranium, all the rest, by other substances with the emission of γ -ray quanta of the energy from 2 to 11 MeV per one absorption. After the capture of neutrons, induced radioactivity often arises, also accompanied by the energy release. Of greatest significance in the nuclear reactor is induced radioactivity associated with the production of U^{239} .

(3.57), since the energy release in the decay of U^{239} and Np^{239} is about 2 MeV per fission.

Table 3.3

Energy of U^{235} Fission by Thermal Neutrons

| Spatial heat distribution | Instantaneous heat liberation, MeV | | Delayed heat liberation, MeV | Total heat, MeV |
|---------------------------|------------------------------------|-----|--|-----------------|
| Localized liberation | Fragments, kinetic energy | 166 | β -particles 7 | 173 |
| Scattered liberation | Prompt γ -ray quanta | 7 | γ -ray quanta of β -decay 6 | 26 |
| | Neutrons, kinetic energy | 5 | | |
| | γ -ray quanta of capture | 8 | | |
| Total heat | 186 | | 13 | ~ 200 |
| Without heat release | — | | Neutrinos 11 | — |

Energy transformed into heat is usually rounded off to the value of 200 MeV per one act of fission, which, recalculated for 1 g of reacted U^{235} , gives:

$$\begin{aligned}
 5 \cdot 10^{23} \text{ MeV} &= 1.94 \cdot 10^{10} \text{ cal} = 8.1 \cdot 10^{10} \text{ J} = \\
 &= 22.5 \text{ MW} \cdot \text{h} \approx 1 \text{ MW} \cdot \text{day} \quad (3.58)
 \end{aligned}$$

The energy released in the heavy nucleus fission is by the order of magnitude higher than the energy of any other nuclear reaction. True, the energy per one nucleon or per unit mass of the substance is somewhat lower than in many other reactions with the participation of light nuclei. It is of interest to note that more than 5% of all the fission energy is carried off with neutrinos and cannot be made use of.

5. Residual energy release. The release of 6.5% of the thermal energy with the time shift with respect to the instant of fission results in some increase of the energy release at the start of the nuclear reactor operation and, what is more important, in the residual energy release after the discontinuation of the fission process. The abundance of radioactive fission products (Sec. 3.6-6) results in the complicated dependence on time of the delayed energy release. Fig. (3.13) shows the averaged total quantity of the energy released with β -particles and γ -ray quanta of β -decay by the time t

after one act of fission, with $t = 0$. The main quantity of the β -decay energy appears rather rapidly. About one third of the whole energy store is being released during 1 min, 60% during 1 hour, about three fourths, during 1 day. However, the subsequent energy release proceeds ever slower. In the case of the reactor which started operating at the time $t = 0$, with the power of one fission per second, the curve in Fig. 3.13 shows the increase in time of the rate of energy release by accumulated fission products or releasing by them the number of megaelectron-volts per second at the time t after the start of operation. In other words, the curve

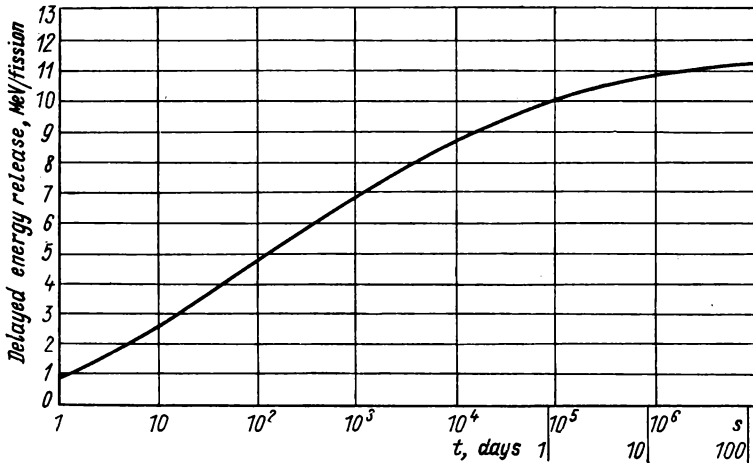


Fig. 3.13. Delayed energy release

represents the law of regeneration of the lacking 6.5% of power to the nominal 200 MeV/s per fission act.

With an infinitely long reactor operation, the number of births and decays of radioactive atoms achieves equilibrium, and the reactor power, its ultimate value. If such a reactor is shut down at some instant of time, $t = 0$, the rate of the delayed energy release decreases in time as the difference between its ultimate value 13 MeV/s and the ordinate at the time instant t after the shutdown (see Fig. 3.13). Thus, in 1 min after the shutdown, the energy release per unit time becomes one third lower, in 1 hour, 60% lower, and in a day, about three fourths lower. If the reactor has been in operation for the finite time t_0 , the delayed energy release at the instant t after the shutdown is the ordinate difference of the curve in Fig. 3.13 at points $t_0 + t$ and t . Because of the residual energy release, uranium spent in reactors requires long holding in special (spent-fuel) storages before its subsequent re-

processing. In some cases it needs forced cooling to avoid uranium slug melt-down.

6. Fission products. When the U^{235} nuclei are fissioned by thermal neutrons, more than thirty different pairs of fragments of different mass are produced. The sum of the mass numbers of paired

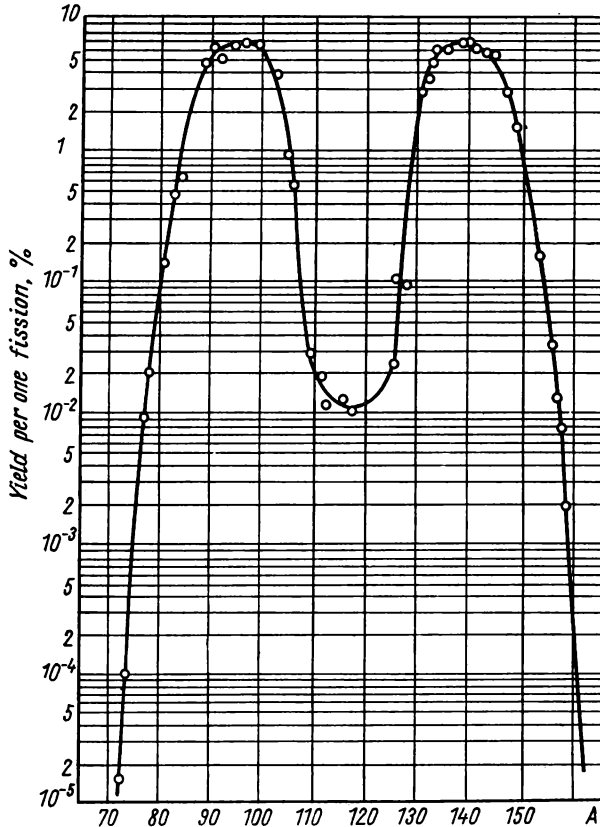
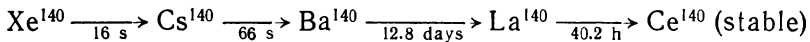


Fig. 3.14. Dependence of the yield of thermal-neutron fission products of U^{235} nuclei on mass number

fragments is 234 as it is the U^{236} nucleus that is fissioned, and the excited fragments emit two neutrons. Fig. 3.14 shows the distribution of the yields of the fission products as a function of the mass number. β -Decay does not change the mass number but only the belonging of the product-atom to the chemical element. Therefore, the yield of the first radioactive atom is the yield of the whole chain with a given mass number. The highest yield is 6%,

i.e., the mass numbers 95 and 139 have the production in six of one hundred fissions. The lightest and the heaviest product-nuclei out of those detected in U^{235} fission have the mass numbers 72 and 161, respectively. Fissioning into equal parts, with $A = 117$, is unlikely, which contradicts the drop model predictions because the structureless drop must most probably be fissioned just into two equal parts. Fissioning into unequal parts is explained in terms of the shell model (Sec. 2.6-3) as a result of the predominant production of pairs with filled shells containing 50 and 82 neutrons. It is typical that with the increase of the bombarding neutron energy, the probability of fissioning into two equal parts is increased and finally becomes maximum, which is in accord with the concept of the applicability of the nuclear models. The character of fission of excited nuclei must, to a less degree, be determined by the possibility of formation of filled shells in the product-nuclei because the order of the nucleons in the shells is inherent in nuclei in their ground or slightly excited states.

The chemical composition of fission products changes due to β -decay. An example of a chain of successive decays may be the following:



If the fission process proceeds sufficiently long at constant velocity, most of the chains become balanced and the chemical composition of fission products remains subsequently unchanged. Each element is represented by many isotopes from different chains. Rare earth elements, a quarter of all fission products, are in the equilibrium state. Most important of the other elements are zirconium, 15%, molybdenum, 12%, and cesium, 6.5%. The xenon and krypton gases constitute 16%. In fissioning of 1 kg of uranium during a long period of time (about 4 years), the volume of these gases is more than 25 m³ under normal conditions.

7. Fission neutrons. The average number of secondary neutrons, ν , per fission act plays a decisive part in the development of a chain reaction. Table 3.4 gives the values of ν for the basic fissile nuclides in thermal neutron fission and for U^{238} in fast neutron fission (Sec. 4.5-6). With the energy increase of the neutron which induces fission, the excitation energy

Table 3.4

The Number of Secondary Neutrons per Fission Act

| Nuclide | ν | $dv/dE, \text{ l/MeV}$ |
|------------|-------|------------------------|
| U^{233} | 2.507 | 0.115 |
| U^{235} | 2.442 | 0.115 |
| Pu^{239} | 2.881 | 0.110 |
| U^{238} | 2.8 | — |

of the fragment-nuclei slightly increases. It results in some increase of the average number of the neutrons emitted by them.

The emission of a neutron by the excited nucleus takes place when, as a result of the energy exchange with other nucleons, the neutron happens to acquire some energy exceeding its bond energy. The excess of the acquired energy over the bond energy is the kinetic energy of the neutron. The distribution of the kinetic energies of the neutrons emitted in such a way is the Maxwellian distribution with the parameter determined by the excitation energy of the nucleus left after the neutron emission, i.e., by "the temperature" of the nucleus. Since fission fragments move at high velocity ($\approx 10^7$ m/s) in neutron emission, the neutron energy spectrum in the laboratory coordinate system does not coincide with the initial Maxwellian distribution in the coordinates of the inertia centre of the neutron-fragment system. Nevertheless, the observed spectra of fission neutrons may be described by the Maxwell law:

$$\frac{1}{n_0} \cdot \frac{dn}{dE} = \frac{a}{v} \sqrt{E} e^{-\frac{E}{T}} \quad (3.59)$$

where T is the distribution parameter expressed in megaelectronvolts, just as the neutron energy E , and a is a constant normalizing the distribution per number of fission neutrons, v . The unity-normalized distribution (after the division by v), is a fraction of neutrons per unit energy interval $(1/n_0)dn/dE$, and n_0 is the total number of the neutrons involved. The distribution parameters (3.59) are not known in advance and are determined experimentally. Table 3.5 lists the values of a and T fitting the observed spectra of fission neutrons to expression (3.59) for fissionable nuclides.

Table 3.5

Parameters of Prompt Neutron Spectra in Thermal Neutron Fission of Nuclei

| Nuclide | a , $\text{MeV}^{-3/2}$ | T , MeV |
|-------------------|---------------------------|-----------|
| U^{233} | 1.888 | 1.306 |
| U^{235} | 1.872 | 1.290 |
| Pu^{239} | 2.121 | 1.333 |

Fig. 3.15 is a U^{235} distribution graph. The efficient temperature of the Maxwell spectrum, T , is in its sense, $(2/3)\bar{E}$, \bar{E} being the mean energy of fission neutrons. In thermal neutron fission of U^{235} , the mean energy of prompt neutrons is about 2 MeV, and the energy of the distribution maximum is about 0.7 MeV. Energies up to 18 MeV were registered in fission neutrons. However, beginning from 10 MeV, there are so few neutrons that

they are of no practical value. The energy spectrum of fission neutrons is, therefore, considered to extend up to 10 MeV. In the lower part of the spectrum, less than 0.5% of all the prompt neutrons have energies lower than 0.05 MeV. The

energy spectra of secondary neutrons of other fissile nuclides are close to the neutron spectrum of U^{235} .

8. Delayed neutrons. The data listed in Table 3.4 are concerned with the total number of secondary neutrons, both prompt and delayed, although the contribution of the latter to the values of ν is negligible. Delayed neutrons emitted in β -decay of some fission products (Sec. 3.3-9) play the decisive part in controlling self-sustaining chain reaction in nuclear reactors, in spite of their very small fraction in the total number of secondary neutrons. From the

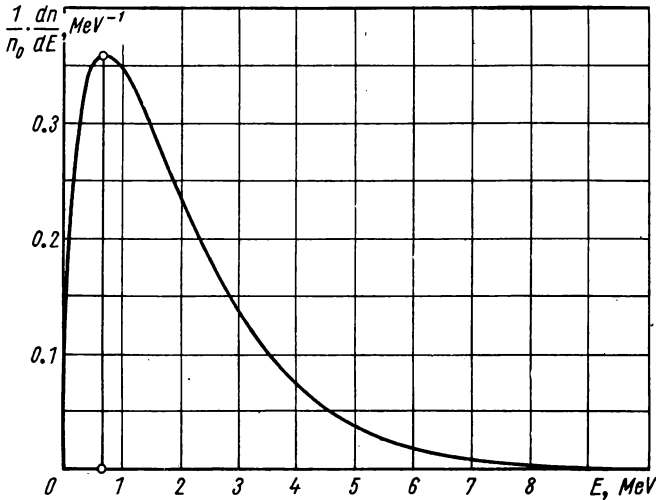


Fig. 3.15. Prompt neutron spectrum in U^{235} fission by thermal neutrons

point of view of control, of some interest are the half-lives of the atoms-precursors of delayed neutrons, and the yields of the delayed neutrons emitted by each precursor, as well as the delayed-neutron energies. Some radioactive precursors have very close half-lives indiscernible in experiments on delayed neutrons. Therefore, delayed neutrons are divided into groups with averaged periods and yields. Six of such groups are usually identified in experiment. In fissioning of different nuclei, the composition of fission products slightly differs, and it tells on the above averaged values in each group. Table 3.6 presents the characteristics of the groups of delayed neutrons emerging in fissioning of the basic heavy nuclides.

For each group, the second column of Table 3.6 gives the range of effective half-lives found for different nuclides subjected to fission. The following columns give the yields of the delayed neutron groups per fission act β_{ji} . The last column presents mean energies of the delayed neutron groups measured for U^{235} . The total sum

Table 3.6

Characteristics of Delayed Neutrons

| Group number | T, s | ${}^{233}\text{U}$ β_{fi} | ${}^{235}\text{U}$ β_{fi} | ${}^{239}\text{Pu}$ β_{fi} | ${}^{232}\text{Th}$ β_{fi} | ${}^{238}\text{U}$ β_{fi} | E_n for ${}^{235}\text{U}$, MeV |
|--------------|-----------|------------------------------------|------------------------------------|-------------------------------------|-------------------------------------|------------------------------------|---------------------------------------|
| 1 | 54-56 | 0.0006 | 0.0035 | 0.0002 | 0.0017 | 0.0005 | 0.25 |
| 2 | 21-23 | 0.0020 | 0.0035 | 0.0018 | 0.0074 | 0.0056 | 0.56 |
| 3 | 5-6 | 0.0017 | 0.0031 | 0.0013 | 0.0077 | 0.0067 | 0.43 |
| 4 | 1.9-2.3 | 0.0018 | 0.0062 | 0.0020 | 0.0221 | 0.0160 | 0.62 |
| 5 | 0.5-0.6 | 0.0033 | 0.0018 | 0.0005 | 0.0085 | 0.0093 | 0.42 |
| 6 | 0.17-0.27 | 0.0032 | 0.0007 | 0.0003 | 0.0021 | 0.0031 | — |
| | β_f | 0.0066 | 0.0158 | 0.0061 | 0.0495 | 0.0412 | |
| | β | 0.00264 | 0.0065 | 0.0021 | 0.022 | 0.0157 | |
| τ_d, s | | 18.4 | 13.0 | 15.4 | 10.1 | 7.68 | |

is given below each column comprising yields. The total yield per fission act is $\beta_f = \Sigma \beta_{fi}$. The fraction of delayed neutrons, β , in the total number of fission neutrons, equal to $\beta = \beta_f/\nu$, is given below. Finally, the last line comprises the mean delay times τ_d , or the averaged life-times of all the delayed neutrons:

$$\tau_d = \frac{\Sigma \beta_i T_i}{\beta \ln 2} = \frac{\Sigma \beta_i \tau_i}{\beta} \quad (3.60)$$

calculated with the use of the experimental values of period T_i for each nuclide [23] and of formula (3.7). Here β_i is the fraction of the i th group and $\Sigma \beta_i = \beta$.

The data of Table 3.6 are obtained for fissile nuclides, in thermal neutron fission, and for threshold nuclides, in fission by neutrons of the fast reactor. The fraction of delayed neutrons is especially low in Pu^{239} , which determines more stringent safety conditions for controlling the plutonium-fuel nuclear reactor as compared to the U^{235} reactor. It is noteworthy that the threshold Th^{232} and U^{238} nuclides, if fissile at all, give rather high yields of delayed neutrons. The initial kinetic energies of delayed neutrons are much lower than the energies of prompt neutrons, which is illustrated by the data for U^{235} .

CHAPTER FOUR

INTERACTION OF MOVING PARTICLES WITH MATTER

Microparticles acquire high kinetic energies in nuclear reactions and transformations. Then, moving in a substance, particles collide with atomic electrons and nuclei and transfer energy to them. The energy of the particles attenuates until it becomes equal to the energy of the thermal motion of the atoms and molecules of the medium. Electrically charged particles interact with atoms with high probability and, therefore, lose their energy very rapidly, traversing short distances until completely decelerated. The intensity of energy losses depends on the velocity of motion, rapidly reducing at high velocities. Fluxes of charged particles, nevertheless, are radiation of low penetrating power. Neutral particles, γ -rays and neutrons, lose their energy in the substance, as a result of rare collisions, and their path lengths in the medium are great. The fluxes of these particles constitute penetrating radiation. In addition, neutrons are absorbed by atomic nuclei and thereby give birth to secondary particles of high energy.

From the point of view of particle's penetration into matter the value of the particle's displacement along the straight line from the point of production to that of stopping or absorption, termed the particle *path or range* R , is important. At the same time, the particle can describe a complicated trajectory the length of which is called the *path length* l . Both the range R and the path length l always imply mean values. The character of the particle's propagation in the substance is not the same and one should discriminate among three fundamental cases.

(1) The motion of the particle up to the first collision, when the particle is absorbed or scattered, leaving the beam. In this case, the attenuation of the particle beam is determined by the exponential dependence (1.54), while the mean path length (1.58) and the mean path have, in fact, one and the same value. Deviations of the paths of separate particles from the mean are great. This is characteristic of narrow neutron- and γ -ray beams in narrow samples of the substance.

(2) Multiple collisions with low energy loss and practically without momentum exchange with target particles. Here the

particles move rectilinearly in the substance and hence, their path coincides with the path length although in this case they are different quantities. The deviations of the paths of separate particles from the mean R are negligible. In this way heavy charged particles, mesons, electrons of high energy (higher than 1 MeV) move in the substance.

(3) Multiple collisions with or without energy losses but with large momentum changes in each collision. Such a character of interaction brings about chaotic motion of particles about the substance accompanied by diffusion. With diffusion, the path length is much greater than the path of the particle, deviations of both the path lengths and the paths of separate particles from the mean values are great. These are neutrons and electrons of low energy (less than 0.1 MeV) in extensive media.

Reactions and transformations of interest for nuclear engineering are accompanied by the emergence of particles of moderate energy usually no higher than 10 MeV. The velocities of heavy particles with such energy lie in the range $(2 \text{ to } 4) \cdot 10^7 \text{ m/s}$, which is much lower than the velocity of light.

4.1. Heavy Charged Particles

1. Ionization energy losses. Heavy charged particles are the nuclei of light atoms, first of all, protons, deutons, α -particles. The principal mechanism of energy losses in the motion of such particles in matter is excitation and ionization of atoms. Electrostatic interaction of the charged particle with atomic electrons is accompanied either by excitation of the atom, i.e., by transition of the atomic electron to a higher energy level, or by ionization, i.e., by the removal of the electron from the atom and by the formation of an ion pair, the remaining positively charged atom and the negative electron. Both excitation and ionization occur on account of the kinetic energy of the moving particle. Subsequently, excited atoms make a transition to their ground energy states, and ions recombine with the production of neutral atoms. Low-energy photons emitted in these processes are absorbed by the substance and their energy is transformed into the energy of the thermal motion of the atoms and molecules of the medium.

The energy spent in excitation is determined by the structure of the atomic energy levels. The energy of the ion-pair production includes the value of the ionization potential, or the bond energy of the corresponding electron in the atom, as well as the kinetic energy with which the electron leaves the atom. The kinetic energy of electrons is usually moderate and is tens of electron-volts. However, in rare cases of collisions of particles with atomic electrons,

the latter acquire high recoil energy. In the limiting case of knock-on collision, the light electrons acquire velocity practically equal to double the velocity of the heavy particle, i.e., the energy of $2mv^2$, where v is the particle velocity, and m , the electron mass. In tracking devices, such electrons leave discernible tracks deflecting from the main track of the particle, and are termed delta-electrons. In fact, this term refers to all the electrons knocked out of the atoms. The kinetic energy of δ -electrons is in its turn spent in the excitation and ionization of other atoms, which is secondary ionization phenomenon. The secondary ionization produces twice as many ion pairs as the primary ionization. Because of the secondary ionization the mean energy lost by the moving particle in the production of one ion-pair in the substance appears moderate.

The excitation or ionization contribution to the total energy lost by the particle depends upon the properties of the environmental atoms. However, all the energy lost by the particle is usually referred to one pair of ions produced in the substance. In experiment, the total energy lost and the number of ion pairs emerging in the gas are detected, and then the mean energy which goes for the production of the ion-pair determined. The most significant conclusion to be drawn from the experiments is the fact that the mean energy of the ion-pair production is constant over a very wide range of moving particle kinetic energies, it is practically the same for α -particles, protons and electrons, and differs but little in different decelerating gases:

| Gas | Energy, eV |
|--------------------------|------------|
| H ₂ | 36 |
| He | 46 |
| N ₂ | 37 |
| O ₂ | 33 |
| Ne | 37 |
| Ar | 26 |
| Kr | 24 |
| Xe | 22 |
| Air | 36 |

The fact that the energy of the ion-pair production is independent of the particle energy allows the energy of the particle to be judged from the number of the ions formed or the number of ion pairs, which appear when particles with a given energy are slowed down, to be calculated. In air, for example, the energy loss of 1 MeV leads to the formation of 28,000 ion pairs.

2. Theory. The energy loss per unit path, $-dE/dr$, depends on the velocity of the particle. The calculation of this value by the quantum mechanics methods gives the following expression for a

heavy particle with charge ze , moving at velocity $v \ll c$

$$-\frac{dE}{dr} = \frac{e^4}{4\pi\epsilon_0^2 m} \cdot \frac{z^2}{v^2} \cdot NZ \ln \frac{2mv^2}{I} \quad (4.1)$$

where the first term in the right-hand side includes the universal constants; the second term, the characteristics of the particle; and the third, the parameters of the medium. NZ is the concentration of electrons in the substance, equal to the product of the number of atoms per unit volume (1.2) per nuclear charge, and I is the mean excitation energy of the atoms of the medium. The latter value represented by the relation

$$I \approx 13Z \text{ eV} \quad (4.2)$$

is given in Table 4.1 for several substances.

Table 4.1

Mean Excitation Energy

| Substance | Z | I , eV | Substance | Z | I , eV |
|-----------|-----|----------|-----------|-----|----------|
| Berillium | 4 | 64 | Copper | 29 | 371 |
| Carbon | 6 | 78 | Lead | 82 | 1070 |
| Aluminium | 13 | 166 | Air | — | 94 |

The particle energy losses are the greater, the lower its velocity and the higher its charge. The quadratic dependence on the charge makes the tracks of particles with different charges readily distinguishable in relevant detecting devices because the width of the tracks is proportional to $(-dE/dr)$. The dependence of the energy loss on the properties of the substance is, first of all, determined by the concentration of electrons with which the charged particle mainly interacts. Collisions with nuclei are extremely rare and do not practically affect the value of the energy loss. For some significant transfer of momentum to the nucleus to occur, the particle should pass quite near the nucleus and be deflected at a high angle θ (1.4).

The properties of the medium are also represented by the mean excitation energy of the atom I in the logarithmic term which results from the averaging of the momentum, transferred by the particle to the electron, over all possible collisions. The transferred momentum cannot be arbitrarily high or arbitrarily low. The possible maximum momentum transferred corresponds to the knock-on collision of the particle with the electron and equals $2v \times \times Mm/(M + m)$, where M is the mass of the particle. The minimum lost momentum p_{\min} is determined by the fact that the electrons in the atom are bonded, and it is calculated according to

the diminution of the particle energy lost on the excitation of the atom: $I = \Delta E = (M/2) \cdot (v_1^2 - v_2^2) = p_{\min} \cdot \bar{v} \approx p_{\min} v$, where v_1 and v_2 are the particle velocities before and after the collision. If the velocity is high, then $v = \bar{v}_1 \approx \bar{v} \approx v_2$, and $p_{\min} = M(v_1 - v_2)$. Integrating with respect to all permissible values of the momentum gives the logarithm difference, i.e., the logarithm of the maximum-to-minimum momenta ratio. In this case, $M > m$, and the logarithm sign means the ratio of the maximum energy of the δ -electron to I .

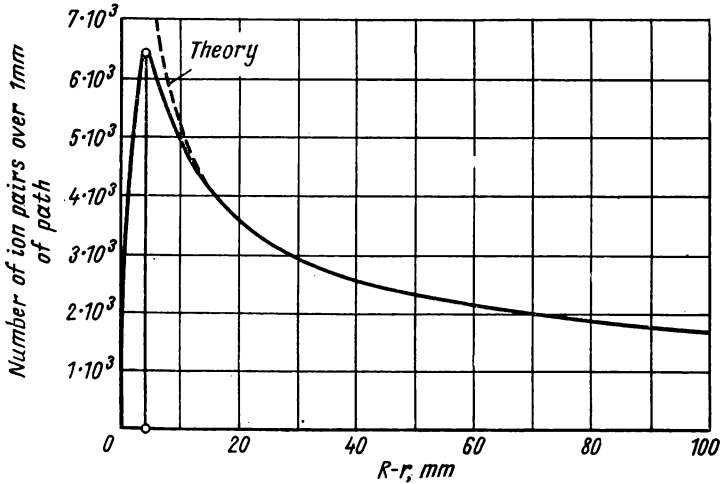


Fig. 4.1. The Bragg curve

3. **Experiment.** Bragg (Britain) was the first to calculate the number of ion pairs emerging at each small segment of the path of the decelerating α -particle. The calculation was performed with the use of an air-filled ionization chamber. Since the ion-pair production requires, on the average, one and the same energy for any particle velocity, the curve of the number of ion pairs against the path r traversed by the α -particle in air, or the Bragg curve, is also the relationship between the specific energy loss and r , $-dE/dr$. In Fig. 4.1 plotted as ordinates are the number of ion pairs originated per 1 mm of path and as abscissae, the residual path of the α -particle, $R - r$, where R is the total path length of the α -particle of a given initial energy in air. The use of the residual path as abscissa for the Bragg curve is convenient since along such a coordinate axis, the zero values of the residual path $R - r$ coincide for α -particles with any total path R . It means that the origin of the Bragg curve appears to be at the same point for particles of different initial energies, i.e., at the end point of the α -par-

ticle path. In addition, the increasing scale $R - r$ is at the same time, although nonproportional, also an increasing scale of the α -particle velocity and energy, which is convenient for the comparison with the relationship (4.1).

Experiment is in fair agreement with theory in case α -particles of the energy higher than 0.75 MeV are slowed down. The residual path of the α -particle of the energy 0.75 MeV is short and in air under normal pressure equals 4 mm. At the latter section, the nature of the dependence of the ionization losses on $R - r$ sharply differs from the theoretical. Instead of an abrupt increase of specific losses with lowering particle velocities, not a less quick fall of the ionization ability down to zero is observed. Such a behaviour of $-dE/dr$ is explained by the effects not considered when obtaining expression (4.1). The velocity of α -particles of the energy 0.75 MeV is equal to $6 \cdot 10^6$ m/s, which approximately coincides with the velocity of the orbital motion of those atomic electrons with which the α -particle most often interacts.

Since the α -particle itself is the nucleus of the atom, it captures electrons moving slowly with respect to it, transforming into the helium singly-ionized atom or into a neutral atom. The α -particle again loses the captured electrons in colliding with other atoms as the velocity of its motion in the substance is still very high. In the last 4 mm of its flight, the α -particle exchanges electrons with the environment more than 1000 times. The capture of electrons reduces the effective charge of the α -particle, and, since $-dE/dr$ is proportional to the square charge, the energy loss per unit path begins to reduce rapidly. In the maximum of the Bragg curve, the effective charge of the α -particle does not equal two but only $z_{eff} \approx 1.5$, ever decreasing with subsequent slowing down. For protons, this effect is substantial, starting with the energy 0.1 MeV where the maximum ionization loss is observed. The residual path of protons with $E = 0.1$ MeV is about 1.5 mm in air.

4. **Paths in gases.** The paths of the motion of slowing down heavy charged particles are linear. In the overwhelming majority of collisions, the energy is transferred to very light electrons and because of this no significant deflection from the direction of the initial motion of the particle is observed. The *path length* of the slowing-down of a heavy charged particle is, therefore, at the same time the *range* R . The path length as well as the range itself in this case are calculated on the basis of the relationship (4.1), consistent with experiment, between the energy lost and the distance traversed:

$$R = \int_0^R dr = \frac{4\pi e_0^2 m}{e^1} \cdot \frac{M}{z^2} \int_0^{v_0} \frac{v^3 dv}{NZ \ln \frac{2mv^2}{I}} \quad (4.3)$$

Here, as distinct from (4.1), all the energy dependence of the elementary range is presented in terms of the particle velocity and, therefore, one more characteristic of the particle emerges, i.e., its mass M , v_0 being the initial velocity of the particle with the range R . The integral in expression (4.3) has no sense in the interval from zero to some velocity v_1 determined by the relation $2mv_1^2 = I$, which is due to the invalidity of (4.1) for any low velocity. It does not, however, cause any difficulty since v_1 lies in the range of velocities where (4.1) contradicts experiment owing to repeated particle neutralization and ionization, and the range computation following (4.3) is valid only if theory agrees with experiment. The residual range, which is not computed by formula (4.3), is usually short compared to the total path.

The range of the heavy charged particle rapidly increases when the initial velocity v_0 is increased, and in accordance with the empirical law, is:

$$R \approx av_0^3 \quad (4.4)$$

where a is the proportionality factor. In actual fact this dependence is stronger [see (4.3)]. The range also depends on the particle constants. With the same initial velocity, the path is directly proportional to the mass of the particle and is inversely proportional to the square of the charge. The dependence on the factor M/z^2 is fulfilled most precisely within the validity limits of expression (4.3). Thus, for example, this factor has the same value for α -particles and protons. Therefore, α -particles and protons with the same initial velocity (not energy) have the same range R if one neglects the correction for the last millimetres of the path.

5. Dispersion. The range R determined by the mean energy loss is the mean path. The differences of the ranges of separate heavy charged particles from the mean path appear small. Ionization losses of heavy particles comprise minute fractions of energy lost in frequent acts of interaction with atoms, which results in low statistical fluctuations of the value of the mean specific energy loss (4.1) and, hence, in the slight spread of the ranges with respect to the mean value. This fact is well observed in experiment (Fig. 4.2). If one detects an α -particle flux (of the same initial energy) as the function of the distance r from the source, then the flux will be the same up to the distance equal to the mean path R .

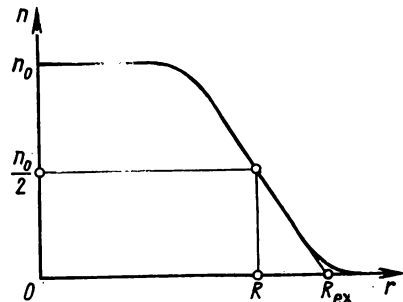


Fig. 4.2. Space distribution of α -particles at the end of the range

In the vicinity of R , there is a transient region where the flux attenuates with the distance r , roughly following the linear law. The distance value corresponding to a double attenuation of the flux is the mean path R . The measure of the deviation from the mean is the relation $(R_e - R)/R$, R_e being obtained by extrapolating the linear region of the α -particle flux attenuation to intersecting with the x -axis. For α -particles and protons with $E = 10$ MeV, this relation is 1% and 1.8%, respectively, slowly increasing with the attenuation of the initial energy of the particles. Since deviations from the mean are small, heavy charged particles of the same sort with similar initial energies are said to have the same range.

6. Paths in condensed media. Most commonly known are the data on particle ranges in air. When recalculating for condensed media, empirical rules are used, for instance the Bragg-Cleaman rule:

$$R = R_A \frac{\rho_A}{\rho} \sqrt{\frac{A_r}{A_{rA}}} = 0.32 R_A \frac{\sqrt{A_r}}{\rho} \quad (4.5)$$

where ρ and A_r are the density and the atomic mass, respectively, subscript "A" meaning air. If R_A is the range of the particle with the given energy in air, then, after substituting the density of the substance in kilograms per cubic metre and its atomic mass into expression (4.5), one obtains the range of the particle in this substance in the same units as the range in air. The ranges of α -particles of the energy up to 10 MeV in solid and liquid media are measured in tens of micrometres. For some solids, plots or tables for the range as an energy function exist [17]

4.2. Fission Fragments

1. Fission fragment parameters. Fission fragments are multiply charged ions of the atoms of average mass numbers with high kinetic energy, forming in nuclear fission (Sec. 3.6). Being accelerated at a very short distance practically equal to the diameter of the atom, the newly formed nuclei entrain slightly more than half the electrons of the original atom and, therefore, have a very large ion charge z . Table 4.2 lists the original characteristics of the light

Table 4.2

Fission Fragment Parameters

| Fragment | A | Z | z | E , MeV | v_0 , m/s | R , mm |
|----------|-----|-----|-----|-----------|------------------|----------|
| Light | 95 | 38 | 27 | 99 | $1.4 \cdot 10^7$ | 22 |
| Heavy | 139 | 54 | 22 | 67 | $0.9 \cdot 10^7$ | 15 |

and heavy fragments with the most probable values of the mass number.

2. Energy loss mechanism. The major loss of energy by fission fragments moving in a substance, as by other heavy charged particles, is due to ionization of the atom. Although the kinetic energies of fission fragments are enormous, the initial velocities are not too high because of the large mass, and the velocity of a light fragment corresponds to the velocity of an α -particle of the energy about 4 MeV. Owing to the high ion charge the specific ionization of fragments is very high, but as the fragment is slowed down the ionization does not increase, as in the case of α -particles or protons, but decreases (Fig. 4.3). This dependence is the consequence of the continuous decrease of the ion charge of the fragment.

In a twenty-times-ionized atom, the bond energy of each added electron is high. Therefore, the interaction of the multiply charged ion with the environmental atoms is, with great probability, accompanied by the transfer of weakly bonded outer atomic electrons to the multiply charged ion. The acquired electrons are not lost by the fission fragment later on, since they are tightly bonded in the moving atomic ion. Accumulation of the missing electrons reduces the ion charge and, hence, the specific ionization of the medium by the fission fragment. This effect exceeds the increase of the specific ionization due to the decrease of the velocity. As a result, the specific loss of the energy of the fission fragment is lowered approximately linearly down to 5 mm of the residual range in air.

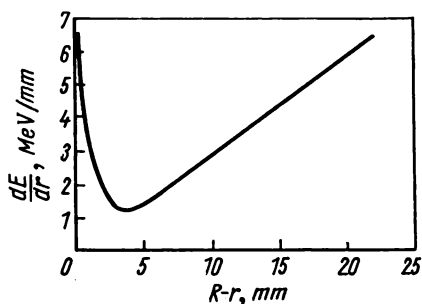


Fig. 4.3. Specific ionization of fission fragments (in air)

When the velocity of the fission fragment approaches about $2 \cdot 10^6$ m/s, the average energy loss per path unit again sharply increases, which also contradicts the relation $-dE/dr$ for α -particles. But in this case, the increase of $-dE/dr$ is associated with the new mechanism of loss, i.e., collisions with atomic nuclei. Such collisions are the more probable, the lower the energy and the larger the nuclear charge of the ion (1.4). Slow fission fragments transfer high momenta to atomic nuclei, losing considerable part of their energy in each act and scattering at sufficiently large angles. Scattering of fragments by nuclei was detected in nuclear photo-emulsions. At the scattering points short tracks of recoil nuclei (to be more precise, of ionized atoms) were observed which, by ana-

logy with recoil electrons, are termed δ -nuclei. At the end of their path, fission fragments undergo some such collisions with nuclei (two on the average), which results in a rapid loss of the remaining energy of the fragments. The exchange with the medium of the last electrons captured by the fragment is of no significance in this case. The minimum value of the specific energy loss by fission fragments falls within approximately the same region of the residual range as the maximum value of the α -particle. But here again $-dE/dr$ of fragments exceeds $-dE/dr$ of α -particles by five times. Consequently, fission fragments strongly ionize the substance along its whole path length.

3. **Ranges.** The paths of fragments in a substance are linear (straight-lines). Only at the ends of the paths, there are possible distortions due to collisions of fragments with nuclei. Nuclear collisions are the basic cause of the rather large spread of fragment ranges relative to the mean value (about 10%), since at the end of the range in a small number of collisions the energy losses are great. In addition, the initial kinetic energy of fission fragments is neither a constant for heavy nuclei in the different fission cases. The ranges of light fragments in a substance are longer than those of heavy fragments. Therefore, the penetration of the fission fragments into a substance is evaluated from the ranges of light fragments. Table 4.3 gives the values of the mean paths of light fragments through some substances, used in reactor designing.

Table 4.3

Ranges of Light Fission Fragments

| Substance | Density, 10^3 kg/m^3 | Range, μm | Substance | Density, 10^3 kg/m^3 | Range, μm |
|-----------|-----------------------------------|-------------------------|-------------------|-----------------------------------|-------------------------|
| Aluminium | 2.7 | 13.7 | Uranium | 10 | 10 |
| Iron | 7.86 | 6.6 | dioxide | | |
| Zirconium | 6.4 | 9.1 | (UO_2) | | |
| Uranium | 18.9 | 6.7 | | | |

4.3. Electrons

1. **Specific properties of electrons.** Electrons are charged particles, therefore, they lose energy predominantly during the excitation and ionization of atoms. Electron interaction with the substance, however, has some peculiarities resulting from that the electron mass is low and moving electrons are identical to atomic electrons from which the greatest part of the energy of the par-

ticles is transferred under the decelerating action of Coulomb forces.

2. Ionization. Losses of the electron energy due to ionization are described by the relation analogous to (4.1). But the expression under the logarithm sign in the formula for electrons is approximately 4 times less than for the heavy particles. First, the maximum momentum for the δ -electron, when $M = m$, is $p_{\max} = 2vMm/(M + m) = mv$, not $2mv$, as in the case when $M \gg m$ (Sec. 4.1-2). Second, the knock-on collision of identical particles is, in fact, equivalent to the absence of any collision since, as a result, the same particle of the same energy continues its motion in the same direction. Therefore, when electrons of high energy exchange collide, the electron of higher energy should be considered as primary and the electron of lower energy, as a δ -electron. Hence, in this case, the maximum energy of the δ -electron is not equal to $mv^2/2$, but to $mv^2/4$, this leading to the corresponding decrease in the maximum momentum of the δ -electron. Third, the quantum mechanical effect of the indistinguishability of identical particles in scattering introduces under the logarithm sign one more factor less than unity. As a result, in formula (4.1), one should take $z = 1$ for electrons and the logarithmic term should be presented as $\ln[(mv^2 \sqrt{e/2})/2I]$, where e is the natural logarithms base.

3. Relativistic electrons. Electrons of kinetic energy higher than 0.01 MeV are already relativistic (see Table 1.1). Ionization losses of such particles are low because of the relationship $1/v^2$ of non-relativistic particles (4.1) and do not practically depend upon velocity since in the given case $v \approx c$. The relation $-dE/dr$ of relativistic charged particles does not, however, remain constant but slowly increases, commencing with kinetic energy approximately equal to $2Mc^2$, which is about 1 MeV for electrons. This increase is not very great and, at an infinitely great energy, it comprises several per cent or tens of per cent of the value of the minimum specific ionization. The specific energy loss by relativistic electrons is determined by the following expression:

$$-\frac{dE}{dr} = \frac{e^4 NZ}{8\pi\epsilon_0^2 m v^2} \left\{ \ln \frac{mv^2 E}{2I^2 (1 - \beta^2)} - (2\sqrt{1 - \beta^2} - 1 + \beta^2) \times \right. \\ \left. \times \ln 2 + 1 - \beta^2 + \frac{1}{8}(1 - \sqrt{1 - \beta^2})^2 \right\} \quad (4.6)$$

where E is the kinetic energy of the electron. When velocities are low, $\beta \rightarrow 0$, this relation transforms into (4.1) with corrections for electrons, and, when $E \gg mc^2$ or $\beta \rightarrow 1$, this relation is

$$-\frac{dE}{dr} = \frac{e^4 NZ}{8\pi\epsilon_0^2 m c^2} \left(\ln \frac{E^3}{2mc^2 I^2} + \frac{1}{8} \right) \quad (4.7)$$

A slight increase of the ionization losses, proportional to the energy logarithm, is explained by the relativistic effect of the distance contraction in the direction of the motion of the body at velocities close to c . As a result, in the coordinate system associated with the medium at rest, the electric field of a moving particle appears to be contracted in the direction of the motion and amplified in the plane perpendicular to the direction of the motion. This amplification of the electric field leads to the possible ionization of the atoms removed farther and farther from the particle path. The increase of the number of atoms with which the moving particle effectively interacts is accompanied by some increase of ionization losses. This increase, however, has a limit in spite of the ever progressing contraction of the Coulomb field with the growth of the particle energy. The charged particle polarizes the medium and its polarization is the greater, the stronger the field. The contraction of the field, therefore, increases the polarization in the transverse direction, which prevents the electric field from penetrating into the depth of the substance. The polarization depends on the density of the substance and shows itself up more strongly in a condensed medium, and weaker, in gas. The influence of polarization on the reduction of ionization losses is, therefore, termed the density effect. The density effect brings ionization losses to saturation in different media at different particle energies over the range (20-50) Mc^2 . Formula (4.7) does not allow for this effect.

Electrons of energy a little higher than 1 MeV are particles with minimum ionization. Under normal conditions, they form in air about four ion pairs per 1 mm of path. In tracking devices, thin tracks of such electrons are readily distinguishable from the tracks of heavy charged particles of the same energy. (For example, protons of the energy 1 MeV produce about 800 ion pairs per 1 mm of path in air). As a result of deceleration, electrons, of course, acquire such velocities at which the specific ionization is as high as the ionization of heavy particles of the energy less than 10 MeV. With these velocities, however, the electron energy content is so small that its path length becomes practically unobservable. In air it is equal to some fractions of a millimetre and is narrower than the diameter of separate silver grains of which the particle tracks are constituted in nuclear photoemulsions, i.e., tracking detectors with the best space resolution. Therefore, electron tracks are always thin, and individually observed electrons are always relativistic particles.

4. Bremsstrahlung. Electrons are characterized by one more mechanism of energy losses, that of braking radiation or bremsstrahlung. According to classical electrodynamics, an accelerating electric charge emits energy proportional to the square of the acceleration. Charged particles being slowed down in the substance,

passing in the vicinity of atomic nuclei, acquire high acceleration due to the Coulomb force proportional to the nuclear charge Z . Since the acceleration is proportional to the force and is inversely proportional to the mass of the particle, the quantity of the emitted energy must be proportional to $(Z/M)^2$, where M is the mass of the particle. Because of such dependence upon the mass the probability of energy emission during deceleration of the heavy charged particle in the nuclear field appears to be 10^6 to 10^7 times lower than during electron deceleration. Therefore, the loss of energy on bremsstrahlung in heavy particles of moderate energies is of no significance.

In fact, slowing down of the electron in the nuclear field is accompanied by the formation of a γ -ray quantum. The energy of this γ -ray quantum is the energy lost by the electron and can be equal to any value down to the value of the initial kinetic energy of the electron, i.e., $h\nu_{\max} = E$. Thus, because of bremsstrahlung, the electron may lose all its energy at once in one act of interaction with the nucleus. The probability of the emergence of a γ -ray quantum of a given energy $h\nu$ is determined by the cross-section pertaining to the atom. This cross-section is calculated by quantum electrodynamic methods and is approximately inversely proportional to the frequency ν of the produced γ -ray quantum. Hence, the product of the photon energy $h\nu$ by the cross-section does not depend upon frequency. It means that in bremsstrahlung with any frequency, on the average, an equal quantity of energy is carried off, though the number of photons is not the same and is inversely proportional to the frequency. The total energy thus lost is proportional to the full range width of feasible frequencies, the minimum of which is $\nu = 0$, and the maximum is determined by the relation $h\nu_{\max} = E$, where E is the initial kinetic energy of the electron. Thus, the total energy spent by the electron on bremsstrahlung is proportional to its kinetic energy. If bremsstrahlung is characterized by the atomic cross-section σ_{rad} with regard to the emission of energy, not of a photon, then such a cross-section does not depend upon energy. In fact, $N\sigma_{rad}$ is a macroscopic cross-section (1.55) which is the probability per unit path of that process to which the cross-section relates, and the probability of the process is the mean fraction of many events of realization of the process (1.48), i.e., in this case it is the mean fraction of the energy being lost per unit path, or $(1/E) \cdot (-E/dr)_{rad}$. Therefore,

$$\left(-\frac{dE}{dr}\right)_{rad} = N\sigma_{rad}E = NZ^2\sigma'_{rad}E \quad (4.8)$$

Since the total emitted energy is proportional to E , the cross-section σ_{rad} does not depend upon energy and is expressed in terms of the universal constants. The dependence of the cross-section

upon Z^2 mentioned above is presented in its explicit form since Z is the characteristic of the substance.

The contribution of bremsstrahlung to the total loss of the energy by the fast electron may be estimated by comparing the specific loss for radiation (4.8) with the loss for ionization (4.7). Since ionization losses of electrons do not practically depend upon energy, the corresponding relation of the values $-dE/dr$ is proportional to the energy and is usually presented in the form

$$\frac{\left(-\frac{dE}{dr}\right)_{rad}}{\left(-\frac{dE}{dr}\right)_{ion}} = \frac{ZE}{800} \quad (4.9)$$

where the numerical factor 800 in the denominator is obtained from the universal constants of the formulae (4.7) and (4.8), has the dimension of energy and is expressed in megaelectron-volts for the electron energy E to be substituted in the same units. As follows from (4.9), ionization losses constitute the basic energy loss mechanism of the electrons moving in the substance with the energy at least lower than 10 MeV. Bremsstrahlung predominates over ionization at high energies and in heavy substances. Thus, for example, in lead ($Z = 82$), only when $E \approx 10$ MeV, relation (4.9) is transformed into unity. Both the numerator and denominator are 1.6 MeV per 1 mm of lead. With lower energy in lead and, the more so, in light substances, ionization loss exceeds the emission loss. However, in slowing-down of electrons of the energy of several megaelectron-volts in heavy substances, the contribution of bremsstrahlung may appear substantial. The produced bremsstrahlung photons may turn out to be more penetrating than the original energy carriers, electrons. Therefore, the bremsstrahlung losses can neither be neglected in the region of moderate energies.

It is well known that X-rays with a continuous energy spectrum are a flux of photons of electron bremsstrahlung. In X-ray tubes, the electron energy is measured in tens of kiloelectron-volts. At such low energies, the major part of the electron energy is not transformed into X-ray photons but into heat because of the ionization of atoms. Therefore, the anticathodes of X-ray tubes must be intensively cooled.

5. Electron scattering. Charged particles moving in the substance are continuously intersecting atomic volumes getting into the range of action of the Coulomb nuclear forces. Interaction with the Coulomb field results in scattering (1.4), which should be considered as a result of the particle collision with the whole atom. Such collision is elastic if the atom is neither excited nor ionized while the particle is passing through it. Elastic collisions are, of course, much more numerous than inelastic collisions especially

at high velocities when the specific ionization is low. In most cases, elastic collisions do not result in considerable deflections of particles from the initial direction of their motion since at energies of the order of megaelectron-volts scattering at a high angle requires a very small impact parameter (see Fig. 1.1). Small impact parameters are extremely unlikely, and with large parameters comparable with the atomic radius, the scattering angles are small, the more so that the Coulomb nuclear field is partially screened by the inner electrons. Therefore, scattering of heavy particles due to elastic collisions is usually small. Only strongly slowed down fission fragments make an exception since they have high intrinsic nuclear charges. With the same impact parameter (1.4), the scattering angle is the larger, the lower the energy of the moving particle, and the higher the nuclear charges of the colliding particles.

On the contrary, light electrons, for some reasons, are subject to efficient scattering and because of it their paths of motion in the substance are not rectilinear. First, owing to their low mass, electrons are actually relativistic particles along all their deceleration path and the transfer of the relativistic energy results in a smaller momentum (1.36). The effect of scattering is rotation of the momentum vector. All other things being equal, the scattering angle is the larger, the lower the value of the momentum. The electron, therefore, scatters at a greater angle than the proton of the same energy and impact parameter does. The greater the contribution of the relativistic constituent to the total mass of the moving particle, the more pronounced is this effect. With the electron energies from 1 to 10 MeV, this brings about the increase of the scattering angle compared to the nonrelativistic formula (1.4) by tens of per cent with small impact parameters and by several times with large impact parameters, i.e., with small scattering angles. Second and most significant, the velocity of electrons of the energy 1 MeV is still close to the velocity of light and the remaining deceleration path of such electrons is very long, while heavy charged particles with a similar energy are at the end of the range. Thus, electrons traverse a substantial part of the path, having low energy, and at elastic collisions, the scattering angles are the larger, the lower the energy of the particle (1.4). Finally, nonelastic collisions of electrons with atoms accompanied by great transfer of energy to δ -electrons or, in fact, the cases of electron-electron scattering are also accompanied by the deflection of electrons at large angles since the masses of colliding electrons are the same.

The above shows that only electrons of very high energy move linearly in the substance. The paths of electrons of the energy ~ 1 MeV are strongly curved and the behaviour of the decelerated

electrons with energies tenths of megaelectron-volts or lower resembles diffusion rather than motion in a given initial direction. Thus, for example, a substantial part of low-energy electrons of β -decay, traversing the surface of a condensed medium, is reflected in a backward direction. This becomes possible owing to the efficient scattering of electrons in the elementary acts of collision with atoms, which is just typical of particle diffusion in a medium.

6. Electron ranges. The length of the path which electrons of a given initial energy traverse during acceleration in the substance, can be calculated on the basis of expressions for the mean specific energy losses (4.6), (4.7) and (4.8). Because of scattering, however, the electron paths in the substance are curvilinear and their mean length is not equal to the range, i.e., to the mean value (measured in a straight line) of the displacement of electrons in space from the point of their emission. The electron path lengths themselves are the values less definite than the paths of heavy particles. Because of great energy losses in the collisions when high-energy δ -electrons and bremsstrahlung photons are produced, the specific loss energy of the electron is subject to significant statistical fluctuations. Therefore, the path lengths of electrons in the substance have the spread from 10 to 15% relative to the mean value.

The calculation of the displacement of decelerated electrons in the substance is complicated because of the difficulties of allowing for multiple scattering. In separate acts of collisions with atoms, scattering is accompanied by electron deflection at predominantly moderate angles, owing to which the electron beam, though diverging, retains the direction of its initial motion at the beginning of the deceleration path. Then, the motion of separate electrons becomes arbitrary and further electron propagation is diffusion. Therefore, the electron ranges are found experimentally and the relation between the range R and the initial energy E of the electrons are determined. The range implies the thickness of the layer of the substance outside of which electrons practically do not pass. The electron ranges can be substantially shorter than the full path lengths of the electrons in the substance. The difference between the range and the path length, however, lessens with the increase of the initial energy of the electron.

The character of attenuation of the electron flux in the substance is defined by scattering and, in addition, depends upon the nature of the source. Even if the source emits monoenergetic electrons, the number of electrons penetrating to a given depth of the substance all the time decreases with the increase of the layer thickness (Fig. 4.4). The peculiarity of such distribution is the existence of a linear region over its greater part. The length of the tail, i.e., the part of the distribution approaching zero, is short

and does not practically depend upon the initial energy. The distance from the start of the deceleration to the intersection of the extrapolated linear part of the distribution with the x -axis (abscissa axis) is, therefore, taken to be the range of monoenergetic electrons. Such a range is termed extrapolated or practical. Empirical relations relating the extrapolated range R_e to the initial energy E are of the form

$$\left. \begin{aligned} R_e &= 0.526E - 0.034, & 0.5 < E < 3 \text{ MeV} \\ R_e &= 0.15E - 0.0028, & 0.03 < E < 0.15 \text{ MeV} \end{aligned} \right\} \quad (4.10)$$

where E is the energy, MeV; R_e , the range in units independent of the density of the substance, g/cm^2 (the product of the linear

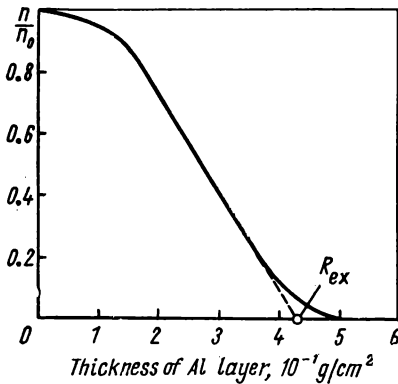


Fig. 4.4. Slowing-down of monoenergetic electrons in aluminium ($E=1$ MeV)

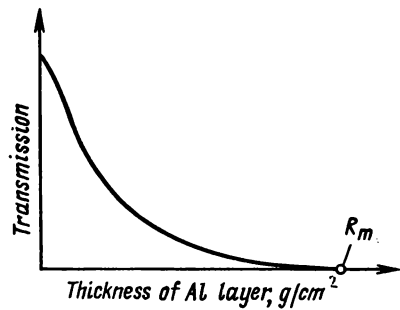


Fig. 4.5. The character of β -particle penetration through aluminium

size, centimetres, by the density, grams, divided by the cubic centimetre). With other energies, the extrapolated range may be found from graphic data [17].

The electron ranges of β -decay are of the greatest practical interest. Beams of such electrons contain many low-energy electrons (see Fig. 3.9), and the highest electron energy is the β -decay energy, (3.30) or (3.31). Since low-energy electrons are most efficiently scattered by the substance and rapidly fall out of the flux propagating in its initial direction, the distribution of β -decay electrons over the thickness of the substance layer proves to be different, nearly exponential (Fig. 4.5):

$$N_e(x) = N_e(0) e^{-\mu x} \quad (4.11)$$

where N_e is the number of electrons in the flux; x is the layer thickness, g/cm^2 ; and μ , the mass absorption coefficient of electrons with a given maximum energy, cm^2/g . The distance to the

point of confluence of the decelerated electrons distribution with the x -axis abscissa axis) is taken to be the range of β -decay electrons and is called the maximum range R_{\max} . The maximum range is associated with the maximum energy of the β -spectrum by the following empirical relations:

$$\left. \begin{aligned} R_{\max} &= 0.542E_{\beta} - 0.133, & 0.8 < E_{\beta} < 3 \text{ MeV} \\ R_{\max} &= 0.407E_{\beta}^{1.38}, & 0.15 < E_{\beta} < 0.8 \text{ MeV} \end{aligned} \right\} \quad (4.12)$$

where E_{β} is the energy, MeV, and R_{\max} is the range, g/cm². It is evident that from the distributions dropping with the increase of the abscissa approximately as an exponent, it is impossible to find the maximum range with sufficient assurance, because the point of confluence of the distribution curve with the x -axis (abscissa axis) is indefinite. Therefore, to obtain the maximum range means comparing the distribution of electrons being decelerated in a reference substance, which is aluminium, with the electron distribution from the reference source of β -particles the maximum range of which in aluminium is known [17]. Relations (4.10) and (4.12) refer to ranges in aluminium, and formulae (4.12) are used in experimental nuclear physics in a most simple determination of the β -decay energies from electron ranges in aluminium.

In dosimetry the problem is opposite, i.e., to determine the ranges in different substances by known electron energies. It follows from experiment that the mass coefficient μ in expression (4.11) is almost independent of the atomic number of the substance, slightly increasing with the increase of Z .

Therefore, electron ranges expressed in grams per square centimetre, by (4.10) and (4.12), may, with sufficient accuracy, be referred to other substances, particularly to light ones. In heavy substances ranges must be somewhat shorter than in light substances, since the value of the mass coefficient is slightly higher. This is in spite of the fact that the path lengths of electrons in heavy substances are just longer than in light ones. The path length is inversely proportional to the concentration of electrons, $NZ = (\rho/A_r)N_A Z$ (4.3), and since (Z/A_r) slightly decreases with the increase of Z (or A_r), then the length of the deceleration path correspondingly increases. The dependence on I , (4.3) and (4.2), also leads to an increase in the overall path with the increase of Z . The range is not, however, determined only by the length of the deceleration path but by scattering as well, which is the more efficient, the higher Z of the nucleus.

Extrapolated ranges of monoenergetic electrons and maximum ranges of β -decay electrons, at the same initial or maximum energy, do not markedly differ since both the former and the latter are at the opposite ends of the distributions in the substance, while

the distribution limits are determined by the initial energy of the most fast electrons in the electron spectrum. Analysis of the numerous data on the ranges of monoenergetic and β -decay electrons enabled the relationship between range and energy for any electron spectrum over a wide energy range from 0.01 to 20 MeV to be ascertained:

$$\left. \begin{aligned} \log R &= 1.265 \cdot \log E - 0.2197 \log^2 E - 0.385, & E < 2.5 \text{ MeV} \\ R &= 0.530E - 0.106, & E > 2.5 \text{ MeV} \end{aligned} \right\} \quad (4.13)$$

where R is the range in aluminium, g/cm², and E , the energy MeV. All the empirical data are consistent with the ranges expressed in formulae (4.13) to a 5 per cent accuracy, and the averaged deviation is less than 0.1 per cent. In the energy ranges, where the simpler formulae (4.10), (4.12) are valid, the results according to these formulae and formulae (4.13) naturally coincide. In literature [10] with reference to the primary source, relation (4.13) is plotted in the interval from 0.06 to 5 MeV. Literature [15] gives a table of ranges (calculated from formulae (4.13)) in linear units both for aluminium and (recalculated by the substance densities) for living tissue and air. In the same handbook a table is given of total path lengths (calculated from formula (4.6)) for a number of substances, from beryllium to gold. Although electron fluxes do not belong to penetrating radiation, their ranges in the substance are much longer than the ranges of heavy charged particles. Thus, the range of electrons of the energy 5 MeV is 21 m in air, 2.5 cm in water, and about 2 mm in lead.

4.4. Gamma-Quanta

1. Definitions. Photons emitted by atomic nuclei and high-energy bremsstrahlung photons are termed γ -quanta. The γ -quantum energy is usually higher than 0.1 MeV and exceeds the bond energy of atomic electrons. Three basic processes determine the absorption of energy carried by high energy photons by a substance, namely, the photoeffect, the Compton scattering and the process of electron-positron pair production. Each of these processes is the interaction of a γ -quantum with the Coulomb nuclear or electron field. As a result of this interaction, the γ -quantum is either completely absorbed or scattered, losing a substantial part of its energy, but in each case, the γ -quantum falls out of the beam in one interaction act. If the losses of the beam particles occur in single collisions, then their flux Φ attenuates with the distance x , following the exponential law (1.54)

$$\Phi(x) = \Phi_0 e^{-\mu x} \quad (4.14)$$

where μ is the linear factor of attenuating the γ -quantum flux, or the corresponding macroscopic cross-section (1.55).

The absolute values of atomic and electron cross-sections with regard to the above processes, except the photoeffect at low energies, are very low. Therefore, γ -quantum fluxes belong to penetrating nuclear radiation. The dependence of the cross-sections of the three processes upon the γ -quantum energy and upon the characteristics of the substance is different and requires individual consideration. However, since the cross-sections are proportional to the interaction probabilities (1.47) and the overall probability is the sum of the interaction probabilities in the separate processes, the linear factors of attenuation of the γ -quantum flux of a given energy in a given substance, relating to different processes, are summed up:

$$\mu = \mu_{ph} + \mu_C + \mu_{pr} \quad (4.15)$$

Here, the subscripts refer to the photoeffect, the Compton effect and the pair-production process, respectively.

The dependence of the cross-sections upon the energy is such that at low energies the photoeffect predominates in photon absorption. With extremely high energies, photons are absorbed in the main on account of the pair-production process. In the intermediate region, the Compton effect is dominant. In hard lead, the Compton effect makes the largest contribution to the attenuation of the γ -quantum flux in the range from 0.5 to 5 MeV. In light aluminium, this range is even wider, from 0.05 to 15 MeV. Hence, the Compton scattering is the basic process of the interaction of γ -quanta of the energy ~ 1 MeV (especially so with the light substances) with a substance. It is just such γ -quanta that are emitted during decays of excited nuclei resulting from spontaneous transformations of atomic nuclei or from nuclear reactions. Reactions of radiative absorption of neutrons, in which the γ -quantum energy often amounts to 5-8 MeV, are an exception.

2. Photoeffect. The photoeffect is the absorption of a γ -quantum by an atom with the emission of one of the atomic electrons. Part of the energy E_γ of the absorbed γ -quantum is spent on the work required for the electron to escape from the atom, and the remaining energy is transferred to the electron as its kinetic energy E_e since the atomic recoil energy is negligible (1.34):

$$\left. \begin{aligned} E_e &= E_\gamma - E_K \\ E_e &= E_\gamma - E_L \\ &\dots \dots \dots \end{aligned} \right\} \quad (4.16)$$

where E_K , E_L , ... are the values of the electron bond energy in K -, L -, and so on, shells, respectively. Electrons of kinetic energy

much lower than the electron rest energy (0.51 MeV) are ejected from atoms predominantly in a direction perpendicular to the motion of γ -quanta, following the $\sin^2\theta$ law, where θ is the angle between the photon and electron momenta vectors [18]. The higher the photon energy, the greater the electron kinetic energy, and the more extended in the direction of the initial motion of photons is the space distribution of the photoelectron momenta. But photoelectrons are never emitted precisely in the direction of the initial motion of the γ -quantum.

Having absorbed the γ -quantum and having emitted the electron, the atom appears to be in its excited state. The release of the atom from its excess energy is accompanied by electron transitions and by emission of characteristic X-ray quanta. Since the absorption of photons of the energy $E_\gamma > E_K$ is predominantly accompanied by the ejection of the K -electron, and the vacancy in the K -shell is, in the main, filled by the L -electron, the energy of these quanta is the difference of the bond energies of electrons in the K - and L -shells and amounts to 0.1 MeV for the heaviest atoms (0.075 MeV in lead). Thus, the photoeffect on heavy atoms (the heavy atom photoeffect) is accompanied by the emission of secondary sufficiently penetrating photons. The energy of secondary photons emitted by light atoms is low and such photons are immediately absorbed by the substance. In addition, light atoms more often emit the Auger-electrons than the X-ray quanta (Sec. 3.4-5).

The photoeffect is only feasible in the atom not in the free electron. The laws of energy and momentum conservation do not hold if one assumes full absorption of the photon by the free electron. In this case, the corresponding relations are of the form

$$\left. \begin{aligned} hv + mc^2 &= m'c^2 \\ \frac{hv}{c} &= m'v \end{aligned} \right\} \quad (4.17)$$

where hv and hv/c are the energy and momentum of γ -quantum, and m and m' , the rest mass and the relativistic mass (1.22) of the electron. It appears that equations (4.17) are compatible only on condition that the velocity of the electron after the collision is equal to the velocity of light, i.e., $v = c$, which is not feasible if the rest mass of the particle is not equal to zero. The transfer of the relativistic mass (the energy motion) is associated with a lower momentum value than the transfer of the rest mass (the rest energy), (1.35), (1.36). Therefore, the transfer of the whole energy and, hence, of the whole momentum by the photon to the particle, the rest mass of which is not zero, proves impossible. To counterbalance the momenta, it is necessary to have a third body which is the atom. Thus, the participation of the atom in the photoeffect

is of principal significance, although this participation does not practically influence the energy balance (4.16) because of the very great difference between the atomic and the electron masses.

The photoeffect is a typical resonance phenomenon. With the greatest probability it occurs when the photon energy is equal

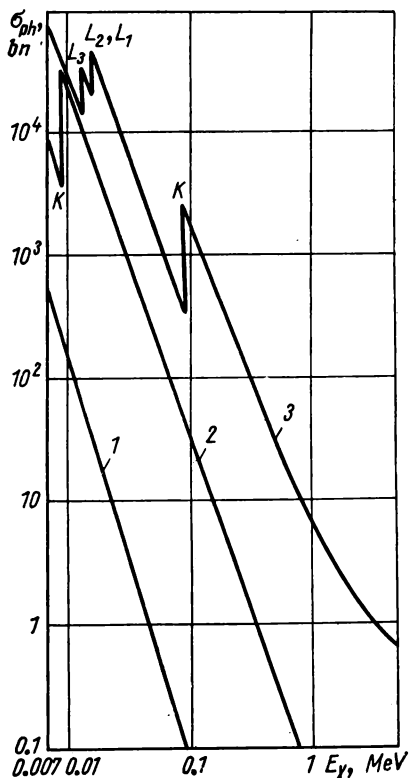


Fig. 4.6. Atomic cross-section with respect to photoeffect
1 — oxygen; 2 — copper; 3 — lead

to the bond energy of the electron in a corresponding shell. The atomic cross-section relative to the absorption of the photon has its maximum. To the left of the maximum, the cross-section for the given shell is zero, and to the right it rapidly decreases with the increase of the photon energy (Fig. 4.6), being added to the photoeffect cross-sections on other shells. Theory in agreement with experiment shows that the photoeffect cross-section varies in inverse proportion to $E_\gamma^{3.5}$. With energy $E_\gamma \gg E_K$, the variation of the cross-section is slowed down and is inversely proportional to the first power of E_γ , but with high energies, the photoeffect is negligible in comparison with the Compton effect. The rapid reduction of the photoeffect cross-section causes the photoeffect to be accompanied by the ejection of mainly K-electrons from the atom (when $E_\gamma > E_K$) because the cross-section contribution by the previous resonances, more remote along the energy scale, on L-,

M- and other shells appears small. The K-shell contribution to the total photoeffect cross-section is not less than 80% on condition that $E_\gamma > E_K$. The photoeffect cross-section strongly depends upon the nuclear charge. Theory predicts this dependence as Z^5 for light nuclei and a somewhat weaker dependence for heavy nuclei. The dependence $Z^{4.6}$ for a sufficiently wide range of atomic numbers, from $Z = 29$ to $Z = 82$, follows from experiment. In this connection, the atomic cross-section relative to the γ -quantum absorption in the

photoeffect may be presented in the following form:

$$\sigma_{ph} = C' \frac{Z^{4,6}}{E_\gamma^{3,5}} \quad (4.18)$$

where C' is the necessary constant of proportionality. Cross-sections (4.18) as a function of energy for lead, copper and oxygen, are shown in Fig. 4.6.

The linear coefficient of attenuation being a macroscopic cross-section (1.55) is equal to the product of the atomic cross-section by the number of atoms per unit volume (1.2):

$$\mu_{ph} = \sigma_{ph} N = C \frac{\rho Z^{4,6}}{A_r} \cdot \frac{1}{E_\gamma^{3,5}} \quad (4.19)$$

where the constant C combines all the values independent of the energy and the characteristics of the substance. Following relation (4.19), one can obtain the linear coefficient $\mu_{ph}(E_\gamma)$ of any substance if μ_{ph} of some substance is known. It is assumed in recalculation that the densities, the atomic masses, and the atomic numbers of the elements of the substances being compared are known.

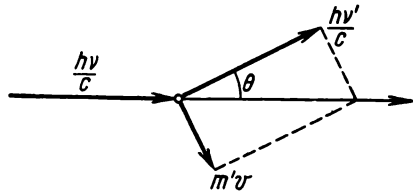


Fig. 4.7. Pulse diagram of elastic scattering of a photon by an electron

3. Compton effect. The Compton effect is the elastic scattering of γ -quanta by free electrons. Usually, these are electrons bonded in atoms, but, first, Compton scattering is feasible on the inherent free electrons and, second, the Compton effect is typical for photon energies much higher than the bond energy of all atomic electrons or, at least, the majority of them. The energy lost in exciting the atoms by knocking electrons out may, therefore, be neglected and the scattering may be considered elastic. Elastic scattering is only characterized by the kinetic energy exchange between colliding particles without loss of energy in excitation and without the transition of energy into the bound state. During elastic collision with the electron at rest, the photon is deflected at some angle θ , and the electron acquires the recoil momentum at the expense of the decrease of the photon energy (Fig. 4.7). Particle energies and momenta after collision are determined by the conservation laws which are in this case expressed by the following equations:

$$h\nu + mc^2 = h\nu' + m'c^2 \quad (4.20)$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c} + m'v \quad (4.21)$$

The law of energy conservation (4.20) includes the rest energy of the electron (1.26) before collision and the total energy (1.23) after collision. The energy of the scattered photon is denoted by $h\nu'$. The law of momentum conservation (4.21) is presented in vector form, momentum vector moduli being written according to relativistic relations (1.27) and (1.25). If from the triangle of the momentum vectors in Fig. 4.7 one obtains the relationship between the squares of their moduli and the angle, and, with the help of relation (1.24), excludes the electron momentum $m'v$ from the obtained equation and from equation (4.20), then the energy of the scattered photon may be expressed through the scattering angle θ :

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{mc^2} (1 - \cos \theta)} \quad (4.22)$$

The kinetic energy of the recoil electron E_e is, naturally, equal to $E_e = h\nu - h\nu'$.

The value of the energy of the scattered quantum is strongly dependent upon the scattering angle, except for the cases of low energies. If $h\nu \ll mc^2$, then with any θ the second term in the denominator (4.22) is negligibly small in comparison with unity and the energy of the photon scattered through any angle $h\nu' \approx h\nu$, i.e., scattering takes place without energy transfer to the electron. In this case, the Compton scattering is actually transformed into the Thomson scattering of low-energy photons by bonded atomic electrons. The Thomson scattering is explained in terms of classical electrodynamics. It is based on the mechanism different from that underlying the Compton scattering, i.e., excitation, by the incident electromagnetic wave, of forced vibrations of the atomic electron which itself emits in this case an electromagnetic wave of the same frequency but in another direction. Averaged over all the directions of the wave emission the electron cross-section with respect to the Thomson scattering does not depend upon the energy and equals $\sigma_{Th} = 0.665$ barn. When the value of the positive electric charge of the atom was not known, experiments with the Thomson scattering of X-rays indicated that the number of electrons in the atom was not great and approximately equalled half the atomic mass (Sec. 1.1-7). If the γ -quantum energy is comparable to or exceeds the rest energy of the electron, then only in scattering through low angles $h\nu' \approx h\nu$. With the increase of the angle θ , the value $h\nu$ rapidly decreases and in the limiting case $h\nu \gg mc^2$, and $\theta = \pi h\nu' = mc^2/2$, that is, in knock-on collision of the γ -quantum with the electron, scattered in the opposite direction is a photon the limiting energy value of which does not depend on $h\nu$, the energy of the incident photon. It is just this photon that counter-balances the momenta when almost all the energy of the

γ -quantum, a particle of the rest mass equal to zero, is transferred to the electron of the rest mass m . It was already pointed out in Sec. 4.4-2 that, when the photon collides with the free electron, the latter cannot acquire the whole energy of the photon, i.e., the photoeffect due to the free electron is not feasible.

The probability of scattering the γ -quantum through a certain angle is determined by the differential scattering cross-section which Klein and Nishina calculated by quantum mechanics

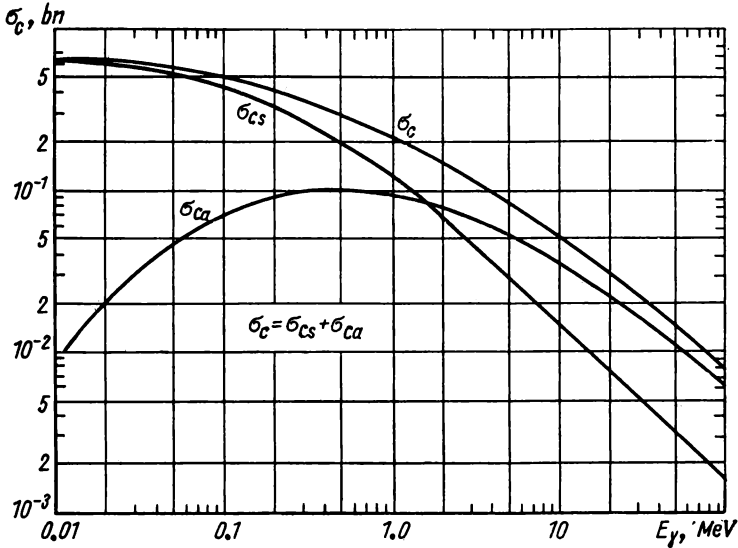


Fig. 4.8. Electron cross-section with respect to Compton effect

methods. Integrating over all possible angles gives the total electron cross-section relative to the Compton scattering

$$\sigma_c = \frac{3}{4} \sigma_{th} \left\{ \frac{(1+\epsilon)}{\epsilon^2} \left[\frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon} \ln(1+2\epsilon) \right] + \frac{1}{2\epsilon} \ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2} \right\} \quad (4.23)$$

Here, $\epsilon = h\nu/mc^2$ is the photon energy in the units of the electron rest energy and σ_{th} is the Thomson scattering cross-section expressed through the universal constants and equal to 0.665 barn. With infinitely low ϵ , the cross-section σ_c turns into σ_{th} (Fig. 4.8) and at $\epsilon \gg 1$ it decreases in inverse proportion to the photon energy $\sigma_c = (3/4) \sigma_{th} (1/2\epsilon) (1/2 + \ln 2\epsilon)$. Theoretical conclusions are in fair agreement with experiment at least in the energy range up to 30 MeV. From the viewpoint of energy transfer by photons, the

Compton effect is a process of partial scattering and partial absorption of energy as part of the energy is transferred to electrons. The fraction of the scattered energy depends upon the photon scattering angle θ and is determined by relation (4.22). If this fraction is multiplied by the calculated differential cross-section scattering at angle θ proportional to the fraction of photons scattered through this angle, and is integrated over all the angles, the overall cross-section of the electron relative to the photon energy scattering σ_{Cs} is obtained. The difference $\sigma_{Ca} = \sigma_C - \sigma_{Cs}$ is the energy absorption cross-section under the Compton effect. The cross-sections σ_{Cs} and σ_{Ca} are also presented in Fig. 4.8. During the Compton effect, γ -quanta are scattered, as in the case of the Thomson scattering, predominantly in the forward direction, $\theta \approx 0$ or in the backward direction, $\theta \approx \pi$. At low energies ($h\nu \ll mc^2$), γ -quanta are scattered forward and backward with approximately equal probability, with almost no energy loss [18]. Therefore, σ_{Ca} is low in the region of low energies. With energies equal to or higher than mc^2 , back scattering is connected with a large loss of energy and its probability in comparison with the probability of forward scattering is enhanced. These circumstances lead to a rapid increase of σ_{Ca} , so that starting from 1.6 MeV, when $\sigma_{Cs} = \sigma_{Ca} = 0.08$ barn, $\sigma_{Ca} > \sigma_{Cs}$ (see Fig. 4.8).

Since the Compton effect is the collision of the photon with the electron, the linear coefficient of attenuation μ_C is equal to the product of the electron cross-section σ_C by the number of electrons per unit volume of the substance NZ , where N is the corresponding number of atoms (1.2),

$$\mu_C = \sigma_C NZ = \sigma_C(E_\gamma) \cdot N_A \frac{Z}{A_r} \rho \quad (4.24)$$

The mass coefficient of attenuation μ_C/ρ (dimension is m^2/kg), derived from (4.24), does not practically depend upon the characteristics of the substance since Z/A_r is approximately constant in all stable nuclides. Light substances have $Z/A_r = 0.5$ because the composition of stable nuclei follows the rule $Z = (A - Z)$ (Sec. 2.4-7). Heavy nuclei have an excessive number of neutrons, but even for U^{238} , $Z/A_r = 0.4$, i.e., this value is constant in all substances, with the accuracy to 20%. If in formula (4.14) the mass coefficient of attenuation is used, then the thickness of the substance layer should also be taken in mass units as $x\rho$ (kg/m^2). Since μ/ρ is the same for all substances, the same values of $x\rho$ for different substances give similar attenuation of the photon fluxes with such energy that the contributions of other processes are small compared to the Compton effect (Sec. 4.4-1). This, in turn, means that the linear sizes of the substance layers of equal attenuation are inversely proportional to the substance densities.

This conclusion is concerned with the Compton effect only and is not valid for low and high γ -quanta energies where the Compton scattering is not predominant. However, even in substances such as aluminium and lead, so different in their atomic mass, the total mass coefficient of attenuation including all the processes is practically the same in the range of E_γ from 1 to 2 MeV (see Figs 4.9 and 4.10).

4. Pair-production process. If the energy of the γ -quantum exceeds $2mc^2$, then the γ -quantum may completely be absorbed by the nuclear Coulomb field, with the production of an electron-positron pair (see Fig. 1.6b). Since the mass of the atom is much greater than the mass of the emerging electrons, all the γ -quantum energy E_γ , except the part which passes into a bound state, is carried off by electrons as their kinetic energy E_{e-} and E_{e+}

$$E_\gamma = E_{e-} + E_{e+} + 2mc^2 \quad (4.25)$$

The electron and positron momenta may be arbitrarily oriented in space, and their kinetic energies may be of different values since a third body participates in the process, the atomic nucleus. The rest energy of the nucleus is much higher than the energy of the γ -quantum, and the momentum carried by the γ -quantum is relatively low (1.35). Therefore, the pair-production threshold in the nuclear field practically coincides with the value $2mc^2$ and equals 1.02 MeV. The pair may be produced in the electron field as well but with much less probability as the electron charge is much smaller than the charges of most nuclei. In the latter case, the threshold is higher and is 2.04 MeV (1.39). The target-electron acquires a high recoil momentum and the pair production results in three quick light particles, two electrons and one positron. Participation of the nucleus or the electron in pair production is of fundamental importance. In field-free space the photon cannot dissociate into an electron and a positron since the laws of energy and momentum conservation are in this case inconsistent. This conclusion can be drawn most directly if the conservation laws are presented by the following expressions [19]:

$$h\nu = m'_-c^2 + m'_+c^2 \quad (4.26)$$

$$\frac{h\nu}{c} \leq p_- + p_+ \quad (4.27)$$

where m' and p are relativistic masses and the moduli of the electron (minus subscript) and positron (plus subscript) momenta. The law of momentum conservation (4.27) is presented as an inequality derived from the triangle of momenta: one side of the triangle is less than or equal to the other two sides. From the law of energy conservation (4.26), with the help of (1.23) and (1.24),

one can obtain $(h\nu)^2 > (p_- + p_+)^2 c^2$, which contradicts (4.27). Equations (4.26) and (4.27) appear to be consistent only on condition that $m_- = m_+ = 0$.

The cross-section of the pair production process referred to one atom is equal to zero when $E_\gamma < 2mc^2$. In the above-threshold region, the cross-section increases at first slowly, but in the range of 3 to 40 MeV it can, with fair precision, be represented by a logarithmic dependence on E_γ . With energies higher than 40 MeV, the cross-section growth is again slowed down, which is first of all true of heavy nuclei as the atomic electrons screen the nuclear field during pair production in the regions of the atom most distant from the nucleus. The ratio of the cross-section to the nuclear charge is quadratic. In the region of the logarithmic dependence, the atomic cross-section with respect to the pair production in the nuclear field σ_p may have the following form

$$\sigma_p = CZ^2 \ln E_\gamma \quad (4.28)$$

where C is the proportionality factor. Then the linear coefficient of attenuation $\mu_p = N\sigma_p$ is equal to (1.2)

$$\mu_p = CN_A \frac{\rho Z^2}{A_r} \ln E_\gamma \quad (4.29)$$

In the energy region up to 10 MeV the dependence on the constants of the substance holds rigorously enough, and expression (4.29) may be used to calculate $\mu_p(E_\gamma)$ of a substance from the known $\mu_p(E_\gamma)$ of the standard. Because of the quadratic dependence of σ_p on the charge value as well as the higher threshold, the pair-production cross-section in the electron field is much less than in the nuclear field. However, at high energies $E_\gamma \gg 50$ MeV, when the effect of screening the nuclear charge by atomic electrons becomes substantial, the contribution of electrons to the pair production process increases. It constitutes $1/Z$ -th part of the total effect since the electron cross-section is by a factor of Z^2 smaller, but there are Z electrons in one atom. Thus, in light substances, the electron contribution to the pair production cross-section may exceed 10% [18]. At lower energies, the electron contribution is substantially lower due to a higher threshold.

The positron produced in the process annihilates in colliding with the electron (Sec. 1.8-2), and transforms the rest energy of particles predominantly into two γ -quanta, each of the energy 0.5 MeV. Thus, although the original γ -quantum is completely absorbed in the pair-production process, the positron annihilation, however, results in the secondary sufficiently hard γ -radiation. Therefore, when considering the problem of protection against

γ -radiation, the coefficient of actual absorption μ_{pa} is determined as

$$\mu_{pa} = \mu_p \left(\frac{E_\gamma - 2mc^2}{E_\gamma} \right) \quad (4.30)$$

μ_{pa} is somewhat different from μ_p only at energies close to the threshold. But even if $\mu_{pa} \approx \mu_p$, the appearance of secondary radiation should not be neglected.

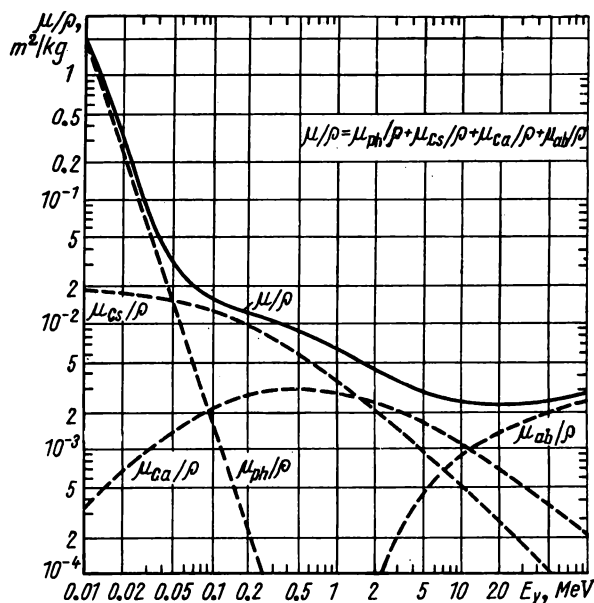


Fig. 4.9. Relation of the mass factor of γ -ray quanta attenuation in aluminium to energy

5. Extended shielding against γ -rays. The sum of linear coefficients of all the processes is the total linear coefficient of attenuation of a γ -ray flux (4.15). The total mass coefficients of attenuation, μ/ρ , as well as their mass components for aluminium and lead are shown in Figs (4.9) and (4.10). The total linear coefficient μ or its mass analog μ/ρ define the exponential attenuation of the γ -ray flux (4.14), but only in the case of narrow beams passing through the samples of a substance of a small cross-section. In this event each act of the photon interaction with the atom or electron, be it absorption or scattering of the photon, results in extracting the photon from the beam. γ -Rays passing through extended shielding walls are a different case. The Compton scattering retains the photon. The pair-production process is also accom-

panied by the formation of secondary photons. Although the energy of scattered or secondary photons is lower than the energy of primaries and the directions of their propagation are arbitrary, nevertheless, some of the γ -quanta reach the boundary of the shield-

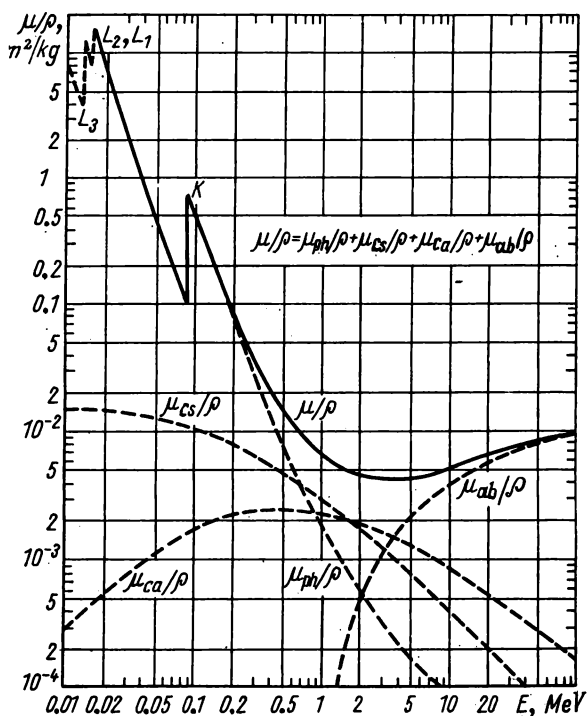


Fig. 4.10. Relation of the mass factor of γ -ray quanta attenuation in lead to energy

ing wall. As a result, the total flux beyond the shielding appears to be higher than the flux calculated according to formula (4.14). If the coefficient μ is composed of only those constituents which are responsible for energy absorption, i.e., $\mu_a = \mu_{ph} + \mu_{ca} + \mu_{pa}$ is used, then on the contrary, formula (4.14) will give an overestimated result.

Scattered and secondary photons have the effective path to the boundary of the shielding much greater than the photons of a straight beam have. The energy of scattered and secondary photons is lower, that is why they are absorbed by the substance more rapidly than the primary γ -quanta. The Compton scattering and the appearance of secondary photons result in something like

γ -rays diffusion in a large volume of the substance. Methods are available of calculating space, energy and angular distributions of photons propagating in a substance [18]. These methods are used for applied purposes of protection against γ -radiation in calculating the build-up factors B which show how many times the actual radiation dose exceeds the dose calculated by formula (4.14) with the use of an ordinary coefficient of linear attenuation μ . Of most practical interest, however, is an experimental determination of the effective factors of γ -quantum absorption, μ_{eff} , in extended shieldings. This method allows the dose rate to be calculated directly from formula (4.14).

μ_{eff} appears to be dependent not only on the energy of γ -rays, Z atoms of the medium, but on the geometry of the absorber as well. Experimental data on μ_{eff} are, therefore, obtained on samples corresponding to the geometrical form of the extended shielding wall and are given in reference books on dosimetry and shielding.

4.5. Neutrons

1. Scattering and reactions. Free neutrons are produced as a result of nuclear reactions. The bond energy of neutrons forming part of the nucleus is measured in megaelectron-volts. Emission of electrons of moderate kinetic energies is, therefore, most unlikely. Owing to this, the initial energies of neutrons are also measured in megaelectron-volts. Neutrons have no electric charge and do not participate in the Coulomb interaction either with atomic electrons or with nuclei. All processes initiated by neutrons are only determined by nuclear forces. Nuclear radii do not exceed 10^{-14} m, which is much lower than the atomic radii 10^{-10} m. Therefore collisions of neutrons with nuclei occur much more rarely than collisions of charged particles with atoms. Neutron path lengths (1.59) from collision with the nucleus to the next collision in condensed media are measured in centimetres or even exceed 10 cm. In addition, it turns out that, although the strongest forces of attraction act between neutrons and nuclei, neutrons as a rule join atomic nuclei relatively seldom and collisions are usually accompanied by scattering. Neutron fluxes, therefore, belong to penetrating nuclear radiation. Eventually, neutrons are absorbed by nuclei usually in a period of time much less than a second so that neutrons have no time to undergo β -decay (1.65). Absorption by nuclei is accompanied by the formation of secondary particles of high energy, namely, γ -quanta, protons and α -particles, and in the case of the fission process, neutrons.

The reason for the weak absorption of neutrons by nuclei is that the absorption is a nuclear reaction, and nuclear reactions undergo

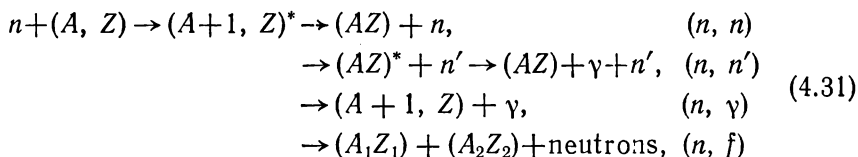
the stage of a compound nucleus. For a compound nucleus to be produced, the energy and spin relations (Sec. 3.5-4) must be strictly obeyed. The neutron resonance widths, or the widths of the energy levels of compound nuclei being formed in the process of neutron capture are small compared to the distances between the levels (Secs 2.7-3 and 2.7-4). They increase with the increase of the kinetic energy of neutrons, and the distances between compound nuclei decrease. Nevertheless, at energies of the order of megaelectron-volts and, the more so, at lower energies, and with light nuclei especially, the probability of the production of a compound nucleus is, first and foremost, determined by the correlation between the width Γ and the distance between the levels, D , and appears relatively small.

Collisions of neutrons with nuclei without production of compound nuclei are a process of elastic scattering termed potential scattering. However, the very fact of producing a compound nucleus does not mean that a neutron absorption reaction has taken place. The type of the reaction is determined by the way of the compound nucleus decay. Since in this case the compound nucleus is formed by absorbing a neutron, its excitation energy (3.43) is at least higher than the bond energy of the neutron, and in this case the excited nucleus decays predominantly emitting a neutron, not a proton or an α -particle. The γ -quantum emission efficiently competes with neutron emission only when the decay of the compound nucleus accompanied by the ejection of a neutron is strongly hindered because of the excitation energy exceeding but very slightly the bond energy of the neutron, i.e., when the compound nucleus is formed in capturing a neutron of the energy about 1 keV or less. Thus, in the main the production of the compound nucleus does not either lead to a nuclear reaction but scattering which is termed resonance scattering in this case.

After resonance scattering, the remaining nucleus is, in its nucleon composition, the original (parent) target nucleus. If this nucleus proves to be in its ground energy state, the resonance scattering is elastic and, in its consequences, does not differ at all from the potential scattering. In both cases, the neutron transfers part of its kinetic energy to the target nucleus as the kinetic recoil energy. If the nucleus, the product of the decay of the compound nucleus, in its turn appears to be excited, the scattering is termed inelastic. Since inelastic scattering is connected with the excitation of the target nucleus on account of the kinetic energy of the neutron and the minimum energy of the nuclear excitation is about 0.1 MeV or higher (Sec 2.7-3), inelastic scattering is possible if the kinetic energy E is higher than the above value, or, to be more precise, if $E > E_1^*$, with E_1^* being the energy of the first excited level of nuclei with which the neutrons interact.

Thus, neutron scattering predominates over their absorption. As follows from experiment, both elastic and inelastic scattering are spherically symmetric, except elastic scattering at energies $E \gtrsim 0.1$ MeV. Since, before absorption, the neutron undergoes many scatterings and each act of scattering is due to the equally probable deflection at any angle, the motion of neutrons in the substance may be described as diffusion. Average displacements of neutrons in space (paths) are, therefore, determined by spatial distributions of the diffusing neutrons. Corresponding neutron distributions can be obtained theoretically as solutions of diffusion equations (Chapter Five).

2. Energy scheme. Of all nuclear reactions resulting in neutron absorption most important is the reaction of radiational capture (n, γ) . If heavy nuclei are considered, the fission process (n, f) should also be allowed for. The possible decay channels of the compound nucleus produced after the capture of the neutron can, therefore, be presented in the following form:



(n, n) and (n, n') denote here elastic (resonance) and inelastic scattering. The energy scheme illustrating the possibility of producing a compound nucleus $(A+1, Z)^*$ and its decay along the first three channels (4.31) is shown in Fig. 4.11. The figure contains schemes of the nuclear levels of the target nucleus (A, Z) , of the compound nucleus $(A+1, Z)^*$ and of the parent nucleus (A, Z) which survives in the event of scattering. To the left of the scheme, a scale of the neutron kinetic energies E_n is presented, its zero point coincides with the value of the ground state energy of the target nucleus, i.e., with the value of the rest energy of the target nucleus-neutron system, $(A, Z) + n$. In this case the total energy of the system $(A, Z) + n$, which is the sum of the rest energy and the kinetic energy, is marked by coordinates along the scale E_n . The total energy is preserved in nuclear interactions and is, therefore, of the same value before the process, during the life-time of the compound nucleus and after its decay. The rest energy of the nucleus $(A+1, Z)$ is lower than the rest energy of the unbound system $(A, Z) + n$ by the neutron bond energy E_{bn} in the nucleus $(A+1, Z)$. Hence, the null count of the excitation energy of the compound nucleus is below the ground state of the target nucleus also by the value of E_{bn} . Therefore, when a neutron of the lowest kinetic energy is absorbed, very high levels of the compound nu-

cleus are excited. The value of the bond energy of the neutron never coincides with the energy of a subsequent excited level of the nucleus. The neutron kinetic energy, however, varies continuously, and any i th level of the compound nucleus of the energy $E_i^* > E_{bn}$

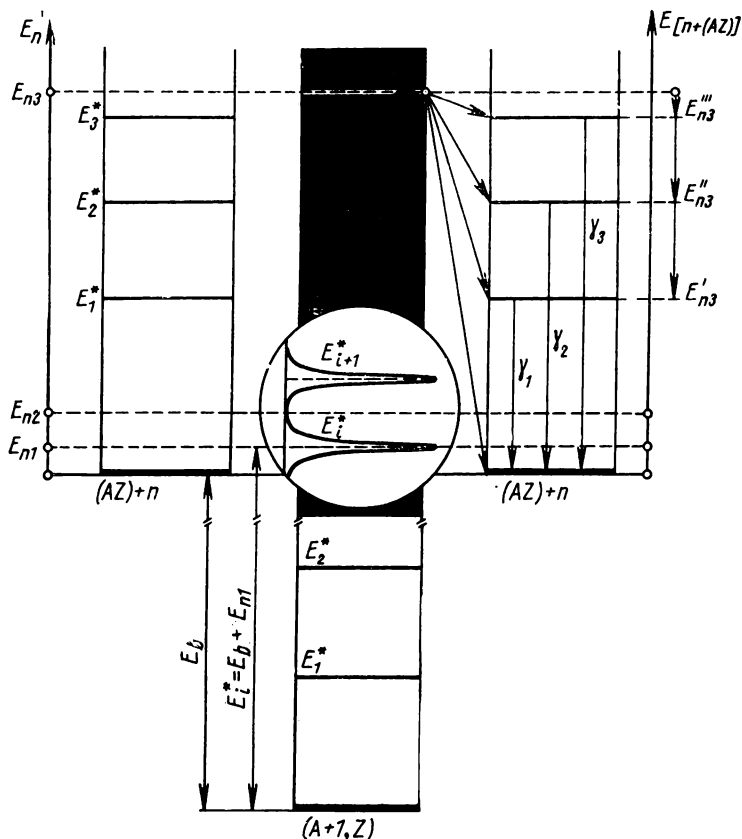


Fig. 4.11. Energy scheme of compound-nucleus production and decay

may be excited by absorbing a neutron of the desired energy E_n if, of course, the spin relations are also satisfied.

If E_{n1} (see Fig. 4.11) is such a kinetic energy that the excitation energy $E^* = E_{bn} + E_{n1}$ brought by the neutron into the nucleus coincides with the energy E_i^* of a nuclear level $(A + 1, Z)$, then there is a possibility of the formation of a compound nucleus $(A + 1, Z)^*$. Radiative capture will take place if the nucleus $(A + 1, Z)^*$ emits γ -rays of the total energy equal to the excitation energy, $E_\gamma = E^* = E_{bn} + E_{n1}$, and transfers into its ground state.

Neutron emission always competes with photo-emission since $E^* > E_{bn}$, and if it occurs, resonance elastic scattering is observed. Part of the excitation energy of the compound nucleus equal to E_{bn} is spent in the work against nuclear forces of attraction and neutron segregation from the nucleus. The remaining energy, E_{n1} , is distributed as kinetic energy between the nucleus (A, Z) and the neutron. The amount of the energy carried off by the neutron depends on the value of the mass of the nucleus (A, Z) and on the scattering angle of the neutron with respect to the direction of bombarding the target nucleus (Sec. 5.1-2). If, with the energy E_{n1} , no compound nucleus has been formed, or if the energy is E_{n2} , when no compound nucleus can be formed, elastic potential scattering takes place. Just as with resonance scattering, the distribution of E_{n1} or E_{n2} between the neutron and the nucleus after scattering is determined by the nuclear mass and the scattering angle.

If the energy of, say, only the first excited level of the target-nucleus (A, Z) is exceeded (in Fig. 4.11, E_{n3} is higher than E_1^* , E_2^* and E_3^*), then inelastic scattering (n, n') is also possible. Inelastic scattering also occurs through the compound nucleus. The decay of the compound nucleus with neutron emission must, however, result in excited states of the nucleus (A, Z). The nucleus (A, Z)* transfers into its ground states by emitting γ -quanta. In Fig. 4.11, it is γ_1 , γ_2 or γ_3 depending upon the excited state in which the nucleus (A, Z) has been formed. In the long run, γ -quanta acquire their energy on account of the kinetic energy of the bombarding neutron, in the particular case, E_{n3} . Therefore, the kinetic energy of the decay products of the compound nucleus ($A + 1, Z$)* is equal to the difference between the initial kinetic energy and the energy of the corresponding excited level of the target nucleus. In the right-hand side of Fig. 4.11, a scale of kinetic energies after inelastic scattering is shown, counted from the value E_{n3} downward. In all cases the relations $E_{n3} = E'_{n3} + E_1^* = E''_{n3} + E_2^* = E'''_{n3} + E_3^*$ are valid. It is easy to see that the kinetic energy after inelastic scattering can be very low, for example, E'''_{n3} in Fig. 4.11. This energy is distributed between the nucleus (A, Z)* and the neutron, as in the case of elastic scattering. If the initial energy of neutrons is very high compared to E_1^* , then, in inelastic scattering of many neutrons, many different levels E_j of the nuclei (A, Z) are excited. Since the energy of inelastically scattered neutrons is mainly determined by the difference $E_n = E_j^*$, then, with a high number of the excited levels E_j^* , neutrons practically have a continuous spectrum of kinetic energies. The mean energy of the inelastic scattering spectrum is sufficiently low. In case of inelastic scattering of neutrons with the initial energy 1 MeV by the nuclei of U^{238} ($E_1^* = 0.045$ MeV), the mean energy of inelastically scattered neutrons is about 0.2 MeV, i.e., on the average, in one act of in-

elastic scattering the energy of the neutron reduces by a factor of 5.

3. The Breit-Wigner formula. Since the compound nucleus is the bound target nucleus-neutron system, one expects that the cross-section of its formation must be determined by the de Broglie wavelength of the neutron (Sec. 1.4-4). The wavelength of the neutron (1.14) is inversely proportional to its velocity and at low energies it can be arbitrarily high. Hence, the cross-section of the compound nucleus formation may also be very large. On the other hand, the

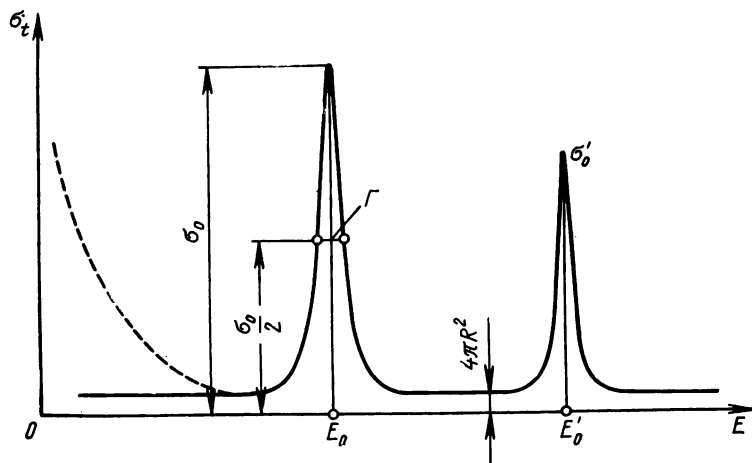


Fig. 4.12. Neutron cross-section to energy relation

compound nucleus is formed only at a particular value of the neutron kinetic energy with its possible deviations within the width of the compound nucleus level Γ . Beyond this narrow energy interval the bound state is not formed and the neutron wavelength is of no significance. The interaction cross-section is determined by the geometrical size of the nucleus. As a result, the ratio of the cross-section to the energy of the neutron acquires a resonance character (Fig. 4.12).

Near the resonance, the cross-section is described by the Breit-Wigner formula. For the cross-section of the radiative capture σ_γ , this formula has the form

$$\sigma_\gamma(E) = \pi \lambda^2 g \frac{\Gamma_n \Gamma_\gamma}{(E - E_0)^2 + \left(\frac{\Gamma}{2}\right)^2} \quad (4.32)$$

Here, E is the kinetic energy, and λ , its associated neutron wavelength (1.14). The statistical factor g (1.21) indicates the percentage of collisions of neutrons with the nuclei (A, Z) , accompanied

by the realization of the total mechanical moment equal to the spin of the compound nucleus level being excited. In this case, g is

$$g = \frac{(2J + 1)}{2(2I + 1)} \quad (4.33)$$

where I is the spin of the target nucleus, J , the spin of the excited level of the compound nucleus and the spin of the neutron is $s = 1/2$. Neutrons with the energy lower than 10^4 eV interact only with nuclei having the mechanical moment of relative motion $l=0$. Therefore, l does not enter into expression (4.33). Such neutrons can only excite levels with $J = I \pm 1/2$ and when $I = 0$, $g = 1$, and for $I > 0$

$$g = \frac{1}{2} \left(1 \pm \frac{1}{2I + 1} \right) \quad (4.34)$$

If I is high, $g \approx 1/2$. The energy E_0 at which one observes the cross-section maximum, Γ , Γ_n , Γ_γ , as well as Γ_f , and the values of the cross-sections at the maximum (for example, σ_0 in Fig. 4.12) are termed resonance parameters. Γ is the total width of the resonance, it is the width of the excited nuclear level of the compound nucleus (Sec. 2.7-4). The remaining values of Γ_i are termed partial widths and have the following meaning.

In accordance with the uncertainty relation (1.20), the total width of the nuclear level is inversely proportional to its life-time, i.e., to the life-time of the compound nucleus, $\Gamma \approx \hbar/\tau$. The mean life-time, in turn, is inversely proportional to the decay constant λ (3.7). The width Γ is, therefore, the decay constant equivalent expressed in energy units. Hence, it describes the probability of the compound nucleus decay in unit time. Since the compound nucleus may decay in many ways (4.31) and each way is characterized by its probability λ_i determining the fraction of the decay in question in the total number of all decays, then each way of decay may be described by an equivalent energy value, i.e., by the partial width of the level Γ_i , proportional to λ_i . Because of the linearity of equation (3.1) presenting the basic decay law in mathematical form, the decay constants λ_i relating to different ways of decay of the same kind of nuclei are summarized and provide the total decay constant $\lambda = \Sigma \lambda_i$. Naturally it could not be otherwise since the physical meaning of the decay constant is the decay probability per unit time, and the probabilities are additive values. Since the widths are proportional to the decay constants λ , they are also summarized

$$\Gamma = \Gamma_n + \Gamma_\gamma + \Gamma_f \quad (4.35)$$

where the partial widths Γ_n , Γ_γ , Γ_f are termed a neutron, a radiative and a fission width and relate, respectively, to the processes of scattering (n, n), radiative capture (n, γ) and fission (n, f).

Thus, the physical sense of the partial width lies in the fact that they, like the decay constants, describe the fractions of the corresponding decays of the compound nucleus. For instance, the fraction of radiative captures is $(\lambda_\gamma/\lambda) = (\Gamma_\gamma/\Gamma)$ and so on, and the total fraction of all possible decays is, naturally, equal to unity. At the same time, the idea of the total width Γ is beyond the notion of a value proportional to the decay probability. The resonance width is the width of the nuclear level and it is related to the decay scattering cross-sections by the uncertainty relation (1.20) which describes some fundamental property of unstable states of microsystems. Each state with a finite life-time has no strictly definite quantized energy value. True, the probability of finding the state has its maximum at the energy E_0 but when the energy differs from E_0 , the probability is not zero although it decreases rapidly. The width Γ is the parameter of the energy distribution of this probability. Hence, Γ is also the distribution parameter of the cross-section value as a function of energy. This is the meaning of the width Γ in the resonance term of the Breit-Wigner formula.

The product of the partial widths Γ_n and Γ_γ in the resonance term of formula (4.32) arises from the fact that the Breit-Wigner formula expresses the cross-section of a reaction through the cross-section of the inverse reaction. The universal principle of the detailed equilibrium reads: if no extraneous circumstances exist, hindering the elementary process, then the probabilities of the direct and inverse processes are equal. Thus, if Γ_n describes the probability of the decay of the compound nucleus $(A+1, Z)^*$ with neutron emission, then Γ_n also describes the probability of the formation of the same compound nucleus when the neutron is absorbed by the nucleus (A, Z) . If, on the other hand, the compound nucleus may decay in several ways, it can be formed in the same number of inverse ways. The compound nucleus $(A+1, Z)^*$ can, for example, be formed either in the absorption of a neutron with energy E_0 by the nucleus (A, Z) or in the absorption of a γ -quantum of the energy $E_\gamma = E_{bn} + E_0$ by the nucleus $(A+1, Z)$. The total width Γ is proportional both to the total probability of the compound nucleus decay and to the total probability of its formation along all the possible channels. Since in the case under consideration the formation of compound nuclei takes place in the bombardment of nuclei by neutrons, the cross-section of any neutron reaction must be proportional to Γ_n . The cross-section of some particular reaction must, in addition, be proportional to the probability of decay along a certain channel. The radiative capture cross-section is, therefore, proportional to the product $\Gamma_n\Gamma_\gamma$.

It follows that expressions for cross-sections of different processes (4.31), or partial cross-sections, only differ in the second factor in the nominator of the resonance term. Substituting the widths

Γ_f , Γ_n or Γ for Γ_γ into (4.32), one obtains expressions for the fission cross-sections $\sigma_f(E)$, the resonance scattering $\sigma_{nr}(E)$ or the total cross-section of the formation of the compound nucleus $\sigma_c(E)$. The partial cross-sections are summarized, (1.49), and their sum is σ_c since the widths are related by (4.35):

$$\sigma_c = \sigma_{nr} + \sigma_\gamma + \sigma_f \quad (4.36)$$

The partial cross-sections are related to the cross-section of the formation of the compound nucleus as follows:

$$\sigma_\gamma = \sigma_c \frac{\Gamma_\gamma}{\Gamma}; \quad \sigma_f = \sigma_c \frac{\Gamma_f}{\Gamma}; \quad \sigma_{nr} = \sigma_c \frac{\Gamma_n}{\Gamma} \quad (4.37)$$

The total cross-section of the neutron interaction with nuclei of a given kind also includes the potential scattering cross-section σ_{np} and is

$$\sigma_t = \sigma_n + \sigma_\gamma + \sigma_f \quad (4.38)$$

where σ_n is the total cross-section of resonance and potential scattering. The cross-section σ_n is not a simple sum of σ_{nr} and σ_{np} since interference is possible between resonance and potential scattering. This effect often results in the distortion of the resonance curve so that the scattering cross-section is smaller on the left of the resonance, and on the right it is higher than the potential scattering cross-section between the resonances [20]. Fig. 4.12 illustrates the total cross-section σ_t in the vicinity of the first of all possible resonances.

The Breit-Wigner formulae are based on the compound nucleus concept and express cross-sections as a function of energy through the parameters of each given resonance not known beforehand. However, with the resonance parameters known, experiment agrees with the predictions of the Breit-Wigner formulae.

4. Resonance parameters. Doppler effect. Resonance parameters are determined experimentally. If one succeeds in determining the precise shape of the curve of the total interaction cross-section, which overlaps a given resonance, it appears to be sufficient for the determination of all the resonance parameters of a non-fissionable substance, i.e., a substance the full resonance width of which is $\Gamma = \Gamma_n + \Gamma_\gamma$. From the curve of the total cross-section one immediately determines E_0 , the cross-section in its maximum σ_0 , and the width Γ (see Fig. 4.12). On the other hand, the total cross-section of the compound nucleus formation, when $E = E_0$, is $\sigma_{c0} = 4\pi\lambda_0^2 \times (g\Gamma_n/\Gamma)$, where λ_0 corresponds to the energy E_0 . With correction for potential scattering, σ_0 is σ_{c0} , therefore, $g\Gamma_n$ is determined by the known σ_0 and Γ , i.e., the neutron width Γ_n is, in fact, determined simultaneously since the factor g is often precisely or

approximately known (4.34). In the case involved, $\Gamma_v = \Gamma - \Gamma_n$, and when measuring only the total cross-section, the capture and scattering cross-sections also appear to be known (4.37). The resonance parameters of fissionable substances are of particular interest for designing nuclear reactors. In this case, one cannot immediately find Γ_v and Γ_f (4.35) by the known Γ and Γ_n . To determine Γ_v and Γ_f , other additional much more complicated experiments to measure either the radiative capture or fission in the given resonance [21] must be undertaken.

To obtain a curve of the total cross-section in resonance requires a set of monoenergetic beams of neutrons, with the energy spread in each beam much less than the resonance width Γ . Only in this case many measurements of the total cross-section value at different energies within the resonance peak can be taken. The techniques of producing monoenergetic neutron beams are well elaborated [21]. If the precise shape of the resonance cross-section curve cannot be obtained, then σ_0 and Γ may be found in two independent relatively simple experiments to assess only the total cross-section [21].

At a normal temperature, the resonance parameters are distorted under the influence of the thermal motion of atoms and molecules of the medium. By analogy with optics, the variation of the shape of the resonance curve due to the thermal motion is termed Doppler effect the essence of which lies in the following. The neutron energy in the Breit-Wigner formula is, in fact, the energy of relative motion in the neutron-nucleus system. With the nucleus at rest, the energy difference in the coordinate system of the centre of mass and in the laboratory system is reduced to the constant coefficient (3.54), and is, therefore, of no significance. Target nuclei always participate in thermal motion, and although the velocities of thermal motion are much less than the velocities of neutrons initiating resonances, nevertheless, the thermal motion has an appreciable effect on the shape of the resonance peak curves, primarily because these peaks are very narrow by the energy scale. As a result, at one and the same neutron energy in the laboratory coordinate system, the energy of relative motion is somewhat higher if the atom is moving towards the neutron at the instant of collision, and somewhat lower when the atom is moving in the same direction as the neutron. Therefore, only part of the neutrons of the energy E_0 interacts with nuclei of the maximum cross-section σ_0 while the rest of them virtually interact with higher or lower energy, i.e., with a smaller cross-section. As a result, the cross-section observed at E_0 appears to be smaller than σ_0 . On the other hand, when the energy differs but slightly from E_0 , neutrons interact with nuclei not only of a small cross-section corresponding to the descending resonance tails, but also with nuclei

of a large cross-section up to σ_0 . This results in an increase of the observed cross-section in the tails of the resonance (Fig. 4.13). The resonance peak appears lower and wider due to the temperature of the medium but the Doppler broadening does not change the value of the area under the resonance peak curve [22]. The data of reference books on measured resonance parameters are corrected for the Doppler effect and are the values referred to the zero temperature of the medium [23].

The distribution of deviations of the relative kinetic energies of neutrons and moving nuclei from the relative energy of neutrons and nuclei at rest is described by the Gauss law [22], with the distribution parameter Δ

$$\Delta = 2 \sqrt{\frac{m_n E k T}{M}} \quad (4.39)$$

termed Doppler width. Here, m_n and M are the neutron and target nucleus masses, respectively; E , to the accuracy of the factor in (3.54), is the kinetic energy of the neutron; k is the Boltzmann constant, and T is the absolute temperature of the medium. The shape of the resonance curve distorted by the atomic thermal motion cannot be represented by a simple expression analogous to (4.32) and valid with $\Gamma \gg \Delta$. Since Δ depends upon T , at higher temperature it may become much higher than the natural resonance width Γ . Here, the distribution of the cross-section as a function of energy is given not by the resonance term of formula (4.32) but by the Gaussian dependence with the parameter Δ , $\sigma(E) \sim \exp[-(E - E_0)^2 \Delta^2]$. The Doppler broadening has an effect upon the absorption of moderated neutrons by uranium in nuclear reactors [24].

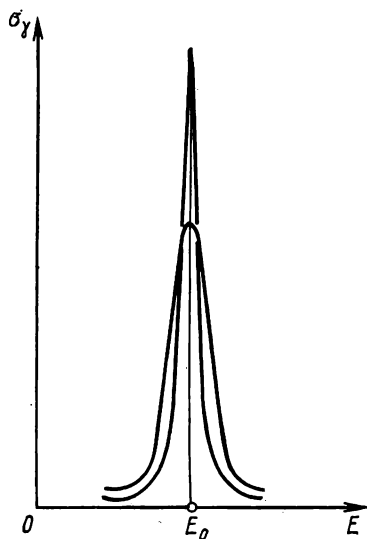


Fig. 4.13. Resonance broadening due to the Doppler effect

Experimental data concerning resonance widths are in fair agreement with the corresponding inferences from theory. The radiative width Γ_γ of the nuclear level increases in proportion to the third power of the excitation energy of the compound nucleus. Since at low kinetic neutron energies ($E < 10^3$ eV), the excitation energy (3.43) is practically a constant value, the radiative widths of all the resonances observed with nuclei of a given kind at low energies do not display any energy dependence. Since during the

decay of the compound nucleus, radiative transitions into quite a lot of intermediate states are possible, the radiative width Γ_γ , which, in fact, reflects the characteristic features of both the initial and many possible finite states, should not depend upon the individual features of any level. Where the radiational width does not depend upon energy, it is a constant value for all the resonances. Neutron widths, on the contrary, increase, in proportion to the neutron velocity $\Gamma_n \sim v$, so that at the lowest kinetic neutron energies usually $\Gamma_n < \Gamma_\gamma$ and at energies of the order of 10^3 eV or higher, $\Gamma_n > \Gamma_\gamma$. If $\Gamma_n > \Gamma_\gamma$, then, with the increase of the neutron velocity, the total width Γ equal to $\Gamma_n + \Gamma_\gamma$ also increases.

Neutron widths display, along with the velocity dependence, random variations of their value determined by the specific features of each nuclear level since with $E < E_1^*$ (Sec. 4.5-2) the compound nucleus decays with neutron emission into the only ground state of the nucleus (A, Z). As a result, higher-energy levels are not necessarily characterized by the high value of Γ_n although in the main it is so. To estimate this feature of the neutron widths, one uses the so-called reduced neutron width Γ_n^0 which is independent of the neutron velocity since it is determined by the following relation

$$\Gamma_n^0 = \frac{\Gamma_n}{\sqrt{E}} \quad (4.40)$$

where E is the value of the kinetic neutron energy, eV. Thus, Γ_n^0 is the neutron width reduced to such a neutron velocity that its energy is 1 eV. The frequency distribution of finding the reduced neutron width of this or that value, obtained experimentally for many resonances of a given nucleus, agrees well with the corresponding theoretical distribution based on the assumption of the random character of spread in the values Γ_n^0 [25], which corroborates the random nature of the fluctuations of Γ_n in different resonances. The nuclear resonance widths Γ_f of fissile nuclides are not energy dependent but, as with the neutron widths, are subject to random fluctuations. Such a behaviour of the Γ_f values is at some variance with the possibility of compound nuclei fissioning in many ways (Sec. 3.6-6). Nevertheless, the character of distributions of the Γ_f values is identical to the distributions of the reduced neutron widths Γ_n^0 [25].

5. Deviations from the Breit-Wigner formula. In formula (4.32) for $\sigma_\gamma(E)$, the energy dependence is given in the denominator of the resonance term in its explicit form and through the energy dependence λ and Γ_n in its implicit form. If all the energy dependence is given in its explicit form, then the radiative capture cross-section

tion will equal

$$\sigma_{\gamma}(E) = \sigma_{\gamma 0} \sqrt{\frac{E_0}{E}} \cdot \frac{1}{\left(\frac{E - E_0}{\Gamma/2}\right)^2 + 1} \quad (4.41)$$

where $\sigma_{\gamma 0}$ is the capture cross-section in the resonance peak

$$\sigma_{\gamma 0} = \sigma_{\gamma}(E_0) = 4\pi\lambda_0^2 g \frac{\Gamma_{n0}\Gamma_{\gamma}}{\Gamma^2} = 4\pi g \frac{\Gamma_{\gamma}}{\Gamma^2} \frac{C}{\sqrt{E_0}} \quad (4.42)$$

the values λ_0 and Γ_{n0} represent the quantities λ and Γ_n when $E = E_0$. They are related to $\lambda(E)$ and $\Gamma(E)$ by $(\lambda/\lambda_0) = (E_0/E)^{1/2}$ and the constant C combines the proportionality constants between λ_0^2 and Γ_{n0} and the energy E_0 . The cross-sections of all the processes associated with the formation of the compound nucleus rapidly decrease with the increase or the decrease of E with regard to E_0 . Beyond the resonances, therefore, the total interaction cross-section of neutrons with nuclei σ_t transforms into the potential scattering cross-section σ_{np} independent of energy (see Fig. 4.12). Theory predicts that the potential scattering cross-section is equal to the quadruple geometrical cross-section of the nucleus, i.e., $\sigma_{np} = 4\pi R^2$, where R is the nuclear radius (2.3). This conclusion does not contradict experiment. Near the resonances, interference of potential and resonance scattering is possible, which decreases or increases the total scattering cross-section, as compared to $4\pi R^2$.

Cross-sections between the resonances, although they are quite low, are not zero especially if the resonances are close to each other, which is characteristic of nuclei with intermediate and high mass numbers (Sec. 2.7-3). In this sense, a specific case is the region of very low kinetic energies of neutrons, below the first neutron resonance. In this region, the cross-sections of processes accompanied by neutron absorption vary in inverse proportion to the neutron velocity v . The law $1/v$ was discovered experimentally and stems from the Breit-Wigner formula. If $E \ll E_0$, the resonance term in (4.41) transforms into a constant number and the radiative capture cross-section changes in proportion to $1/\sqrt{E}$ or $1/v$. Under some circumstances, the increase of the absorption cross-section may appear so large that its absolute value will exceed the potential scattering cross-section, and the total cross-section in the region of low energies will also display an increase (the dotted line in Fig. 4.12). The latter case is either associated with the first resonance lying quite low, as is usual with intermediate and heavy nuclei, or with a very wide resonance. The wide resonance means a very short life-time of the compound nucleus (1.20). It is this property that is characteristic of exoenergetic reactions which result in the production of charged particles (n, p) and (n, α) .

If the difference between the energies of the initial and finite states is great, the decay always proceeds especially rapidly. The exoenergetic reactions (n, p) and (n, α) are observed with some light nuclides and, although the light nuclide nuclei have their first resonances at a very high energy of the order of tenths of megaelectron-volt, their resonance widths are of the same values. Thus, the first resonance has its effect on the whole energy region below the resonance. In these cases, with $E \ll E_0$, the reaction cross-section also obeys the law $1/v$, for instance, the cross-section of the (n, α) -reaction on B^{10} (see Fig. 4.17).

The value of the radiative capture cross-section in the resonance maximum (4.42) also decreases with the increase of the resonance energy E_0 . True, $\sigma_{\gamma 0}$ depends also upon the neutron width, and with different levels, these values are subject to random fluctuations. It is not necessary, therefore, for every higher-lying resonance to have a lower value of absorption cross-section than for the preceding resonance at low energy. But the general tendency remains invariable. The highest cross-section in the maximum is, as a rule, characteristic of the first resonance while the height of the resonance peaks decreases with the increase of the neutron energy. With sufficiently high energies, when $\Gamma_n \gg \Gamma_\gamma$ and $\Gamma \approx \Gamma_n$, the radiative capture cross-section in the resonances decreases much more rapidly (4.42) since here $\Gamma \sim v$ as well. The levels of heavy nuclei overlap even in the region of neutron energies of the order of 10^4 eV or higher. If the nuclear levels have been overlapped, the compound nucleus is formed with equal probability at any neutron energy E , and the radiative capture cross-section decreases with the increase of the energy continuously and faster than in the region of low energies, at least in proportion to $1/E$.

If in (4.41) one takes E to be equal to $E_0 \pm (\Gamma/2)$, the cross-section $\sigma_\gamma = (1/2)\sigma_{\gamma 0}$ as the factor $(E_0/E)^{1/2} = 1$, since Γ is quite low as compared with E_0 . It means that the resonance width Γ is the peak-width of the cross-section, taken in the middle of its height, whether of the cross-section peak σ_γ , σ_{nr} or $\sigma_t \approx \sigma_c$.

6. Energy regions. Neutrons produced in nuclear fission have energy not higher than 10 MeV. Subsequently, during inelastic or elastic scattering, the kinetic energy of the neutrons lowers down to the energy of the thermal motion of atoms and molecules of the medium. The neutron energy distribution in thermal equilibrium with the medium is the Maxwellian distribution (4.44). However, since neutrons are absorbed by atomic nuclei of the medium, some may be in the thermal energy region only if other neutrons continuously arrive from higher energy regions. Therefore, although the Maxwellian distribution is not restricted on the side of high energy values, in the case of neutrons a practical limit to the distribution can be pointed: it is the energy at which a flux of mo-

derating neutrons is equal to the flux of neutrons already brought into equilibrium with the medium. In nuclear thermal reactors, this energy is about 0.2 eV.

The character of the neutrons interaction with nuclei varies in the energy range from 0 to 10 MeV. Therefore, it is common to divide the entire possible energy range of neutrons in the nuclear reactor into three regions characterized by their peculiar interactions, namely, the ranges of fast, intermediate and thermal neutrons. The boundaries between the regions are conventional and the processes characteristic of one region are not excluded in the others:

| | |
|-------------------------------|----------------|
| fast region | 0.1—10 MeV |
| intermediate region | 0.2— 10^5 eV |
| thermal region | 0—0.2 eV |

The thermal region, though negligibly small, is rather significant. Neutrons, which have reached thermal equilibrium, no longer change their mean energy and, hence, accumulate in the thermal region where they are, in the long run, absorbed. Nuclear reactors are respectively termed fast, intermediate and thermal, depending upon the energy region in which the greatest number of neutrons are absorbed.

7. Fast neutrons. The energy of 99% fission neutrons lies in the fast region. The total cross-section in this region approximately equals $\sigma_t \approx 2\pi(R + \lambda)^2$, where R is the radius of the nucleus (2.3) and λ , the wavelength of the neutron of the energy E (1.14). The main feature of the fast region is that since the radiative capture cross-section σ_γ is very small here, the total cross-section is the scattering cross-section σ_s , and the latter is the sum of the elastic σ_n and inelastic $\sigma_{n'}$ scattering cross-sections: $\sigma_t \approx \sigma_s = \sigma_n + \sigma_{n'}$. The thresholds of inelastic scattering lie just within this region or a little below its boundary with the intermediate region (Sec. 4.5-2). Near the threshold, $\sigma_{n'}$ rapidly increases but then becomes constant and its value may comprise up to one third of σ_s (Fig. 4.14). As the energy of the bombarding neutron increases, the excitation energy of the second compound nucleus $(A, Z)^*$, (4.31), which may decay, also increases, either emitting one more neutron, which does not already correspond to inelastic scattering but to the reaction $(n, 2n)$, or splitting in the case of the heaviest nuclei. The competition of these processes causes a decrease in $\sigma_{n'}$ observed in fissile nuclei at the energy 6 MeV, i.e., near the photofission thresholds (Sec. 3.6-1). The lighter the nucleus, the higher, as a rule, the threshold $\sigma_{n'}$ and the higher the energy region where $\sigma_{n'}$ begins decreasing.

Elastic scattering also has its peculiarity in the fast region. At low neutron kinetic energies, at least lower than 10^4 eV, the neutron wavelength is much greater than the nuclear radius and scattering

takes place in the same way as by the point nucleus. In this case, the mechanical moment of the relative motion of interacting neutrons and nuclei can only be equal to zero, $l = 0$. In experiment, this shows up as the spherical symmetry of scattered neutrons in the centre-of-mass coordinate system, i.e., as the constancy of the differential cross-section of scattering $d\sigma_n/d\theta$ for all the scattering angles θ . As the kinetic energy increases and the wavelength decreases, it becomes possible for the neutron to interact with the peripheral regions of the nucleus with mechanical moment relative to the centre $l = 1, 2$ and so on. Such scattering is characterized by the selectivity of scattering angles: when $l = 1$, neutrons are

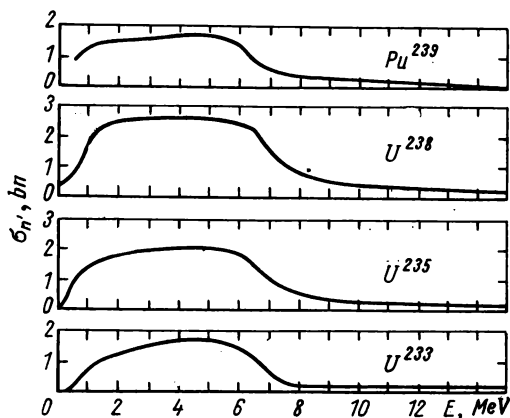


Fig. 4.14. Inelastic scattering cross-section

scattered predominantly in the forward direction. With other values of l , the probability of scattering at other angles becomes greater. The higher the neutron energy, the greater the contribution of non-spherically symmetrical scattering. Such scattering appears first in heavy nuclei, and in light nuclei at the energy $\approx 10^5$ eV. Fig. 4.15 shows the dependence $d\sigma_n/d\theta$ as a function of the cosine of the scattering angle for U^{238} at different energies. Such behaviour of $d\sigma_n/d\theta$ reduces the mean scattering angle. This results in the increase of neutron paths in the substance during diffusion, with no change in the total scattering cross-section (Sec. 5.3-3).

As for fission cross-sections, the fast region is characterized by the fact that here the fission cross-sections of the even-even heavy nuclides, such as U^{238} , Th^{232} and others, are not equal to zero. The nuclear fission thresholds of such nuclides are in the range of 1 MeV. Beyond the threshold, the cross-section rapidly reaches its plateau and then increases again at the energy about 6 MeV due

to the decrease of σ_n . The fission cross-sections of *fissile* nuclides in the region of fast energies are minimum as are all the cross-sections relating to the processes of neutron absorption. But if the radiative capture cross-sections continuously decrease with the increase of the energy approximately proportional to $1/E$, the fission

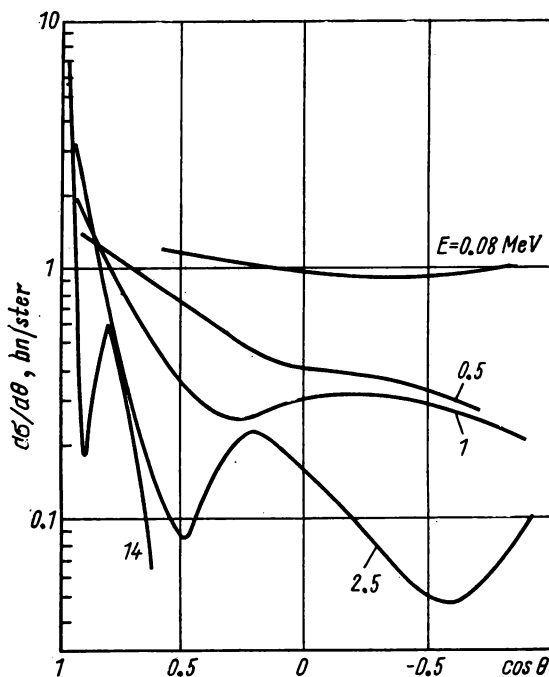


Fig 4.15. The $d\sigma/d\theta$ cross-section of U^{238}

cross-sections practically remain invariable in the entire fast region. At energies about 6 MeV, the fission cross-sections also somewhat increase due to the decrease of σ_n (Fig. 4.16). Thus in this region, the relationship between the fission cross-section and the radiative capture cross-section in fissile nuclides significantly changes for the benefit of fission, and the value $\alpha = \sigma_v/\sigma_f$ becomes much lower than unity.

8. Intermediate neutrons. Resonance phenomena are the most characteristic feature of the intermediate energy region, therefore it is also called resonance region and intermediate neutrons are termed resonance neutrons. Light nuclei have their first resonances at the energy from 10^4 to 10^5 eV, and the nuclear level overlapping is observed only in the range of nuclear energies ~ 10 -15 MeV, so that these nuclei display their typical resonance picture not in the

intermediate but in the fast region in the cross-section-to-energy relationship. However, as was mentioned in Sec. 4.5-5, of most interest are low-energy resonances. But such resonances are observed only with nuclei of intermediate and high mass numbers with $A \gtrsim 50$ since the probability for the level to be found in the region from, say, 0 to 100 eV is negligible at the intervals between

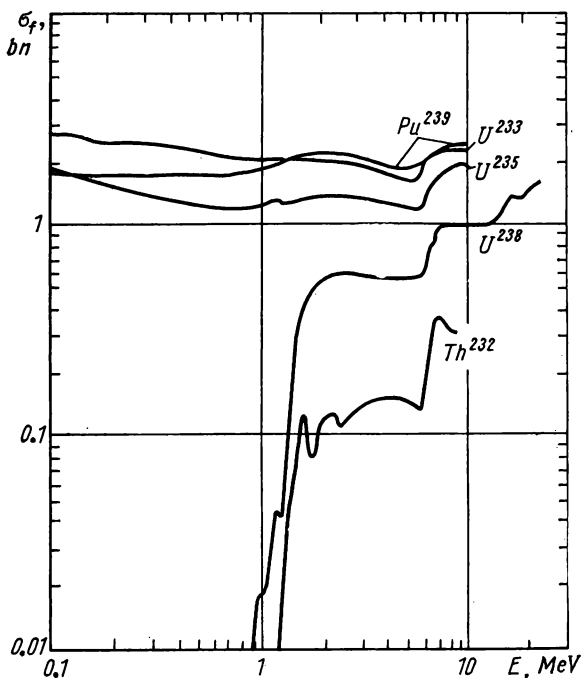


Fig. 4.16. Fission cross-section of heavy nuclei

the levels of the order of 10^4 eV. The measured resonances of intermediate and heavy nuclei, including the highest resonances, lie in the intermediate energy region.

Since the neutron width Γ_n decreases with the decrease of the neutron velocity, $\Gamma_n \ll \Gamma_\gamma$ at low energies and the low-lying resonances with $E_0 \leq 10^2$ eV are the capture resonances, i.e., in these resonances $\sigma_\gamma \gg \sigma_n$ (4.37). At higher energies, on the contrary, $\Gamma_n \leq \Gamma_\gamma$ and resonances observed when $E_n > 10^3$ eV are scattering resonances, i.e. $\sigma_n \gg \sigma_\gamma$. The highest absolute values of cross-sections in resonances are observed at the lowest energies just where the capture fraction is large compared to scattering. Thus, intermediate neutrons must be most effectively absorbed by the first resonances, i.e., in the lowest part of the intermediate region.

Nevertheless, one should not forget that the radiative capture resonances are always narrow since the γ -quantum emission by nuclei implies a long life-time of the compound nucleus. The widths of the capture resonances, or of the resonances with $\Gamma \approx \Gamma_\gamma$ are measured by tenths or even hundredths of electron-volts. Neutrons become intermediate as a result of slowing down in elastic scattering, and in certain acts of collision with nuclei, they lose large portions of their energy ($\gg \Gamma$), so that their energies seldom coincide with the resonance energies. Between the resonances, the absorption cross-section is very low and, therefore, scattering, as a whole, prevails over capture in the intermediate region as well, although the resonance capture may exert a noticeable influence on the balance of neutrons being moderated. Intermediate neutrons are more effectively absorbed by substances with large reaction cross-sections (n, α) or (n, p) , (Sec. 4.5-5). In nuclear reactors, of such absorbers boron is used the total cross-section of which is shown in Fig. 4.17. Although the (n, α) -reaction cross-section σ_α diminishes in proportion to $1/v$, it is much slower than in the case of absorbers with their resonances near the thermal region.

The fission width Γ_f is subject to random fluctuations. A contrary picture is, therefore, possible, although fission usually prevails over radiative capture in the resonances of fissile substances. In this connection, the value $\alpha = \sigma_\gamma/\sigma_f$ in the intermediate energy region is also subject to changes. It is, on the average, higher than in the thermal region. The value α substantially decreases only when reaching the energy $\sim 10^5$ eV (see Fig. 7.1).

Light nuclei ($A < 50$) have no resonances in the whole intermediate region or in its larger part. Their total cross-sections are, therefore, the potential scattering cross-sections. The radiative capture cross-sections are, usually, very low but they always obey the law $1/v$ and increase when approaching the thermal region.

9. Thermal neutrons. Energy distribution of thermal neutrons in weakly absorbing media is the Maxwellian distribution. The average energy of thermal neutrons proves to be somewhat higher than the average energy of the thermal motion of the molecules of the medium. The latter means that neutrons do not virtually reach thermal equilibrium with the medium. The reason for this is the continuous absorption of neutrons, which is the more efficient, the lower their velocity, and their replenishment on account of moderating neutrons of higher energies. The neutron distribution is, nevertheless, very close to the Maxwellian distribution but corresponding to a higher temperature than the temperature of the medium. This "neutron temperature" T_n is evaluated by the following relation:

$$T_n = T \left(1 + 0.92A \frac{\Sigma_a}{\Sigma_s} \right) \quad (4.43)$$

where T is the temperature of the medium, Σ_s and Σ_a are the macroscopic cross-sections of thermal neutron scattering and absorption at the energy kT and A , the mass number of the atoms of the moderator. A fraction of thermal neutrons in unit volume in

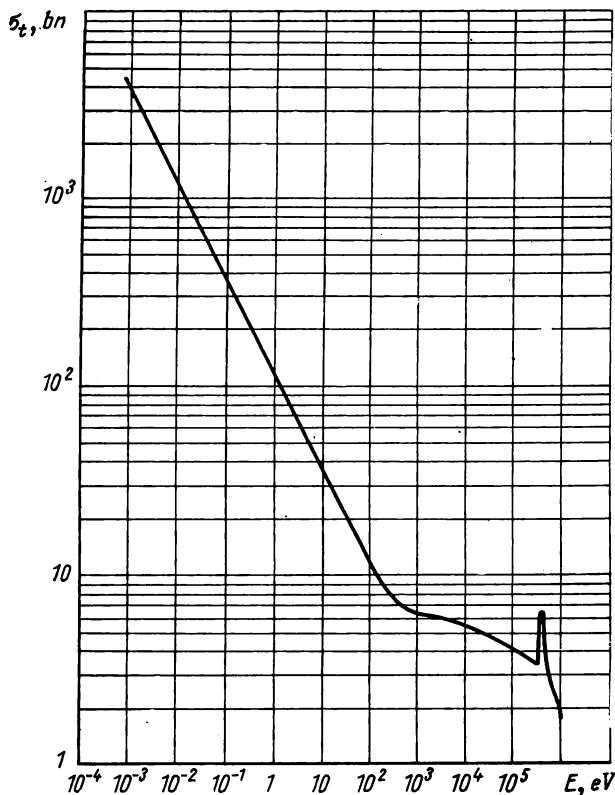


Fig. 4.17. Total cross-section of natural boron (19.8% B¹⁰)

unit energy interval, having the energy E , or the Maxwellian distribution of neutrons by their energies normalized to unity, is of the form:

$$\frac{1}{n_0} \frac{dn}{dE} = F(E) = \frac{2\pi}{(\pi k T_n)^{3/2}} \sqrt{E} \cdot e^{-\frac{E}{k T_n}} \quad (4.44)$$

where n_0 is the total number of thermal neutrons in unit volume of the substance, and $k = 1.38 \cdot 10^{-23} \text{ J/}^\circ\text{K} = 8.62 \cdot 10^{-5} \text{ eV/}^\circ\text{K}$ is the Boltzmann constant. Distribution (4.44) is shown in Fig. 4.18. The most probable energy, i.e., the energy in the distribution maximum,

is equal to $kT_n/2$ and the average energy of the Maxwellian spectrum is $(3/2)kT_n$. The standard thermal energy is, however, taken to be $E_{th} = kT_n$, which is convenient since kT_n is the Maxwell distribution parameter. Sometimes kT_n is said to be the energy of the most probable neutron in the Maxwell velocity distribution, $(1/n_0) (dn/dv)$. This is correct but it has nothing to do with the choice of the standard energy. If $\Sigma_a \ll \xi\Sigma_s$, $T_n = T$ and at the normal temperature of the medium $T = 293^\circ\text{K}$, then the standard energy $kT = 0.025$ eV. The velocity of such neutrons is 2200 m/s.

The cross-sections of the thermal region are usually referred to the standard energy at normal temperature. And although neutrons

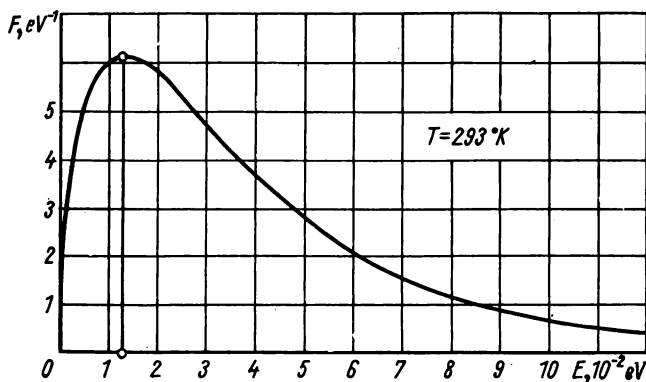


Fig. 4.18. Maxwellian distribution

have different energies and the reaction cross-sections vary according to the law $1/v$, it does not make calculations of the reaction velocities in the thermal region difficult. The number of neutron absorptions in unit volume in unit time is $\Phi\Sigma_a$ (1.56) where Φ is the neutron flux (1.50), and Σ_a , the macroscopic absorption cross-section (1.55). The number of neutrons in unit energy interval with the energy E in distribution (4.44) is $n_0F(E)$, and the flux density $\Phi(E) = n_0vF(E)$. If the absorption cross-section varies following the law $1/v$, the relation $\sigma_{st}/\sigma_a(E) = v/v_{st} = \sqrt{E}/\sqrt{E_{st}}$ exists between the standard cross-section σ_{st} with $E_{st} = 0.025$ eV and the cross-section with any other energy. The number of neutron absorptions Y over the whole thermal region is, therefore, equal to

$$\begin{aligned}
 Y &= \int_0^\infty \Phi(E) \Sigma_a(E) dE = \int_0^\infty n_0 v F(E) \cdot N \frac{\sigma_{st} v_{st}}{v} dE = \\
 &= n_0 v_{st} N \sigma_{st} = \Phi_{st} \Sigma_{st} \quad (4.45)
 \end{aligned}$$

where the integration is performed over all the theoretically possible energy values in the Maxwellian spectrum and $\int_0^{\infty} F(E) dE = 1$. Thus, in the case of the law $1/v$, the number of absorptions over this whole region is equal to the number of absorptions in the flux with the same total number of neutrons but with each neutron having its standard velocity. This result is independent of the choice of the standard velocity.

If the first resonance is so close to the thermal region that its left tail overlaps this region, then the absorption cross-section in the thermal region changes slower than under the law $1/v$. The cross-section with $E = 0.025$ eV is, nevertheless, assumed standard in this case. The total number of absorptions Y_x is not, however, equal to $\Phi_{st}\Sigma_{st}$, which is allowed for by introducing the factor f :

$$Y_x = \int_0^{\infty} n_0 v F(E) N \sigma_a(E) dE = \Phi_{st} \Sigma_{st} \cdot f \quad (4.46)$$

where, by virtue of (4.46) and (4.45), f is:

$$\begin{aligned} f &= \frac{\int_0^{\infty} n_0 v F(E) N \sigma_a(E) dE}{\int_0^{\infty} n_0 v F(E) N \frac{\sigma_{st} v_{st}}{v} dE} = \frac{\int_0^{\infty} \sqrt{E} F(E) \sigma_a(E) dE}{\sigma_{st} \sqrt{E_{st}}} = \\ &= \int_0^{\infty} \frac{\sigma_a(E)}{\frac{\sigma_{st} \sqrt{E_{st}}}{\sqrt{E}}} F(E) dE \quad (4.47) \end{aligned}$$

The factor f is the ratio averaged over the Maxwellian spectrum, of the actual absorption cross-section to the cross-section changing according to the law $1/v$ and, with $E = E_{st}$, equal to the actual cross-section $\sigma_a(E_{st}) = \sigma_{st}$, i.e., f allows for the deviation of the cross-section dependence from the law $1/v$ and in the case of $\sigma \sim (1/v)$, $f = 1$. The factor f , in turn, depends upon the neutron temperature T_n and upon the real upper limit of the neutron energy distribution [24].

The neutron flux $\bar{\Phi}$ averaged over the Maxwellian spectrum is expressed through Φ_{st} by the relation $\bar{\Phi} = (2/\sqrt{\pi}) \cdot \Phi_{st}$, where the factor $2/\sqrt{\pi}$ is determined by the choice of the standard energy. Since in the equation of thermal neutron diffusion, the flux Φ is not energy dependent and, hence, is a Maxwellian spect-

rum-averaged flux $\bar{\Phi}$, the term describing absorption in (5.43) should be corrected for the average flux. As a matter of fact, absorption is given by expressions (4.45) and (4.46). It means that $Y = \Phi_{st} \Sigma_{st} = \bar{\Phi} (\sqrt{\pi}/2) \Sigma_{ast} = \bar{\Phi} \cdot \Sigma_a$ where $\Sigma_a = \Sigma_{ast} (\sqrt{\pi}/2)$. The tabular cross-section for the standard energy must, therefore, be multiplied by the factor $\sqrt{\pi}/2$ to give the correct number of absorbed neutrons in the product with $\bar{\Phi}$.

In the thermal region, the contribution of partial cross-sections to the total cross-section of the neutron interaction with the substance is fully determined by the proximity of the first resonance. The width of the resonance, of course, plays its part too but the first resonances are usually the radiative capture resonances and their widths are very small. Therefore, if the first resonance is observed at an energy not higher than 1 eV, its left tail overlaps the thermal region and the capture cross-section proves to be much larger than the scattering cross-section, so that $\sigma_t \approx \sigma_v$. Owing to the increase of the absorption probability while the neutron velocity is decreasing, the capture cross-section and, hence, σ_t increase in the limits of the entire thermal region. In this case a deviation from the law $1/v$ is observed since while approaching the resonance, the cross-section does not only stop decreasing but increases in the resonance limits. Many cases occur among nuclides with the mass numbers $A > 100$ of such cross-section behaviour in the thermal region. The corresponding substances are strong absorbers of thermal neutrons. Fig. 4.19 shows as an example the total cross-section of cadmium an isotope of which, Cd^{113} , with 12% content in the natural mixture of isotopes, possesses the feature in question. Because of the resonance of Cd^{113} at the energy 0.18 eV, the cross-section of the radiative capture of thermal neutrons by cadmium is so large that the 1 mm thick cadmium layer is quite opaque to thermal neutrons. All fissile nuclides are among such nuclides. True the absorption of neutrons by fissile nuclides is, in the main, accompanied by the fission process. The contribution of the radiative capture is relatively low and is determined by the value $\alpha = \sigma_v/\sigma_f$. The fission cross-sections of U^{235} and Pu^{239} over a wide energy range are shown in Fig. 4.20 and the standard thermal cross-sections are given in Table 4.4.

If the first resonance is at the energy higher than 5 eV, then, although the radiative capture cross-section increases within the entire thermal region according to the law $1/v$, it usually does not exceed the potential scattering cross-section and such substances, therefore, have $\sigma_t \approx \sigma_n$. This type includes the even-even heavy nuclides U^{238} and Th^{232} having the first resonances at 6.8 and 22 eV respectively (see Table 4.4 and Fig. 4.21).

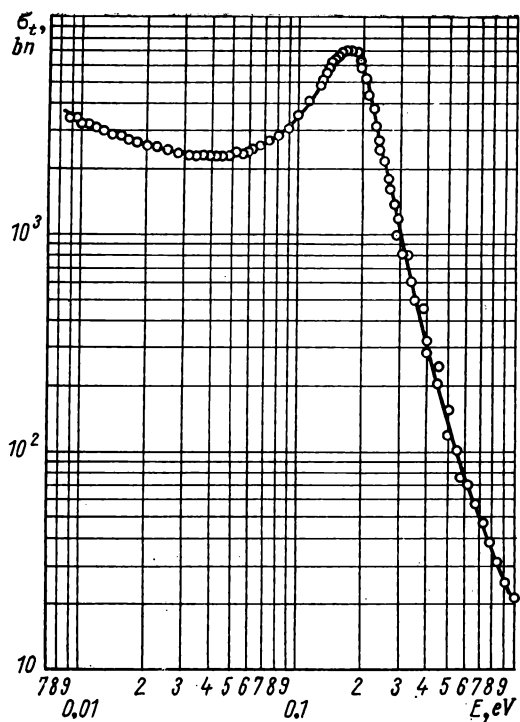


Fig. 4.19. Total cross-section of cadmium

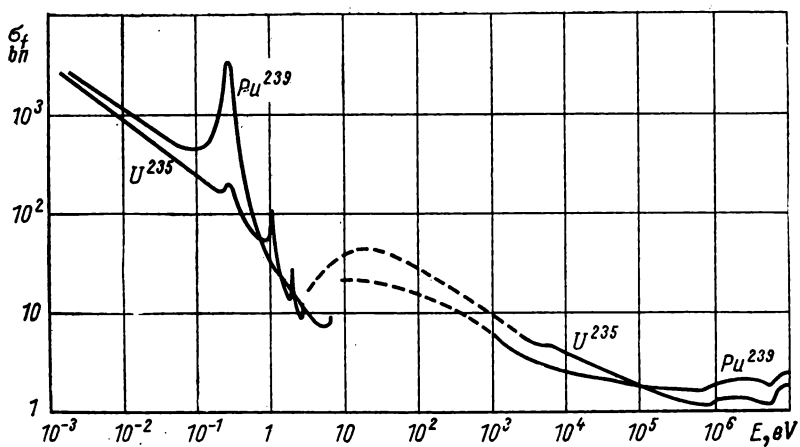


Fig. 4.20. Fission cross-sections of U^{235} and Pu^{239} (The dotted line is the resonance region)

Table 4.4

**Thermal Cross-Sections of Fissile
and Fissionable Nuclides, Barn**

| Nuclide | $\sigma_a = \sigma_f + \sigma_\gamma$ | σ_f | $\sigma_n = \sigma_s$ | $\alpha = \sigma_\gamma / \sigma_t$ |
|----------------------|---------------------------------------|------------|-----------------------|-------------------------------------|
| U ²³³ | 577.6 | 530.6 | 10.7 | 0.0885 |
| U ²³⁵ | 678.5 | 580.2 | 17.6 | 0.1694 |
| Pu ²³⁹ | 1012.9 | 741.6 | 8.5 | 0.3659 |
| Th ²³² | 7.4 | 0 | 13 | — |
| U ²³⁸ | 2.73 | 0 | 13.8 | — |
| Natural ura- nium | 7.68 | 4.12 | 13.8 | — |

The above characteristic features of U²³⁵ and U²³⁸ predetermine the possibility of obtaining a self-sustained chain thermal neutron reaction with the use of natural uranium. Excited nuclei produced in the capture of neutrons of U²³⁵ or U²³⁸ have substantially different excitation energies (Sec. 3.6-2), due to which the intervals between the neutron resonances prove to be much shorter in the case of the U²³⁵ neutron bombardment than in the case of the U²³⁸ neutron bombardment, since the higher the excitation energy of the nucleus, the shorter the intervals between the levels. For this reason, the first U²³⁵ resonance turns out to be much lower and, hence, the absorption cross-section in the thermal region is much higher than with U²³⁸. As a result, on condition of preliminary moderation of fission neutrons down to the thermal energy, U²³⁵ in spite of its low concentration in natural uranium proves to be competing with U²³⁸ in absorbing neutrons and can sustain a chain reaction.

The absorption cross-sections of light nuclides whose nuclei initiate exoenergetic reactions accompanied by charged particle emission are very high and follow the law $1/v$ in the thermal region. Table 4.5 lists the neutron absorption cross-sections at the energy $E = 0.025$ eV for some elements with a natural mixture of isotopes.

10. Coherent scattering. In cases where $\sigma_t \approx \sigma_n$ and the energy is so low that the de Broglie neutron wavelength (1.13) is comparable to the distance between the crystal planes, diffraction neutron scattering is observed in crystals. If a neutron flux of any set of energies is incident on a single crystal at the angle ψ to the planes separated by the distance d , neutrons of the wavelength λ defined by the Bragg-Wolf condition are reflected at the same angle

$$2d \sin \psi = n\lambda \quad (4.48)$$

Table 4.5

Thermal Absorption Cross-Sections

| Element | Reaction | σ_a , barn | Element | Reaction | σ_a , barn |
|-----------------|-------------------------------|-------------------|---------|----------------------|-------------------|
| H | (n, γ) | 0.332 | K | (n, γ) | 2.07 |
| H ² | (n, γ) | 0.0005 | Cr | (n, γ) | 3.1 |
| He ⁴ | — | 0 | Fe | (n, γ) | 2.6 |
| He | He ³ (n, p) | < 0.01 | Ni | (n, γ) | 4.6 |
| Li | Li ⁶ (n, α) | 71 | Cu | (n, γ) | 3.85 |
| Be | (n, γ) | 0.01 | Zr | (n, γ) | 0.185 |
| B | B ¹⁰ (n, α) | 760 | Nb | (n, γ) | 1.16 |
| C | (n, γ) | 0.0038 | Mo | (n, γ) | 2.7 |
| N | N ¹⁴ (n, p) | 1.8 | Ag | (n, γ) | 64.8 |
| O | (n, γ) | < 0.0002 | Cd | (n, γ) | 2540 |
| Na | (n, γ) | 0.52 | Hf | (n, γ) | 102 |
| Mg | (n, γ) | 0.069 | Pb | (n, γ) | 0.17 |
| Al | (n, γ) | 0.24 | U | $(n, \gamma) + (nf)$ | 7.68 |

where $n = 1, 2, 3, \dots$ is the order of reflection. This phenomenon is made use of in crystal monochromators to separate monoenergetic neutron beams from fluxes with a continuous spectrum over the energy range from 0.1 to 10 eV. Diffraction is possible in scattering of coherent waves, therefore the cross-section describing scattering under condition (4.48) is termed coherent. Even under condition (4.48), however, not all neutrons are scattered coherently since there are some factors hindering coherent scattering, such as arbitrary orientation of nuclear spins, thermal motion of ions, crystal lattice defects due to both the absence of ions in the nodes and multi-isotopic composition of chemical elements, because of which the filled nodes appear to have nuclei of some different type, and others. The scattering cross-section in the thermal region is, therefore, the sum of the coherent σ_{cr} and noncoherent σ_{ncr} scattering, $\sigma_n = \sigma_{cr} + \sigma_{ncr}$. The contribution of each partial constituent is determined by the nature of the substance.

The probability of scattering neutrons from the plane with the given d is the lower, the smaller the angle ψ , i.e., the lower λ . In terms of cross-sections, it means that with the increase of the energy the coherent scattering cross-section decreases. The high order of reflection is always associated with the shorter wavelength and is, hence, less probable than the first order with $n = 1$. At the same time, the total number of neutrons participating in the coherent process at first does not almost change with the energy increase and the decrease of σ_{cr} means that part of neutrons freely

pass through the crystal taking no part in the scattering. Scattering at the angle α under condition (4.48) and passing through the crystal without scattering are one and the same process of interaction of coherent neutron waves with the crystal. With the decrease of energy, on the contrary, σ increases. But with $(\lambda/2) > d$, under condition (4.48) coherent scattering becomes impossible and σ_c immediately vanishes. If the contribution of σ_c to the total cross-section σ_t was large, the total interaction cross-section of neutrons with the substance sharply reduces and the substance becomes transparent to neutrons.

Ordinary crystal bodies are polycrystals and, therefore, the selectivity of the scattering of neutrons of a given energy at a certain

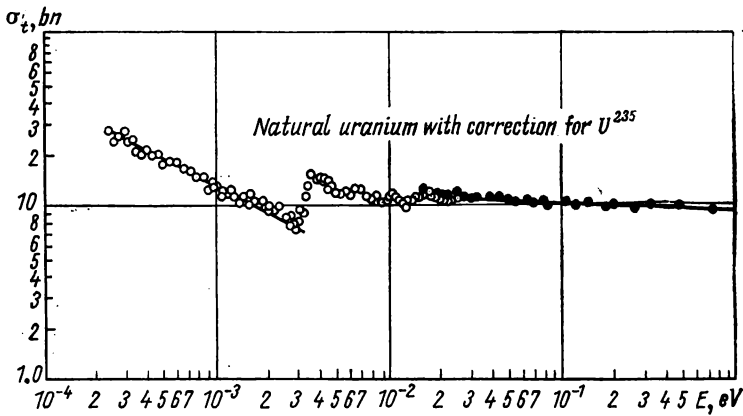


Fig. 4.21. Total cross-section of U^{238} at low energy

angle cannot be observed in such substances. The energy dependence of the coherent cross-section, however, has its effect upon the dependence of the total scattering cross-section since neutrons of a given energy always find a crystal with which they interact following condition (4.48). Apart from the main crystal planes with the longest interplane distance other planes having smaller values of d exist. The cross-section of neutron scattering by a polycrystalline body, therefore, displays a number of maxima abruptly falling off when the energy decreases and gradually lowering in the direction of higher energies (Figs. 4.21 and 4.22). In the range from 10^{-2} to 10^{-3} eV, $\lambda/2$ becomes greater than the main interplane distance and the cross-section sharply decreases. This small cross-section is $\sigma_t = \sigma_v + \sigma_{ncr}$. If the noncoherent scattering cross-section is less than the capture cross-section, $\sigma_t \approx \sigma_v$ and increases following the law $1/v$ with further energy decrease, which is observed, for example, with U^{238} (Fig. 4.21). If the absorption cross-

section is very small as, for example, in the case of graphite (see Table 4.5), then $\sigma_t \approx \sigma_{ncr}$. In this case, one also observes the increase of the cross-section when the energy attenuates following the law $1/v$. This is, however, due to some new interaction mechanism between neutrons and the crystal lattice.

In the energy region $\sim 10^{-3}$ eV, the neutron energy is much lower than the average energy of thermal motion. Hence, the possibility of energy transfer from the crystal lattice to the neutron of exceedingly low energy. The energy transfer is accompanied by the momentum transfer, i.e., by neutron scattering. The probability

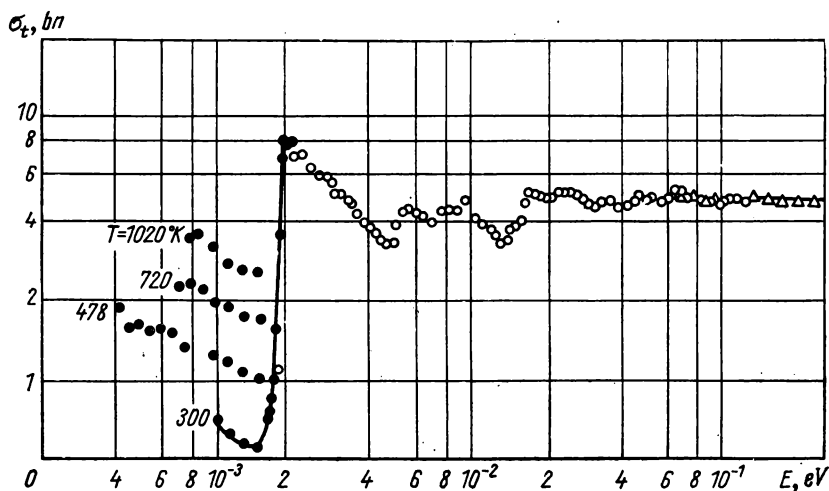


Fig. 4.22. Total cross-section of carbon in thermal region

of the process is proportional to the residence time of the neutron in the crystal or to $1/v$. Since this process is the energy transfer from the crystal lattice, it should be the more probable, the greater the difference between the neutron energy and the average atomic energy, i.e., the higher the temperature of the medium. Actually when $\sigma_{ncr} \ll \sigma_v$, the cross-section σ_t , remaining inversely proportional to velocity, increases with the increase of the temperature of the medium (see Fig. 4.22). It is in this circumstance that the given case differs from the case of $\sigma_v \gg \sigma_{ncr}$ when σ_t is also proportional to $1/v$ but does not exhibit any temperature dependence.

With the increase of the energy up to the values of the order of 0.1 eV or higher, the wavelength of the neutron reduces and the number of neutrons participating in coherent interaction decreases. The contribution of σ_{cr} to the total scattering cross-section σ_n decreases on account of the increase of the noncoherent compo-

nent σ_{ncr} . Beyond the thermal region, the scattering cross-section $\sigma_n = \sigma_{ncr}$. Near the resonances, however, the possibility arises of interference of potential and resonance scattering, causing the distortion of the shape of the resonance peaks, i.e., coherent scattering. Neutrons possess wave properties, and coherent scattering is always possible on condition that the object of interaction is of a spatial structure the linear parameter of which is comparable with the de Broglie wavelength of the neutron λ or λ (Sec. 1.4-4, -5).

11. **Effects of chemical bond.** Scattering of intermediate and thermal neutrons is spherically symmetrical in the centre-of-mass system of coordinates, i.e. the differential cross-section $d\sigma/d\theta$ of neutron scattering at any angle θ is constant. In the laboratory

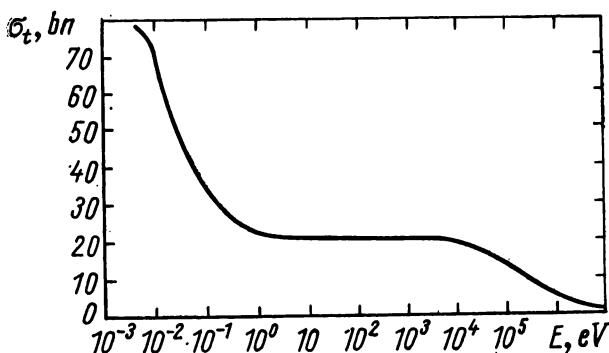


Fig. 4.23. Total hydrogen cross-section

system of coordinates, however, because of the transfer of momentum to the nucleus, scattering is not spherically symmetrical and neutrons are mostly scattered in the forward direction (Sec. 5.1-3), that is, $d\sigma/d\psi$ depends upon the angle and is maximum in the ultimate case of scattering at the angle $\psi = 0^\circ$, which corresponds to a glancing blow, practically without any transfer of momentum to the nucleus. The maximum value of $d\sigma/d\psi$ is $[(A + 1)/A]^2$ times greater than $d\sigma/d\theta$. A being the mass number of the target nucleus. With scattering at large angles, $d\sigma/d\psi$ is much lower than $d\sigma/d\theta$ and the cross-section summed up over all the angles in both reference systems is the same.

The bond energy of atoms in molecules and crystals is measured in units of fractions of electron-volts. If the energy of the neutron colliding with the nucleus is lower than the bond energy of the atom in the molecule, the atom cannot be knocked out of the molecule and the neutron momentum in scattering is transferred to the whole molecule. Since the mass of the molecule is usually much greater than the mass of the neutron, scattering actually takes

place without any transfer of momentum and becomes spherically symmetrical. It follows that $(d\sigma/d\psi)_{bd}$ on the bound nucleus does not now depend upon ψ , and when $\psi = 0^\circ$, it must coincide with $d\sigma/d\psi$ on the unbound nucleus since when $\psi = 0^\circ$, scattering never results in momentum transfer. When $\psi = 0^\circ$, $d\sigma/d\psi$ is $[(A + 1)/A]^2$ times greater than $d\sigma/d\theta$. Therefore, $(d\sigma/d\psi)_{bd}$ summed up over all the angles gives the total scattering cross-section σ_{bd} which is $[(A + 1)/A]^2$ times greater than the cross-section of scattering on an unbound nucleus. Thus, the chemical bound effect results in an increase of the scattering cross-section. The asymmetry of neutron scattering in the laboratory system of coordinates, is especially great for light nuclei. Therefore, the increase of the scattering cross-section with the change-over to collisions with bound nuclei is also of particular significance with light nuclei. When $A = 1$ (hydrogen), the scattering cross-section is four-fold. Scattering on bound nuclei becomes spherically symmetrical only if the neutron wavelength exceeds the molecular sizes, i.e., at energies of the order of 10^{-2} eV. The chemical bond effect, however, becomes already pronounced at energies about 1 eV. At such energies, neutrons are moderated on account of molecular excitation and the character of the scattering becomes complex. Observation of σ_{bd} in the thermal region is hindered by the coherent scattering phenomenon (Sec. 4.5-10). Therefore, the cross-section of scattering by bound nuclei is usually calculated by multiplying the cross-section by unbound nuclei, measured at an energy of several electron-volts, by the factor $[(A + 1)/A]^2$. In bound hydrogen, the coherent cross-section σ_{cr} is very small and the increase of the scattering cross-section from 20 to 80 barns is well observed in the thermal region (Fig. 4.23).

CHAPTER FIVE

MODERATION AND DIFFUSION OF NEUTRONS

5.1. Moderation of Neutrons

1. Mechanism of moderation. Scattering is observed mainly in collisions of neutrons with nuclei. In the scattering event, the nucleus acquires recoil momentum and the energy of the scattered neutron decreases correspondingly. The decrease of the kinetic energy means the reduction of the velocity of motion. The process is, therefore, termed moderation of neutrons. Moderation ceases only upon reaching the energy region of the thermal motion of the particles of the medium when neutrons lose and acquire their energy with equal probability. Energy losses in scattering are the greater, the less the difference between the masses of the neutron and the nucleus, and therefore neutrons are moderated most efficiently when colliding with nuclei of the lightest atoms. Moderation due to the transfer of only kinetic energy to nuclei is elastic scattering moderation. In inelastic scattering (Sec. 4.5-1), neutrons are moderated still stronger, however, not to infinitely lower energies. Inelastic scattering thresholds lie with the energy $E \approx 0.1$ MeV, and the average energy of inelastic neutron scattering cannot be much lower than the threshold energy. Inelastic scattering is, therefore, significant only in moderation of fast neutrons. Since inelastic scattering thresholds are the lower, the heavier the nucleus, nuclei with high and intermediate mass numbers are mainly responsible for inelastic moderation.

Moderation by light nuclei is used to produce thermal neutrons. The efficiency of moderation depends upon the mean loss of energy in one act of collision of the neutron with the nucleus. After several such collisions, neutrons of moderate initial energies, such as fission neutrons, acquire such energies at which scattering by light nuclei is spherically symmetrical (Sec. 4.5-7). Thus, the overwhelming number of collisions in moderation is characterized by equally probable scattering through any angle in the centre-of-mass coordinate system and such an assumption takes account of a single scattering event.

2. Scattering event. Figure 5.1a gives \mathbf{v}_1 and \mathbf{v}_2 as the velocity vectors of the neutron before and after scattering at the angle ψ in the laboratory coordinate system. The target nucleus is at rest

before collision, and after collision it has the velocity v_{A2} . In the centre-of-mass coordinate system, the total momentum of colliding particles is zero, i.e., the momenta of the neutron and the nucleus are equal and oppositely directed

$$v_n = Av_A \quad (5.1)$$

where v_n and v_A are the velocity moduli of the neutron and the nucleus, and A , the mass number of the nucleus. In relation (5.1), the absolute mass values are of no significance, the atomic masses are, without any discernible error, replaced by the mass numbers, and the mass number of the neutron is unity. The centre-of-mass coordinate system is convenient in that in collision the absolute values of momenta-vectors do not vary, it is only their orientation that is changed. This follows from the fact that the total momentum is zero. In scattering, the total momentum may be zero either when the momentum moduli of both particles are retained or when they simultaneously increase or decrease proportionately. The latter would, however, contradict the law of energy conservation.

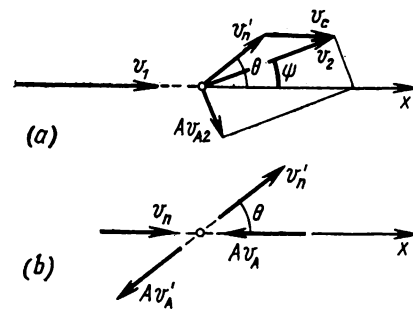


Fig. 5.1. Neutron and nuclear pulse diagram in the laboratory (a) and in the centre-of-mass coordinate system (b)

scattering, the total momentum may be zero either when the momentum moduli of both particles are retained or when they simultaneously increase or decrease proportionately. The latter would, however, contradict the law of energy conservation.

Hence, in the centre-of-mass coordinate system (Fig. 5.1b)

$$v'_n = v_n, \quad v'_A = v_A \quad (5.2)$$

Since the target nucleus is at rest, the velocity of the centre-of-mass coordinate system v_c in the laboratory coordinate system is just the inverse velocity of the nucleus in the centre-of-mass system, i.e., $v_c = -v_A$. The moduli of these velocities are equal

$$v_c = v_A \quad (5.3)$$

and v_c coincides with v_1 in direction. Hence, on the basis of the theorem of velocity summation at the translational motion before collision

$$v_1 = v_n + v_c \quad (5.4)$$

and after collision

$$v_2 = v'_n + v_c \quad (5.5)$$

It follows from (5.1) through (5.4) that

$$v_c = \frac{v_1}{A+1} \quad (5.6)$$

and

$$v'_n = \frac{A}{A+1} v_1 \quad (5.7)$$

and from the velocity triangle (5.5), taking into account (5.6) and (5.7), it follows

$$v_2^2 = v_1^2 \frac{A^2 + 2A \cos \theta + 1}{(A+1)^2} \quad (5.8)$$

where θ is the scattering angle in the centre-of-mass system corresponding to the angle ψ . From (5.8) immediately follows the relationship between the neutron energy after scattering, E_2 , and the energy before scattering, E_1

$$\frac{E_2}{E_1} = \frac{v_2^2}{v_1^2} = \frac{1}{2} [(1 + \varepsilon) + (1 - \varepsilon) \cos \theta] \quad (5.9)$$

where

$$\varepsilon = \left(\frac{A-1}{A+1} \right)^2 \quad (5.10)$$

The maximum neutron energy after the collision with the nucleus $E_{2 \max} = E_1$, with the scattering angle $\theta = 0$, i.e., virtually in the absence of scattering. The minimum energy $E_{2 \min} = \varepsilon E_1$, with $\theta = \pi$, i.e., with knock-on collision and backward scattering. The value ε is determined only by the mass number A . With $A = 1$, $\varepsilon = 0$ and the minimum neutron energy in the knock-on collision with the nucleus of the light hydrogen atom is zero. All the kinetic energy of the moderated neutron is carried off by the proton. With high values of A , $\varepsilon \approx 1$, even in knock-on collision, the energy E_2 practically coincides with E_1 and neutrons very slowly lose their energy. Thus, the drop of energy in one scattering event is the greater, the greater the scattering angle, and the lower the mass number, i.e., the lighter the atoms of the scatterer. The quantitative characteristic of the moderation efficiency should be the value of the mean energy loss in a scattering event, which can be obtained after averaging E_2 over all possible scattering angles.

3. Scattering law. Because of the momentum transfer to the target nucleus, scattering is not spherically symmetrical in the laboratory coordinate system. This directly follows from the relationship between the scattering angles in both reference systems. In the velocity triangle (5.5) is projected on to the x -axis (see Fig. 5.1a)

$$v'_n \cos \theta + v_c = v_2 \cos \psi \quad (5.11)$$

and (5.6) through (5.8) are taken into account, then one can obtain the relationship between the angles

$$\cos \psi = \frac{A \cos \theta + 1}{\sqrt{A^2 + 2A \cos \theta + 1}} \quad (5.12)$$

The scattering angles are lower in the laboratory coordinate system than in the centre-of-mass system although the ultimate values 0 and 180° coincide, except for the case of $A = 1$. It means that if in the centre-of-mass system the probability density of neutron scattering in any direction, determined by the spherical coordinates (φ, θ) , into a single solid angle is constant, i.e., $d\omega/d\Omega_\theta = 1/4\pi$, with $1/4\pi$ being the number normalizing the probability density per unity, then, in the laboratory coordinate system, $d\omega/d\Omega_\psi$ should depend upon the angle ψ . (The scattering probability does not depend upon the spherical coordinate φ .) As a result, the average cosines of the scattering angle appear to be different in both systems. In the centre-of-mass system

$$\overline{\cos \theta} = \int_{\Omega_0} \cos \theta \frac{d\omega}{d\Omega_\theta} d\Omega_\theta = \frac{1}{2} \int_0^\pi \cos \theta \sin \theta d\theta = 0 \quad (5.13)$$

i.e., scattering, on the average, takes place through the angle $\theta = 90^\circ$ or to a plane perpendicular to the direction of neutron motion before scattering. Since the function differential does not depend upon the independent variables, i.e., $(d\omega/d\Omega_\theta)d\Omega_\theta = (d\omega/d\Omega_\psi) \cdot d\Omega_\psi$, the average cosine in the laboratory coordinate system, $\overline{\cos \psi}$, may be calculated by expressing in through $\cos \theta$ (5.12):

$$\begin{aligned} \overline{\cos \psi} &= \int_{\Omega_0} \cos \psi \frac{d\omega}{d\Omega_\psi} d\Omega_\psi = \int_{\Omega_0} \cos \psi \frac{d\omega}{d\Omega_\theta} d\Omega_\theta = \\ &= \frac{1}{2} \int_0^\pi \frac{A \cos \theta + 1}{\sqrt{A^2 + 2A \cos \theta + 1}} \sin \theta d\theta = \frac{2}{3A} \end{aligned} \quad (5.14)$$

where Ω_0 is a full solid angle just as in (5.13), and $d\Omega_\theta = \sin \theta d\theta d\varphi$ is an element of the solid angle in spherical coordinates equal to $2\pi \sin \theta d\theta$ after integrating over φ . The mean cosine of the scattering angle can be a measure of scattering asymmetry. At arbitrarily large A , $\overline{\cos \psi} \rightarrow \overline{\cos \theta}$ and scattering becomes spherically symmetric in the laboratory coordinate system. The kind of asymmetry involved is determined by the momentum transfer to the target nucleus, and if the nuclear mass is large in comparison with the neutron mass, practically no momentum transfer takes place. The greatest asymmetry is observed with $A = 1$, i.e., when neutrons are scattered by the nuclei of the light hydrogen atom, by protons. In this case, expression (5.12) gives $\overline{\cos \psi} = \overline{\cos(\theta/2)}$, and the angle $\theta = \pi$ in the centre-of-mass system corresponds to scattering in the laboratory system at the angle $\pi/2$

with the ultimate zero value of the momentum since, with $\theta = \pi$, $E_2 = \varepsilon E_1 = 0$. Hence, when colliding with protons, neutrons cannot be scattered backward, always $\psi < \pi/2$, and $\cos \psi = 2/3$, which corresponds to the angle $\psi = 48^\circ$.

By virtue of (5.2), the kinetic energy of colliding particles does not change in the centre-of-mass system. If, in addition, scattering is spherically symmetric in this reference system, then, in the laboratory coordinate system, any value of energy after scattering, E_2 , appears to be equally probable over the range of $E_{2\max}$ to $E_{2\min}$ (Fig. 5.2). Although, when the scattering angle θ approaches $\pi/2$,

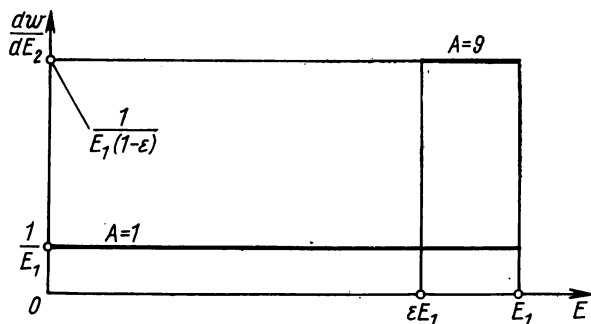


Fig. 5.2. Probability density distribution of neutron energy upon scattering

the increment of E_2 by the absolute value increases in proportion to $\sin \theta$ $dE_2 = -[E_1(1 - \varepsilon)/2] \sin \theta d\theta$ [see (5.9)]. However, in the three-dimensional space, the solid angle $d\Omega_\theta = \sin \theta \cdot d\theta d\varphi$ also increases as θ approaches $\pi/2$. Therefore, the increment dE_2 referred to the unity of the solid angle $dE_2/d\Omega_\theta$, is constant and does not depend either on φ or on θ . Since scattering in any direction (φ, θ) is equally probable [$d\omega/d\Omega_\theta = 1/(4\pi)$], the probability density $d\omega/dE_2$ is also constant. As follows from the relationship between the differentials $d\omega d\Omega_\theta$ and dE_2 , the absolute value of the probability density is

$$\frac{d\omega}{dE_2} = \frac{1}{E_1(1 - \varepsilon)} \quad (5.15)$$

and the integral of $d\omega/dE_2$ over all possible values of E is equal to unity, according to the definition of probability.

4. Logarithmic energy decrement. The constancy of $dE_2/d\Omega_\theta$ means that E_2 decreases linearly from $E_{2\max}$ to $E_{2\min}$ when the co-ordinate θ of the solid angle unity is changed. The quantity being changed linearly, its mean value is $E_2 = [(E_2)_{\max} + (E_2)_{\min}]/2 = E_1(1 + \varepsilon)/2$ and the mean energy loss

$$\overline{\Delta E} = \overline{E_1 - E_2} = E_1 - \overline{E_2} = \frac{1 - \varepsilon}{2} E_1 \quad (5.16)$$

is always the same fraction of the initial energy E_1 . It means that while E_1 is great, the energy loss in elastic scattering occurs in large portions. But the lower the energy, the smaller, correspondingly, the amount of energy being lost. In describing neutron moderation, the average loss of the energy logarithm is usually used rather than the average loss of energy. The increment of a logarithm is inversely proportional to the energy $d \ln E_1 = dE_1/E_1$. And since the energy increment is proportional to E_1 in moderation (5.16), the average logarithmic loss of energy must not depend upon E_1 either. This value is readily calculated as long as $d\omega/dE_2$ is known:

$$\begin{aligned}\xi &= \overline{\Delta \ln E} = \overline{\ln E_1 - \ln E_2} = \overline{\ln \frac{E_1}{E_2}} = \\ &= \int_{\epsilon E_1}^{E_1} \ln \frac{E_1}{E_2} \frac{d\omega}{dE_2} dE_2 = 1 + \frac{\epsilon}{1-\epsilon} \ln \epsilon \quad (5.17)\end{aligned}$$

The quantity ξ is termed *logarithmic energy decrement* and is only determined by the mass number of the nuclei of the scatterer (5.10).

Making use of ξ , one can calculate the average number of collisions of the neutron with nuclei, which results in moderation from

the initial energy E_0 to any lower energy, for instance, down to the energy of thermal motion, E_{th} . The total change of the energy logarithm is $\ln E_0 - \ln E_{th} = \ln (E_0/E_{th})$. And as ξ is the average change of the energy logarithm in one event of scattering, then the average number of collisions N in moderation is

$$N = \frac{\ln \frac{E_0}{E_{th}}}{\xi} \quad (5.18)$$

If fission neutrons are meant, their average initial energy is $E_0 = 2 \cdot 10^6$ eV (Sec. 3.6-7) and the thermal neutron energy is $E_{th} = 0.025$ eV, under normal conditions (Sec. 4.5-9) so that the total change of the energy logarithm is 18.2. Table 5.1 contains the values of ξ , calculated from (5.17), and the average numbers of collisions with nuclei of some nuclides in moderation of fission neutrons down to thermal energy. It turns out that for the energy to change by a factor of 100 million, neutrons must have, on the

Table 5.1

Constants Characterizing Neutron Moderation

| Nuclide | A | ξ | $N = \frac{18.2}{\xi}$ |
|------------------|-----|--------|------------------------|
| H ¹ | 1 | 1.000 | 18 |
| H ² | 2 | 0.725 | 25 |
| He ⁴ | 4 | 0.425 | 43 |
| Be ⁹ | 9 | 0.209 | 86 |
| C ¹² | 12 | 0.158 | 114 |
| O ¹⁶ | 16 | 0.120 | 150 |
| U ²³⁸ | 238 | 0.0084 | 2170 |

average, only 18 collisions with the nuclei of the light hydrogen atoms but more than 2000 in scattering by the uranium nuclei.

5. Moderating power. In moderation of neutrons by a substance, it is not only the value of the average energy loss in one event of collision that counts but the number of such collisions in unit volume of the substance, equal to $\Phi\Sigma_s$ (1.56), where Σ_s is the macroscopic scattering cross-section, is also of significance. The product $\xi\Sigma_s$ is called the moderating power of the substance since it allows for the both factors mentioned above which are referred to the same neutron being moderated. The moderating power determines the efficiency of neutron moderation by each unit volume of the substance. The higher the moderating power, the smaller the volume of the substance required to moderate neutrons down to a given energy. Table 5.2 lists the moderating powers of some substances. Since the macroscopic cross-sections related to nuclei of different types in unit volume are summed up, the moderating powers of chemical compounds and mixtures of the substances $(\xi\Sigma_s)_m$ are also found as the sums of the moderating powers of the components:

$$(\xi\Sigma_s)_m = \sum_i \xi_i \Sigma_{s,i} \quad (5.19)$$

6. Moderation ratio. If neutron moderation is not an aim in itself and the thermal neutrons obtained must be of some use, for instance, be transferred to uranium in a thermal reactor, then one more requirement is imposed on the moderating substance: moderated neutrons should not be absorbed by the moderating substance. Otherwise stated, the macroscopic cross-section of the absorption of the thermal neutrons of the moderator $\Sigma_{a,th}$ should be as low as possible. The ratio $\xi\Sigma_s/\Sigma_{a,th}$ characterizes the ability of the substance to moderate neutrons and to retain them upon moderation, and it is termed moderation ratio. The moderation ratios are presented in Table 5.2. There are a number of light substances meeting all the requirements imposed, used as neutron-moderators in nuclear reactors. Other light substances, such as lithium and boron, are very strong neutron absorbers in reactions with the emission of charged particles (see Table 4.5). They cannot, therefore, be used as moderators.

7. Properties of moderators. The moderating properties of homogeneous substances are the best and they are quite prominent among the properties of other moderators. Ordinary hydrogen has the highest ξ and, which is most significant, an abnormally large scattering cross-section $\sigma_s = 20$ barns over a sufficiently wide energy range (see Fig. 4.23). Concentration of hydrogen atoms in chemical combinations with other elements is also high. To illustrate the effect of the concentration of atoms on the moderating power of the substance, Table 5.2 presents helium which does not

Table 5.2

**Moderating Powers and Moderation Ratios
of Substances**

| Substance | Chemical formula | ρ , 10^3 kg/m^3 | $\xi \Sigma_s$, m^{-1} | $\frac{\xi \Sigma_s}{\Sigma_a \text{ th}}$ |
|-----------------|---------------------------------|-----------------------------------|----------------------------------|--|
| Water | H ₂ O | 1 | 135 | 61 |
| Polyethylene | (CH ₂) _n | 0.92 | 161 | 61 |
| Heavy water | D ₂ O | 1.10 | 18.8 | 5700 |
| Beryllium | Be | 1.84 | 15.5 | 125 |
| Beryllium oxide | BeO | 2.96 | 12 | 170 |
| Graphite | C | 1.60 | 6.1 | 205 |
| Helium | He | 0.000178 | 0.009 | 45 |
| Uranium | U | 18.7 | 0.33 | 0.009 |

form chemical compounds and is only in its gaseous state under normal conditions. Although ξ of helium differs but slightly from ξ of hydrogen, the moderating power of helium is negligible in comparison with $\xi \Sigma_s$ of condensed media. The moderating power of other gases, hydrogen included, is also quite low. The heavier the substance, the lower, as a rule, its moderating power due to the lesser ξ . Substances heavier than hydrogen are not used as moderators. Table 5.2 presents the moderating power of uranium to compare it with light substances.

Moderation ratios are the main characteristics of moderating materials for nuclear reactors. Heavy water is the best moderator from the standpoint of the whole sum of requirements placed on moderators. The moderation ratio of heavy water is extraordinarily high as both oxygen and heavy hydrogen, deuterium, absorb neutrons but negligibly (see Table 4.5). The high value of this constant makes heavy water most attractive for use in nuclear reactors.

However, heavy water is disadvantageous in that it is very expensive since it is produced from the light water containing 0.015% heavy water by the isotope separation techniques. Carbon is the second in the list. It is the heaviest of all the moderators, is of low moderating power, and of a very low cross-section of thermal neutron absorption. Hydrogenous substances appear to be the worst in their moderation ratio due to the substantial absorption of neutrons by hydrogen (see Table 4.5). Finally, it turns out that with light water as moderator, a chain self-sustained reaction cannot be produced with the use of natural uranium while with other moderators it is quite possible. But the high moderating power of water, its low cost, availability and some other valuable properties are points in favour of the reactors operating with en-

riched uranium and a water moderator. In addition, water is an excellent shielding material in protection against neutron fluxes of a moderate initial energy, for instance, against neutrons from nuclear reactors. The problem of shielding against neutrons is one of moderating them. And the moderating power of water is high although it decreases rather rapidly in the energy range $E > 0.1$ MeV due to the decrease of σ_s (see Fig. 4.23).

5.2. Spectrum of Slowing-Down Neutrons

1. Neutron balance. The simplest picture of slowing down is observed in a medium consisting only of hydrogen atoms, $A = 1$, $\xi = 1$, and with no neutron absorption. In this case, upon each collision, the neutron is preserved and with equal probability acquires any energy lower than the initial one (see Fig. 5.2). If Q neutrons of the energy E_0 are produced per unit volume in unit

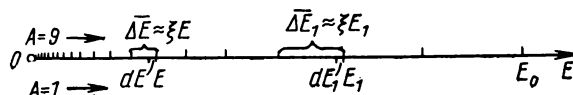


Fig. 5.3. Energy scale. The average energy drop in slowing-down for $A=9$ is shown

time, then their energy distribution in slowing down can be obtained by considering the balance at any intermediate energy. Although in the process of slowing down neutrons are scattered over the volume of the substance, the number of neutrons of a given energy per any unit volume will be the same in the case of sources distributed uniformly over any large volume. The flux (1.50) of slowing-down neutrons is usually referred to a single energy interval and proves to be the function of energy $\Phi(E)$. If the flux is defined, the number of events of scattering neutrons of the energy E per unit volume in unit time per unit energy interval is $\Phi(E) \cdot \Sigma_s(E)$ (1.56), where $\Sigma_s(E)$ is a macroscopic scattering cross-section (1.55) with the energy E .

The neutron balance at the energy E is the equality between the number of neutrons entering the interval dE and leaving it due to scattering (Fig. 5.3). Since $\xi = 1$, neutrons of any $E_1 > E$ can acquire the energy E in scattering, and the fraction of neutrons that have scattered into dE from dE_1 is merely dE/E_1 , since all the neutrons which have come into collision with nuclei at E_1 are equally probably (5.15) distributed over the interval of 0 to E_1 (see Fig. 5.2). The number of collisions at E_1 in the interval dE_1 is $\Phi(E_1) \cdot \Sigma_s(E_1) \cdot dE_1 = F(E_1) \cdot dE_1$, where F is the function of the number of collisions. The number of neutrons which have en-

tered dE from dE_1 is $F(E_1)dE_1 \cdot (dE/E_1)$, and the number of neutrons from the whole interval of E to E_0 is $\left(\int_E^{E_0} F(E_1) \frac{dE_1}{E_1} \right) \cdot dE$.

In addition, the source emits Q neutrons per second and the same number of neutrons undergo their first scattering per second. $Q(dE/E_0)$ neutrons enter the interval dE . The sum of neutrons that have entered dE is equal to the number of neutrons that have left, i.e., $F(E) dE$:

$$F(E) dE = dE \int_E^{E_0} \frac{F(E_1)}{E_1} dE_1 + \frac{Q}{E_0} dE \quad (5.20)$$

2. Fermi spectrum. The equation obtained is reduced to a differential one by differentiating since the integral in (5.20) does not contain the independent variable in the sub-integral expression

$$\frac{dF}{dE} = -\frac{F}{E} \quad (5.21)$$

The solution of the equation is

$$F(E) = \frac{C}{E} \quad (5.22)$$

and the integration constant C is determined by substituting the solution into (5.20), with $E = E_0$, i.e., it follows from the initial condition $F(E_0) = C/E_0 = Q/E_0$. It means that $C = Q$, and the ultimate solution for the flux is of the form

$$\Phi(E) = \frac{Q}{\Sigma_s \xi E} \quad (5.23)$$

During moderation in media with $\xi < 1$ at energies near E_0 some peculiarities arise in the distribution of $\Phi(E)$ [26]. But when E decreases, these peculiarities disappear and the flux distribution is described by the expression analogous to (5.23):

$$\Phi(E) = \frac{Q}{\Sigma_s \xi E} \quad (5.24)$$

Expression (5.24) is the dependence of the flux Φ upon energy or it is the spectrum of slowing-down neutrons (the Fermi spectrum). Since the scattering cross-section is approximately constant in the intermediate energy region, one may say that the flux of slowing-down neutrons is inversely proportional to energy, i.e., it increases when the energy decreases. The reason for the increase of the flux is that neutrons, while slowing down, lose on the average the same fraction of the energy they possess at a given

moment. While E is large, large are the energy portions $\overline{\Delta E}$ being lost on the average in one scattering event (see Fig. 5.3) and in this case $\overline{\Delta E}$ may include a lot of single intervals to which the flux $\Phi(E)$ is referred. When E becomes small, small are $\overline{\Delta E}$ and for a single energy interval to be passed, many scattering events are required. The smaller E , the slower moderating neutrons are shifted along the energy scale, and just this means an increase of the flux per single energy interval. It is significant in this case that the number of neutrons intersecting any energy coordinate E in unit time be constant and equal to the power of the source.

3. Slowing-down density. The function describing the number of neutrons slowing down in unit time per unit volume per energy value E is termed slowing-down density $q(E)$ and is related to the flux by the following relation

$$q(E) = \Phi(E) \Sigma_s \cdot \xi E \quad (5.25)$$

The fact that relation (5.25) corresponds to the definition of q stems from the following considerations. The value ξ is the mean change of the energy logarithm in one scattering event (5.17), $\xi = \overline{\Delta \ln E} \approx \overline{\Delta E}/E$. It means that $\overline{\Delta E} = \xi E$ is the value of the energy lost, corresponding to the mean decrement of the energy logarithm in one scattering event, so that if the interval $\overline{\Delta E}$ is between E and $E + \overline{\Delta E}$, then each scattering event in this interval is accompanied by the decrease of the neutron energy below the value E . $\Phi \Sigma_s$ is the number of events of neutron scattering in unit time per unit volume referred to a single energy interval. Then, $\Phi \Sigma_s \overline{\Delta E} = \Phi \Sigma_s \xi E$ is the number of scattering events in the given interval with the average decrement of energy necessarily lower than E , i.e., it is the slowing-down density q . It follows from the comparison of (5.25) and (5.24) that in this case, $q(E) = Q$, i.e., q is a constant value equal to the power of the neutron source. If the number of neutrons passing through some value of energy E in unit time, i.e., $q(E)$, is lower than the power of the source Q , it means that neutrons are lost at some value higher than E . This corresponds to slowing-down in the presence of absorption.

4. Absorption taken into consideration. In the case of $\xi = 1$, the expression for the flux $\Phi(E)$ may easily be obtained despite the presence of absorption. Now, the function $F(E)$ is

$$F(E) = \Phi(E) [\Sigma_s(E) + \Sigma_a(E)] \quad (5.26)$$

where Σ_s and Σ_a are the macroscopic cross-sections of scattering and absorption, respectively. Of the total number of collisions (5.26) only the fraction $\Sigma_s/(\Sigma_s + \Sigma_a)$ brings about scattering. In all other cases, neutrons are absorbed and are not considered. The

neutron balance in the interval dE near any value of the energy E (5.20) is presented in the following form

$$F(E) = \frac{Q}{E_0} + \int_E^{E_0} \frac{\Sigma_s(E_1)}{\Sigma_s(E_1) + \Sigma_a(E_1)} \frac{F(E_1)}{E_1} \cdot dE_1 \quad (5.27)$$

which implies that, with the energy E_0 , absorption is still absent since E_0 belongs to the fast region. The integral equation (5.27) is once more reduced to the differential one

$$\frac{dF}{dE} = - \frac{\Sigma_s}{\Sigma_s + \Sigma_a} \cdot \frac{F}{E} \quad (5.28)$$

the solution of which is

$$- \int_{F(E)}^{F(E_0)} \frac{dE}{F} \int_E^{E_0} \frac{\Sigma_s}{\Sigma_s + \Sigma_a} \cdot \frac{dE}{E} \quad (5.29)$$

or

$$\ln \frac{F(E)}{F(E_0)} = \int_E^{E_0} \frac{\Sigma_s}{\Sigma_s + \Sigma_a} \cdot \frac{dE}{E} \quad (5.30)$$

Since $F(E_0) = Q/E_0$ (5.27), then

$$F(E) = \frac{Q}{E_0} e^{\int_E^{E_0} \frac{\Sigma_s}{\Sigma_s + \Sigma_a} \cdot \frac{dE}{E}} \quad (5.31)$$

Finally, because of the parity

$$\frac{\Sigma_s}{\Sigma_s + \Sigma_a} = 1 - \frac{\Sigma_a}{\Sigma_s + \Sigma_a} \quad (5.32)$$

$$F(E) = \frac{Q}{E} e^{-\int_E^{E_0} \frac{\Sigma_a}{\Sigma_s + \Sigma_a} \cdot \frac{dE}{E}} \quad (5.33)$$

And if with some energy value $\Sigma_a \ll \Sigma_s$, the flux is still equal to $\Phi(E) = F(E)/\Sigma_s$, i.e.,

$$\Phi(E) = \frac{Q}{\Sigma_s E} e^{-\int_E^{E_0} \frac{\Sigma_a}{\Sigma_s + \Sigma_a} \cdot \frac{dE}{E}} \quad (5.34)$$

5. The resonance escape probability. Thus, in the case of absorption of neutrons in the slowing-down process an additional exponential factor smaller than unity appears in the expression for the

flux. It is significant that the same factor appears in the expression for the slowing-down density $q(E)$ [26]. The latter means that not all neutrons reach the energy E but only some part of them which were not absorbed at higher energies. In the general case of slowing-down in a medium with $\xi \neq 1$, this factor $\varphi(E)$ is of the form

$$\varphi(E) = e^{-\int_E^{E_0} \frac{\Sigma_a}{\Sigma_s + \Sigma_a} \cdot \frac{dE}{\xi E}} \quad (5.35)$$

and represents the fraction of neutrons that have escaped absorption in slowing-down to the energy E . Since in the intermediate energy region, the absorption cross-sections are small, except the capture resonances (Sec. 4.5-8), it is the resonances which make the greatest contribution to absorption in slowing-down. For this reason, the coefficient φ is called the resonance escape probability.

5.3. Thermal Neutron Diffusion

1. **Diffusion event.** Free neutrons produced in nuclear transformations repeatedly collide with atomic nuclei before being subsequently absorbed, and in each collision event they are scattered at

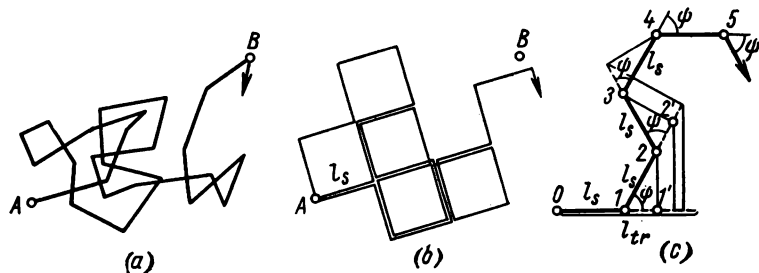


Fig. 5.4. The plane-diffusion picture for a real (a), an average (b) and an average neutron with nonspherically symmetric scattering (c)

an arbitrary angle. The motion of neutrons is, eventually, a chaotic motion in a medium, and the translation at the chaotic motion is dispersion. The trajectory of the neutron motion appears to be as shown in Fig. 5.4a. Since the neutron cross-sections are small and, hence, the free-path lengths of neutrons between collisions are great (1.59), neutrons cover long distances in the process of diffusion. Collision of the neutron with the nucleus is a random event and the trajectory of a separate neutron cannot be predicted theoretically. The description of the diffusion process is performed by statistical methods—with the neutron sources given, one finds the space distribution of the average number of neutrons in unit

volume, the distribution of neutrons by their energies and by the directions of their motion in each elementary volume, and the time-dependent distributions. When applied to a single neutron, this description gives the density distribution of the probability for the neutron to have this or that set of independent variables, such as space coordinates, energy and velocity at any instant of time.

A detailed description of the diffusion process is quite complicated and requires the use of the Boltzmann kinetic equation. In homogeneous media with low absorption, however, isotropic diffusion takes place, which is characterized by the fact that at any point of the volume the motion of diffusing neutrons is equally probable in any direction. In this case, there is no dependence upon the directions of the velocity vectors, and neutron diffusion is rather precisely described by a simpler diffusion equation the solutions of which are known and well studied. Large volumes of a pure moderator provide examples of media in which real space distributions of neutrons coincide with the solutions of the diffusion equation. It is true that such a coincidence is only observed in regions with no disturbance of diffusion isotropy, which is not always the case. Near the boundary of the body and vacuum (under normal conditions, air is such a rarified medium that it is considered to be vacuum from the standpoint of neutron diffusion), near a strong absorber or a source of neutrons, diffusion is nonisotropic since the number of neutrons moving in the direction of the vacuum, or of the absorber, or from the source of neutrons, is always higher than the number of neutrons moving in the opposite direction. However, if the volume of regions with disturbances of diffusion isotropy is small in comparison with the entire volume of the body, such corrections for solutions of the diffusion equation may be introduced that make these solutions applicable in many cases of practical significance, including the case of describing neutron diffusion in nuclear reactors.

In isotropic diffusion, the neutron velocity is considered to be a scalar quantity and it is, therefore, fully defined by energy. In the general case in the process of diffusion, neutrons have their energy reduced so that solutions of the diffusion equations are the functions of energy. However, upon reaching the thermal region the neutron energy does not on the average change, although in certain collisions it may either decrease or increase, the probability of its possible values being determined by the Maxwell distribution (4.44). Isotropic diffusion of neutrons of constant energy is the simplest case. The average number of neutrons in unit volume of the substance, or the neutron density n , and hence the neutron flux Φ only depends upon the space coordinates and, perhaps, upon time, and the neutron velocity v is merely the factor of proportionality between Φ and n (1.50).

2. Average neutron. A number of physical parameters characterizing the neutron motion in space, for instance, the path between collisions, the scattering angle and the velocity of thermal motion, are not constant. At the same time, they have no regular time dependence but assume different values from collision to collision just because collisions with nuclei are random. These parameters determine the coefficients in the diffusion equation, which are constant. The latter is achieved by averaging the parameters in question over a great number of collisions. A neutron with its parameters averaged is always of the same velocity. Each time from collision to collision, it traverses the same distance l_s (1.59). It has a constant number of collisions per unit time, equal to v/l_s and the scattering angle always equal to $\pi/2$ (Sec. 5.1-3). The diffusion of such a neutron, as if it were not in a three-dimensional space but in a plane, is shown in Fig. 5.4*b*. In the three-dimensional space, the average neutron is each time scattered along the plane perpendicular to the direction of its motion before scattering. A single neutron has a low probability of multiple repetition of the same deviations of the scattering parameters from the mean, and the diffusion of the overwhelming majority of neutrons is correctly described by appropriate equations with constant coefficients determined by the mean values of the elementary characteristics of the scattering event.

3. Transport length. A moderate-energy neutron is each time scattered at the angle $\pi/2$. This corresponds to the spherical symmetry of scattering. Scattering by light nuclei is, however, not spherically symmetrical due to the transfer of a large portion of the neutron energy to the nucleus (Sec. 5.1-3). Scattering is neither spherically symmetrical in collisions of fast neutrons of mechanical moments $l > 0$ with nuclei (Sec. 4.5-7). The case of non-symmetric scattering is reduced to the case of spherical symmetry by introducing the transport length, or the transport free path of the neutron l_{tr} , which is the effective displacement of the neutron in the direction of motion till the next scattering event which corresponds to the rotation of the neutron trajectory by the angle $\pi/2$. Fig. 5.4*c* illustrates the physical sense of this quantity. If the average neutron is scattered at an angle lower than $\pi/2$, one may assume that after the first scattering event the neutron will reach point 2, having been scattered at the angle $\pi/2$ but having traversed (in the direction of its initial motion) a somewhat longer path than l_s , namely, $l_s + l_s \cos \psi$, with $\cos \psi$ being the average cosine of the scattering angle. The correction introduced does not, however, allow for the same effect in subsequent scattering events. The path before the second scattering has the same correction which shifts the point 2 to the position 2' so that the first path before scattering at the angle $\pi/2$ to the point 2' becomes equal to

$l_s + l_s \overline{\cos \psi} + l_s (\overline{\cos \psi})^2$, etc. In neglecting the neutron absorption, the number of subsequent collisions may be arbitrarily high and the above correction taken into account in all collisions results in the following expression:

$$l_{tr} = l_s [1 + \overline{\cos \psi} + (\overline{\cos \psi})^2 + \dots] = \frac{l_s}{1 - \overline{\cos \psi}} \quad (5.36)$$

The scattering length l_s , corrected for the non-sphericity of scattering, is the transport length l_{tr} .

Each characteristic path length of the neutron before the corresponding collision may be compared to its macroscopic cross-section [see (1.59) through (1.61)] and, hence, the transport length l_{tr} may be compared to the transport cross-section $\Sigma_{tr} = 1/l_{tr}$ the relation of which to the macroscopic scattering cross-section Σ_s follows from (1.59) and (5.36):

$$\Sigma_{tr} = \Sigma_s (1 - \overline{\cos \psi}) \quad (5.37)$$

It is evident that for the macroscopic cross-sections, expression (5.37) is of the form

$$\sigma_{tr} = \sigma_s (1 - \overline{\cos \psi}) \quad (5.38)$$

When intermediate neutrons are scattered from nuclei, the value $\cos \psi$ is simply expressed through the mass number (5.14). If the scattering law cannot be theoretically obtained, the probability $d\omega/d\Omega_\psi$ for (5.14) is found in experiment by determining the angular distribution of the scattered neutrons. The experiment is analogous to Rutherford's experiment on alpha-particle scattering (Sec. 1.2-3). The results of such experiments are usually presented in the form of the relationship of the differential scattering cross-section and the scattering angle $\sigma_s(\psi) = d\sigma_s/d\Omega_\psi$ since the cross-section is the probability of collision, related to the number of nuclei per unit area of the target N_1 , i.e., $\sigma_s(\psi) = (1/N_1) (d\omega/d\Omega_\psi)$ [see (1.47)]. Here, the total scattering cross-section is

$$\sigma_s = \int_{\Omega_0} \sigma_s(\psi) d\Omega_\psi = 2\pi \int_0^\pi \sigma_s(\psi) \sin \psi d\psi$$

and according to (5.38), the transport cross-section is

$$\sigma_{tr} = \int_{\Omega_0} \frac{d\sigma_s}{d\Omega_\psi} (1 - \cos \psi) d\Omega_\psi = 2\pi \int_0^\pi \sigma_s(\psi) (1 - \cos \psi) \sin \psi d\psi \quad (5.39)$$

Thus, neutron diffusion in non-spherically symmetric scattering takes place as if it were spherical symmetry but the scattering

cross-section were by a factor of $(1 - \overline{\cos \psi})^{-1}$ lower and the corresponding path between collisions by a factor of $(1 - \overline{\cos \psi})^{-1}$ higher. It should also be pointed out that asymmetry in one scattering event does not have any effect upon diffusion isotropy in the medium since the scattering asymmetry is observed relative to some chosen direction of the neutron motion while diffusion assumes their chaotic motion. All these considerations are concerned with neutrons whose energy is higher than the average energy of thermal motion. In the thermal region, the centre-of-mass coordinate system and the laboratory system coincide, and $l_{tr} = l_s$ in spherically symmetrical scattering.

4. **Diffusion current.** If the neutron density $n(r)$ is not constant over the whole volume of the substance, which is always the case, leakage of neutrons out of the higher-density regions into the regions of lower density takes place. This fact is elucidated in Fig. 5.5 by the pattern with one space coordinate. The neutron density is higher in the short interval near r_1 than in the interval near r_2 . And since all neutrons are in motion, the number of neutrons being scattered from r_1 to r_2 is also higher than in the opposite direction.

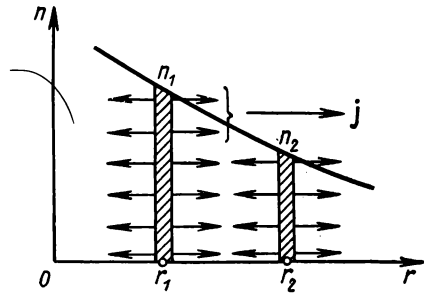


Fig. 5.5. Neutron diffusion current

Thus, the difference between the neutron densities gives rise to a diffusion current \mathbf{j} . In case of isotropic diffusion each elementary volume scatters the same number of neutrons in all directions, which in Fig. 5.5 is denoted by an equal number of the respective arrow-lines in the positive and negative directions of the axis r . In this case the diffusion current or the number of neutrons which, due to diffusion, flow through a unit area in unit time in the direction of a lower density, proves to be proportional to the drop of the neutron density per unit path, or to the density gradient

$$\mathbf{j} = -D' \text{grad } n \quad (5.40)$$

where the minus sign means that the derivative of the decreasing function is negative and the \mathbf{j} -vector is directed towards decreasing n , and D' is the proportionality constant (diffusion coefficient) indicating that current depends upon the nature of the substance as well. If diffusion is non-isotropic, diffusion current cannot be expressed by relation (5.40) since the inflow of neutrons from r_1 to r_2 with one and the same gradient may be heavier or weaker, depending upon the space distribution of neutrons being scattered

by each elementary interval. Thus, if r_2 were at the boundary of the body and vacuum, then neutrons would be scattered in a lower number from r_2 to the left than to the right, into the vacuum, and, with the same density gradient, the current would prove to be heavier than with equally probable scattering into r_2 . Relation (5.40) is the basic assumption of the simple theory of diffusion, which is equivalent to the assumption of the diffusion everywhere isotropic.

In the case of spherical symmetry of scattering, the diffusion coefficient D' is expressed through the free-path length l_s and through the neutron velocity and in the general case through l_{tr} and v , $D' = l_{tr}v/3$. Since in isotropic diffusion, v is a scalar, i.e., the mean projections of the velocity vector are throughout equal to zero and, hence, v does not depend upon the space coordinates, v can be put under the sign of the space coordinate differentiation, and relation (5.40) be written with respect to the neutron flux $\Phi = nv$:

$$\mathbf{j} = -D \text{grad } \Phi \quad (5.41)$$

where D is the diffusion coefficient for the flux

$$D = \frac{l_{tr}}{3} \quad (5.42)$$

Although the scalar value of the flux Φ and the diffusion current \mathbf{j} have the same dimension, there is a principal difference between them. Φ is proportional to the density n and indicates the number of neutrons traversing a unit area in unit time at a given point in space due to the chaotic motion of neutrons about the medium. The diffusion current \mathbf{j} is a vector indicating in which direction and how many neutrons flow through a unit area in unit time at a given point in space due to the difference between the counter-components of the current Φ in this direction. If in some space area the neutron density does not depend upon the space coordinates, then $\mathbf{j} = 0$, while Φ always equals nv .

5. Diffusion equation. The diffusion equation is an equation of the neutron balance in an arbitrary unit volume of the substance. Three phenomena are known causing disappearance or appearance of neutrons in a given volume. These are: inflow out of the neighbouring volumes, absorption and birth of neutrons in the volume involved. The increase or decrease of the number of neutrons in unit time, i.e., $(\partial n / \partial t) = (1/v) \cdot (\partial \Phi / \partial t)$ with a plus or minus sign, is determined by the predominance of this or that of the processes in question. If the vector of the diffusion current \mathbf{j} is defined, then the divergence of the vector is the decrease of the number of neutrons in the unit volume in unit time due to diffusion. Hence, the increase in diffusion $-\text{div } \mathbf{j} = D \text{div grad } \Phi = D \Delta \Phi$, where D

is put beyond the differentiation sign div , since it does not depend upon the space coordinates, and $\Delta\Phi$ is the notation of the total second derivative of the flux over the space coordinates, $\Delta\Phi = (\partial^2\Phi/\partial x^2) + (\partial^2\Phi/\partial y^2) + (\partial^2\Phi/\partial z^2)$. The number of neutrons absorbed per unit volume in unit time is $\Phi\Sigma_a$ (1.56), where $\Sigma_a = \Sigma_{ast}(V\pi/2)$ (see Sec. 4.5-9), and the number of thermal neutrons produced is the number of neutrons that have acquired their thermal energy in slowing-down per unit volume in unit time, i.e., the slowing-down density (5.25) taken with the thermal energy E_{th} , $q_{th} = q(E_{th})$. The total increase with absorption subtracted, is the total increase in unit time per unit volume

$$D\Delta\Phi - \Sigma_a\Phi + q_{th} = \frac{1}{v} \frac{\partial\Phi}{\partial t} \quad (5.43)$$

The relation of the neutron balance (5.43) is termed diffusion equation with respect to the flux Φ which is the function of space coordinates and time, $\Phi(\mathbf{r}, t)$; D , Σ_a and v are constant coefficients, and the source function q_{th} also depends in the general case upon the space coordinates and time.

6. Boundary conditions. The diffusion equation is a partial differential equation. With the t -dependence of the flux, the solution while applied to a certain physical problem, assumes a particular form if the initial condition in time is defined. In a stationary case, a boundary-value problem is solved and the form of its solution is determined by the conditions defined at the boundary of the body within which the equation is valid. If the separation of the variables is possible, then, with respect to each variable, the equation is an ordinary second-order differential equation and its general solution contains two integration constants. The constants are determined by the given values of the function at the boundary points from the symmetry conditions of the problem or the requirement for the limited neutron flux function over all the entire range of variation of the space variables. If in the medium, a point source is given with the intensity Q of neutrons per second, then the relation between the number of neutrons at any point in space and the value Q is set on the basis of the source condition. Neutrons diffuse from the source into the medium and their number traversing the sphere surface of the radius r with its centre at the source point is $Q' = 4\pi r^2 j$, where j , the diffusion current (5.41) expressed through the function Φ , is a general solution of equations with indefinite integration constants. Since in each small volume neutrons are absorbed, $Q' < Q$ but in the limit Q' tends to Q and one of the integration constants is defined from the relation

$$Q = \lim_{r \rightarrow 0} 4\pi r^2 j \quad (5.44)$$

If the medium is finite, then beyond its boundaries there may be either vacuum or any other dense medium. Each dense medium is characterized by its diffusion constants, and the solution is found separately in each medium, Φ_1 and Φ_2 . At the boundary points, $r = r_0$, the requirement is imposed that fluxes and diffusion currents obtained for these points from different solutions should coincide, i.e.,

$$\Phi_1(r_0) = \Phi_2(r_0) \quad (5.45)$$

$$D_1 \left(\frac{d\Phi_1}{dr} \right)_{r=r_0} = D_2 \left(\frac{d\Phi_2}{dr} \right)_{r=r_0} \quad (5.46)$$

Relations (5.45) and (5.46) are called conditions for the joining of solutions. Empty space has no chaotic diffusion and the flux Φ vanishes. Null conditions are, therefore, set at the boundary of the body and vacuum. At the same time, the concentration of neutrons in the vacuum near the body which scatters neutrons is, of course, not equal to zero. It is, however, low since in vacuum neutrons do not behave as they do in a solid medium. In vacuum, neutrons move without collisions. The velocity with which neutrons move away from the source in empty space is the velocity of their motion v , and in a condensed medium this is the ratio of the average displacement in space during diffusion to the life-time of the neutron, or the neutron mobility which is tens of times lower than v . Therefore, in transition from the condensed medium into vacuum, the function of the neutron density must decrease abruptly. This implies both relatively low concentration of neutrons beyond the boundaries of the medium and its inequality to zero at the boundary of the body and vacuum. Nevertheless, the null boundary condition is usually used, and the coincidence of the neutron distribution with the true distribution is achieved by an artificial approach: the neutron flux is considered to be vanishing beyond the boundaries of the body. There is every reason for that since the decreasing function in case of a discontinuous change at some point can always be extrapolated to zero in a continuous manner.

The coordinate of the point where the flux should be considered vanishing is usually found from the solution of the kinetic equation in some simplest case. The fact is that neutrons in the medium, from collision to collision, traverse finite paths and, therefore, in the vicinity of the boundary at a distance as short as l_{tr} , there is a lack of neutrons moving into the body since vacuum does not reflect neutrons. It means that the diffusion isotropy in the boundary layer is disturbed and the diffusion equation is not valid. The precise change-over of the diffuse flux Φ into the directional flux beyond the boundaries of the body can only be obtained as the solution of the kinetic equation allowing for the distribution of the directions of the neutron velocity vectors. Thus obtained, the distri-

bution of the neutron flux from those inner regions where the diffusion theory is also valid is linearly extrapolated beyond the boundaries of the body. The point where the extrapolated flux vanishes is taken to be a conventional boundary of the body (Fig. 5.6). Now, the solution of the diffusion equation coincides with the true distribution of the flux over the whole volume of the body but only on condition that the flux vanishes at the extrapolated boundary of the body

$$\Phi(r_{ex}) = 0 \quad (5.47)$$

The flux beyond the boundaries of the body has, of course, no physical sense and in a small boundary region does not reproduce its true plot. The difference between the coordinates of the extrapolated r_{ex} and the geometrical r_0 of the body boundaries $\delta = r_{ex} - r_0$ is found from the solution of the kinetic equation, and for the plane boundary, it is

$$\delta = 0.71 \cdot l_{tr} \quad (5.48)$$

Hence, the null boundary condition in the simple diffusion theory is of the form

$$\Phi(r_0 + 0.71 l_{tr}) = 0 \quad (5.49)$$

where r_0 is the coordinate of the geometrical boundary of the body, and l_{tr} , the transport length (5.36).

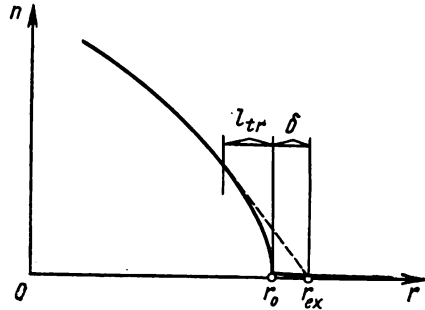


Fig. 5.6. Neutron density at the boundary of the body with vacuum

7. Neutron ranges. The range is the mean displacement in space along a straight line from the point of the thermal neutron production to the absorption point. In the simplest case, all the points of production coincide and only the coordinates of the absorption points should be averaged. Such an assumption implies a point source of the thermal neutrons. Since neutron diffusion is described by the second-order differential equation, the balances of neutrons moving throughout the volume of the substance, for instance, the fraction of absorption in volume or the fraction of leakage, are expressed in terms of the quadratic values of displacement. Hence, the ranges of diffusing neutrons are of no practical interest. It is the root-mean-square ranges R^2 , that are of significance. If the distribution function of the probability of the neutron absorption in unit volume in the vicinity of the point r , $f(r) = d\omega/dV$, is known, then

$$R^2 = \int_{V_\infty} r^2 f(r) dV = 4\pi \int_0^\infty r^4 f(r) dr \quad (5.50)$$

where integrating is carried out over an infinite volume of the medium V_∞ , and the volume element in spherical coordinates with no dependence upon the angular coordinates is $dV = 4\pi r^2 dr$. The probability distribution function can be obtained both experimentally and by solving diffusion equation (5.43). In the latter case, $f(r)$ depends upon the coefficients of the equation as upon parameters, and after calculating (5.50), R^2 appears to be expressed through these coefficients, i.e., in the long run, through the macroscopic cross-sections, the transport and the absorption ones. The function $f(r)$ is the law of the *range* distribution of neutrons. Because of the diffusion, the total path length of the neutron in the substance, l_a , is much longer than its range, $\sqrt{R^2}$, and it is determined by the macroscopic absorption cross-section, $l_a = 1/\Sigma_a$ (1.60). Since the neutron absorption is a unitary event then, as is always the case, the distribution of neutrons by their *path lengths*, l_a , is determined by expression (1.54), where σ is the absorption cross-section σ_a .

8. Point source in infinite medium. When applied to the case in question, diffusion equation (5.43) is simplified. It is assumed that the point source emits thermal neutrons and, hence, there are no volume-distributed sources, $q_{th}(r) = 0$. It is also assumed that the neutron emission rate Q is constant in time. It means that the problem is stationary and $d\Phi/dt = 0$. As a result, the diffusion equation proves to be the following:

$$\Delta\Phi - \kappa^2\Phi = 0 \quad (5.51)$$

and it means that in diffusion, as many thermal neutrons are absorbed as appear in unit volume in unit time. Here, $\kappa^2 = \Sigma_a/D$. In considering diffusion in three-dimensional space, it is always assumed that the function Φ depends upon the three space coordinates. The number of independent variables may, however, be less if the geometrical conditions of the problem cause the space symmetry of the solution. In the case involved, the problem has spherical symmetry, and if one uses the spherical coordinate system, placing its origin at the point of the source, then the flux function will depend only upon one coordinate, i.e., upon the distance from the source r . In this case equation (5.51) will be of the form

$$\frac{d^2\Phi}{dr^2} + \frac{2}{r} \cdot \frac{d\Phi}{dr} - \kappa^2\Phi = 0 \quad (5.52)$$

where the first two terms are the radial part of $\Delta\Phi$ in spherical coordinates and the angular coordinate derivatives are equal to zero. By substituting $\Phi = u/r$, equation (5.52) reduces to

$$\frac{d^2u}{dr^2} - \kappa^2u = 0 \quad (5.53)$$

Equation (5.53) is analogous to equation (5.51) but, by contrast, contains the function of only one independent variable. In this case, its solution is expressed by elementary functions.

The form of the general solution of (5.53) is defined by the sign before the second term. If this is a minus sign, as in the present case, the solution is the sum of the hyperbolic sine and cosine, or, which is the same, the sum of the corresponding exponents

$$u(r) = Be^{-\kappa r} + Ce^{\kappa r} \quad (5.54)$$

But if this is a plus sign, the solution is the sum of the ordinary sine and cosine

$$u(r) = B \sin \kappa r + C \cos \kappa r \quad (5.55)$$

where B and C are the integration constants. Upon changing to the function Φ , one has

$$\Phi(r) = B \cdot \frac{e^{-\kappa r}}{r} + C \cdot \frac{e^{\kappa r}}{r} \quad (5.56)$$

Since in the problem the medium is infinite, from the condition of the limited flux function (Sec. 5.3-6) $C = 0$. The constant B is defined from the source condition (5.44);

$$Q = \lim_{r \rightarrow 0} 4\pi r^2 \left(-D \frac{d\Phi}{dr} \right) = 4\pi D \cdot B \quad (5.57)$$

and the ultimate solution is of the form

$$\Phi(r) = \frac{Q}{4\pi D} \cdot \frac{e^{-\kappa r}}{r} \quad (5.58)$$

9. Diffusion length. The stationary equation of diffusion with no sources (5.51) contains one constant coefficient $\kappa^2 = \Sigma_a/D$ and, hence, its solution should depend upon κ as upon a parameter. The dimension κ is the inverse length, and the value $L = 1/\kappa$ is called the diffusion length of thermal neutrons. The diffusion length is expressed through the macroscopic cross-sections or the path lengths l_{tr} (5.36) and l_a (1.60)

$$L^2 = \frac{1}{\kappa^2} = \frac{D}{\Sigma_a} = \frac{1}{3\Sigma_{tr}\Sigma_a} = \frac{l_{tr}l_a}{3} \quad (5.59)$$

The diffusion length is simply related to R^2 . Since the distribution of the neutron flux near the point source has been found, the mean square of the neutron translation, R^2 , can directly be calculated. The number of neutrons absorbed in unit volume in unit time in the vicinity of the point r is $\Phi(r)\Sigma_a$ and, hence, in diffusion all these

neutrons had the range r . Thus, the mean-root-square range is

$$R^2 = \frac{\int_{V_\infty} r^2 \Phi(r) \Sigma_a dV}{\int_{V_\infty} \Phi(r) \Sigma_a dV} = \frac{\frac{Q \Sigma_a}{4\pi D} \int_0^\infty r^2 \frac{e^{-\frac{r}{L}}}{r} 4\pi r^2 dr}{\frac{Q \Sigma_a}{4\pi D} \int_0^\infty \frac{e^{-\frac{r}{L}}}{r} 4\pi r^2 dr} = 6L^2 \quad (5.60)$$

where the denominator is the total number of neutrons absorbed in the whole volume in unit time, i.e., Q , since the problem is stationary, and the same number of neutrons is absorbed by the substance in a second as is emitted by the source. It follows from (5.60) that the square diffusion length is equal to one sixth of the mean square translation of the thermal neutron in diffusion. Like R^2 , its part L^2 also characterizes the neutron translation in the substance and hence the fraction of thermal neutrons leaving the boundaries of the body of a finite volume. The latter should immediately be understood qualitatively. The fraction of neutrons leaving the boundaries of the medium must depend both upon the geometrical dimensions of the body and upon the magnitude of the neutron translation in diffusion. This fraction is the greater, the smaller the dimensions of the body, and the higher L .

The diffusion length is the most significant diffusion constant of the medium, and for all moderating materials used in reactor engineering it has been measured experimentally. Unfortunately, the diffusion length cannot be obtained in the simplest experiment with a point source by measuring $\Phi(r)$ and calculating the mean square translation R^2 (5.60), since no point sources of thermal neutrons exist. The produced neutrons are always fast and they become thermal only while slowing down in a large volume of the substance. However, in a body of such a geometrical form and with a source for which an analogous solution of the diffusion equation may be obtained, the comparison of the real distribution of the neutron flux with the theoretical distribution makes it possible to determine the parameter of the diffusion theory, i.e., the length L which fits the real distribution to the solution of the diffusion equation. The most simple technique of measuring distributions of $\Phi(r)$ is the activation of thin foils of substances of a high absorption cross-section, in which the absorption of neutrons results in radioactivity with a half-life convenient to measure. The value of the foil activity is proportional to the number of absorptions $\Phi \Sigma_a V_0$ (1.57), i.e., to the neutron flux. Hence, the space distribution of the activity value of the foil irradiated at different points of the body volume is, to within the proportionality constant, the distribution

of the neutron flux. Table 5.3 lists the values of the diffusion constants of moderating materials.

Table 5.3
Diffusion Constants of Moderators

| Moderator | Density, 10^3 kg/m^3 | l_{tr} , cm | L , cm | Albedo |
|------------------|-----------------------------------|------------------|----------|--------|
| H ₂ O | 1 | 0.49 | 2.72 | 0.8 |
| D ₂ O | 1.1 | 2.88 | 160 | 0.98 |
| Be | 1.84 | 1.6 | 21 | 0.90 |
| BeO | 2.96 | 1.65 | 29 | 0.92 |
| C | 1.6 | 2.6 | 54 | 0.94 |

10. **Albedo.** If in an arbitrary direction x , the component of the scalar flux Φ_x is isolated, it turns out that a greater part of the neutrons of this component comes back after collisions with the nuclei of the atoms of the medium, forming a counter flux with the component in the opposite direction, Φ_{-x} through a unit area of the plane perpendicular to x . It means that the medium reflects neutrons. The qualitative characteristics of the reflection effect is the internal reflection factor, or albedo, determined by the following relation:

$$\beta = \frac{\Phi_{-x}}{\Phi_x} \quad (5.61)$$

The values of the components of the scalar fluxes in the directions of x and $-x$ may be calculated following the diffusion theory [26] and they are not only determined by the diffusion constants of the medium but by the vicinity of the boundary of the body with the vacuum to the plane within the volume, at the points of which the albedo is defined. Such a relationship must be understood if one remembers that neutrons are not reflected from the planes but from the volumes of the substance. If the distance to the boundary with the vacuum is short in comparison with the diffusion length, then many of the neutrons constituting Φ_x reach the boundary and leave the volume of the body, while some part of them could scatter backward in the case of an infinite volume of the substance. Hence, the vicinity of the boundary lowers Φ_x and, thus, lowers the albedo. The maximum albedo value is possessed by a body of an infinitely large volume. In this case, β depends only upon the diffusion constants of the medium and, on condition that $(d/L) \ll 1$, it is approximately equal to [26]:

$$\beta \approx 1 - 4 \frac{D}{L} = 1 - \frac{4}{3} \cdot \frac{l_{tr}}{L} \quad (5.62)$$

The albedo is the higher, the lower l_{tr} , i.e., the less the distance at which the trajectory of the neutron is deflected at the angle $\pi/2$. And the albedo is the higher, the greater L , i.e., the greater the depth of the substance from which the neutron can make its way back without being absorbed. The albedo values in an infinite medium are very high (see Table 5.3). Even light water absorbing neutrons appreciably reflects 80% incident neutrons. Heavy water reflects 98% neutrons. It is true that the moderating layer equivalent in its reflection ability to an infinitely extended medium should have the depth of the order of $2L$, which is 5.4 cm for H_2O and 320 cm for D_2O . Because of the high reflection factor, in the process of diffusion each neutron repeatedly, on the average more than ten times, intersects an initial conventional plane within the body volume. The adjacent volumes of the substance multiply exchange the same portions of neutrons, which results in a low mobility of neutrons and forms those high concentrations of neutrons in condensed media which are experimentally observed and in comparison with which the concentrations in directed neutron fluxes in empty space are negligibly low. For the same reason, the diffusion current j is usually low in comparison with the scalar neutron flux Φ which describes their chaotic motion.

5.4. Diffusion of Slowing-Down Neutrons

1. Diffusion during slowing-down. In each collision with the nucleus, neutrons of the energy higher than thermal lose their energy and, before achieving the energy equilibrium with the particles of the medium, undergo a sufficiently high number of collisions (see Table 5.1). From collision to collision, just as in the thermal region, neutrons on the average cover the distance l_s (1.59) and before the efficient deflection of the trajectory of motion by the angle $\pi/2$, neutrons are displaced by l_{tr} (5.36). The scattering cross-section Σ_s and the mean cosine of the scattering angle $\cos\psi$ determining l_s and l_{tr} are, in general, energy dependent. In the intermediate region (Sec. 4.5-8) where most of collisions take place during slowing-down of fission neutrons, these values are, however, practically not constant. Since in each collision with the nucleus, the neutron scatters at an arbitrary angle, translation of slowing-down neutrons in the medium is also diffusion. As distinct from thermal neutrons, diffusion of slowing-down neutrons results in the decrease of energy. In addition, the number of collisions is limited at least in the case of the average-energy neutron. Therefore, if instead of distribution of the characteristics of a separate collision, one uses their mean values and obtains equations describing the behaviour of an average neutron, the solutions of the equation will be the less consistent with real neutron

distributions by space and energy coordinates, the less the number of collisions required in slowing-down to thermal energy.

2. Theory of continuous slowing-down. If fast neutron sources are uniformly distributed over the volume of the substance, the diffusion of slowing-down neutrons is isotropic. If the neutron source is a point one, the anisotropy is observed only in the vicinity of the source at a distance of the order of l_{tr} and with neutrons of only the initial energy since the distribution of the directions of the velocity vectors becomes equally probable just after the first collision with spherically symmetrical scattering. Absorption in slowing-down is low, especially in moderating substances with no capture resonances. Therefore, the diffusion of slowing-down neutrons can also be described by a simple diffusion equation. At the boundary of the body with the vacuum a peculiarity arises which is allowed for by introducing an extrapolated boundary of the body (5.49), just as in the case of thermal neutron diffusion.

In this case the average neutron has one more averaged characteristic, the value of the mean loss of the energy logarithm ξ (5.17). This circumstance is an additional assumption when one considers diffusion of slowing-down neutrons. On the basis of it, one can associate the chronologic time from the very instant of the fast neutron birth, with its energy. The average neutron collides with the nucleus after the path l_s , each time decreasing the energy logarithm by ξ . If the neutron velocity is v , then the number of collisions in unit time is $dv/dt = v/l_s = v\Sigma_s$. Since ξ is the change of the energy logarithm per one collision, $\xi = d \ln E/dv = (1/E) \cdot dE/dv$, the change of the energy of the slowing-down neutron in unit time is

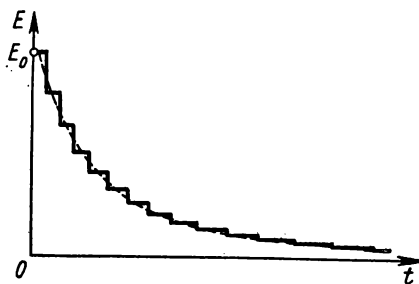


Fig. 5.7. Energy loss in time with the average neutron slowing-down

$$\frac{dE}{dt} = \frac{dE}{dv} \cdot \frac{dv}{dt} = \xi E v \Sigma_s \quad (5.63)$$

Relation (5.63) is a functional link between time and energy, expressed through differentials. Although with time the average neutron loses its energy in finite portions (Fig. 5.7), expression (5.63) determines the continuous relationship between energy and time, which is shown by the dotted line in the figure. Hence, (5.63) substitutes the discrete picture of slowing-down for continuous moderation. In this connection, expression (5.63) is the basis of the

theory of continuous slowing-down. However, (5.63) is an inevitable step after the losses of energy in different portions were substituted for the mean logarithmic loss ξ , i.e., the average neutron is taken to be the slowing-down neutron. Slowing-down of such a neutron differs but little from continuous moderation. It means that to describe the diffusion of slowing-down neutrons, one may now use diffusion equation (5.43) which is also an equation of continuous diffusion. The matter is, that mathematical equations containing no parameters of discrete processes as coefficients of the distribution functions, do not describe such processes. The application of the equations is based on the close similarity of the continuous and the discrete pictures under strictly defined conditions. Here, relations of the type of (5.63) do always and necessarily exist and provide a transition from the discrete to the continuous model, so that (5.63) is not the only peculiarity of the theory of continuous slowing-down. The same is the expression for diffusion current (5.41) in obtaining diffusion equation (5.43). In (5.41) only l_{tr} is left of the discrete nature of neutron diffusion and it enters appropriately in the expression for the diffusion coefficient (5.42). Therefore, the diffusion equation is, in fact, also an equation of a continuous, not a discrete diffusion. This equation cannot, of course, give a correct description of the neutron density in the vicinity of the boundary of the body with vacuum where the behaviour of the neutron density is first and foremost determined by the discrete nature of collisions and paths. The discontinuity of the neutron density at the boundary with vacuum also stems from the simple theory of diffusion as the consequence of the discontinuity of the neutron mobility (Sec. 5.3-6), i.e., from the boundary condition. Thus, operation with the diffusion equation is valid as long as diffusion is isotropic and the slowing-down neutron is an average neutron.

3. Neutron-age equation. It is assumed in the simplest case that all fast neutrons are produced with the same initial energy E_0 . If the source emits some portion of neutrons in unit time, then, with time and as the distance from the source increases, these neutrons will be distributed over an ever increasing volume so that in the vicinity of an arbitrary point with the space coordinates \mathbf{r} the number of neutrons out of this portion in unit volume, or the neutron density n_1 , will depend upon the chronologic time t counted from the moment of emission, $n_1 = n_1(\mathbf{r}, t)$. Since absorption is absent during slowing-down, and a neutron source exists only with energy E_0 , which corresponds to the time $t = 0$, i.e., there are no sources distributed by the time of slowing-down, diffusion equation (5.43) written with respect to the neutron density $n_1(\mathbf{r}, t)$ takes the following form

$$D\nabla^2 n_1 = \frac{\partial n_1}{\partial t} \quad (5.64)$$

where D is the diffusion coefficient for the flux (5.42). Now with the help of relation (5.63), one can substitute variables in equation (5.64). Since the source has emitted the above portion of neutrons in unit time, then $n_1(\mathbf{r}, t)$ is practically some dn/dt , i.e., the number of neutrons in unit volume and in unit time. The differential of any function does not depend upon independent variables and, therefore, if dn/dE is $n_2(\mathbf{r}, E)$, then

$$n_1(\mathbf{r}, t) dt = n_2(\mathbf{r}, E) dE \quad (5.65)$$

from whence

$$n_1(\mathbf{r}, t) = n_2(\mathbf{r}, E) \frac{dE}{dt} = n_2 v \Sigma_s \xi E = q(\mathbf{r}, E) \quad (5.66)$$

The product $n_2(\mathbf{r}, E) v = \Phi(\mathbf{r}, E)$ is a neutron flux in a unit energy interval and, hence, $n_1(\mathbf{r}, t)$ in the coordinates (\mathbf{r}, E) is the slowing-down density q , as it was previously determined in (5.25). It is no wonder since $n_1(\mathbf{r}, t)$ in its original meaning is just the slowing-down density. Virtually, $n_1(\mathbf{r}, t)$ is the number of neutrons in unit volume, appearing at the point \mathbf{r} due to slowing-down of some original portion of neutrons, emitted by the source in unit time. Since slowing-down of the average neutron is considered, subsequently, all the neutrons of this portion in any unit volume in unit time go above the value t in time and, hence, lower than the value E in energy. Consequently, $n_1(\mathbf{r}, t)$ is the slowing-down density in the coordinates (\mathbf{r}, t) . Since $(\partial n_1 / \partial t) = (\partial n_1 / \partial E) \cdot (\partial E / \partial t)$, and n_1 as a function of energy has the symbol q introduced previously, then, as a result of substituting the variables, equation (5.64) is reduced to the following form

$$D \Delta q(\mathbf{r}, E) = \xi E \Sigma_s \frac{\partial q(\mathbf{r}, E)}{\partial E} \quad (5.67)$$

To get rid of the coefficients, a new independent variable, the neutron age τ , is introduced:

$$\tau(E) = \int_E^{E_0} \frac{D}{\Sigma_s} \cdot \frac{dE}{\xi E} \quad (5.68)$$

and after one more substitution of the variables, the age equation is obtained:

$$\Delta q(\mathbf{r}, \tau) = \frac{\partial q(\mathbf{r}, \tau)}{\partial \tau} \quad (5.69)$$

with the density function having its previous symbol. Generally speaking, as follows from (5.68), the differentials $d\tau$ and dE have different signs since τ increases with the decrease of energy. This circumstance does not, however, make the algebraic signs in the right and the left sides of equation (5.69) different because in in-

intermediate operations, when passing from dt to dE , the minus sign has already once been omitted since the derivative of dE/dt is virtually negative. The signs of dt and $d\tau$ in (5.64) and (5.69) are the same.

4. Neutron age. Equation (5.69) was derived by Fermi. In its form, it is similar to the equation of thermal conductivity the right side of which has age- instead of time-differentiation. However, age bears a relation to chronologic time of the moderating neutron only because it increases with time. As follows from (5.68) or (5.69),

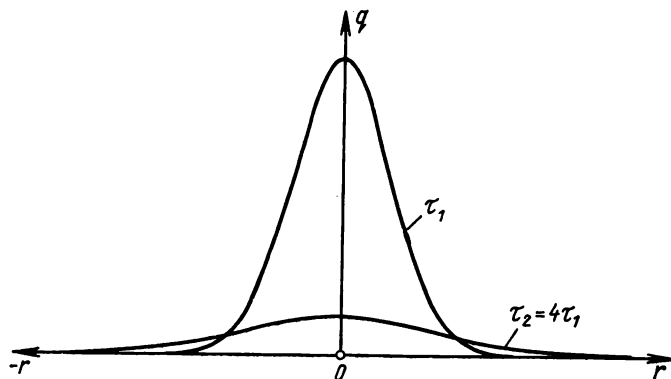


Fig. 5.8. Space distribution of slowing-down neutrons

age dimension is a real dimension. The physical sense of age is identical to the square length of thermal neutron diffusion. Age describes the translation of slowing-down neutrons in space and is simply related to the mean-square translation during slowing-down, R^2 . The mean-square translation R^2 is most simply calculated if the space coordinate of the origin of all slowing-down neutrons diffusion is the same, i.e., the fast neutron source is of the point type. The solution of the age equation for the case of the point source in an infinite homogeneous medium is of the following form [26]:

$$q(r, \tau) = \frac{Q}{(4\pi\tau)^{3/2}} \cdot e^{-\frac{r^2}{4\tau}} \quad (5.70)$$

where Q is the source intensity and the slowing-down density q depends upon one space coordinate by virtue of the spherical symmetry of the problem. Fig. 5.8 shows the relationship $q(r)$ for two values of the second variable τ . If τ is small, i.e., the energy is high, then neutrons are in the vicinity of the source point. In the process of slowing-down and with the increase of τ , space distri-

bution increasingly diffuses. Finally, upon achieving the thermal energy E_{th} , relation (5.70) gives the distribution of thermal neutron sources, $q_{th}(r) = q(r, \tau_{th})$, near the point source of fast neutrons. With each energy value, $\tau(E)$ is the parameter of the space distribution of neutrons of this energy. It is natural that, in calculating the mean square translation, it will be expressed in terms of this distribution parameter. Since the slowing-down density, $q(r, \tau)$, is the number of neutrons of the age τ in unit volume in the vicinity of the point r of the portion Q , the square range of these neutrons is $r^2 \cdot q(r, \tau)$, and the mean square R^2 should be summed up over all the Q neutrons of the given age, distributed over the infinite volume, and be referred to one neutron:

$$R^2 = \frac{1}{Q} \int_{V_{\infty}} r^2 q(r, \tau) dV = \frac{1}{Q} \cdot \frac{Q \cdot 4\pi}{(4\pi\tau)^{3/2}} \int_0^{\infty} r^4 e^{-\frac{r^2}{4\tau}} dr = 6\tau \quad (5.71)$$

Thus, like the square length of diffusion (5.60), the neutron age τ is also one sixth of the mean-square space translation of slowing-down neutrons and is sometimes called the square length of slowing-down.

5. Age measurement. The neutron age $\tau(E)$ determines the space distribution of slowing-down neutrons at any energy, and in finite media it also determines the fraction of neutrons leaving the volume of the substance upon reaching the boundaries of vacuum due to diffusion, i.e., the leakage of neutrons out of the medium. As far as its application to the design of thermal reactors, of some special interest is the age of neutrons slowing down to thermal energy and having the initial energy of fission neutrons. Henceforth, the word age will always imply this value. The age of all moderating materials is measured in experiment.

Since it is not a hard job to make a fast neutron source of a small volume, the age measurement is usually reduced to the measurement of the mean square translation of neutrons in the medium with a point source. For example, a small flat target of U^{235} irradiated by a beam of thermal neutrons is a point source of fission neutrons. To obtain the mean square translation, it is necessary to experimentally find the space distribution of neutrons of a given energy, which is performed with the use of thin foils activated by neutrons. The activation is proportional to the number of absorbed neutrons (3.8), i.e., to the flux $\Phi(r, E)$. However, the flux is proportional to the slowing-down density q (5.25) and the distribution of the activation value over the volume of the substance is the distribution of the slowing-down density q . Unfortunately, it is impossible to directly measure the age of neutrons slowing to thermal energy since there are no detectors capable of discriminating the neutrons which have just achieved E_{th} from those which became

thermal long ago and which have diffused from the adjacent volumes to this point. The age is, therefore, measured at some energy higher than the thermal one, using substances possessing capture resonance (Sec. 4.5-8). For example, the In^{115} isotope has a powerful capture resonance at the energy 1.46 eV and is activated in neutron absorption. True, the capture cross-section of indium, as of all other substances with low-lying capture resonances, is high in the thermal region (Sec. 4.5-9). But if indium doped with cadmium absorbing thermal neutrons is irradiated, then absorption and, hence, activation will only concern neutrons whose energy is approximately 1.46 eV since the capture resonance width is low. If $\mathcal{J}(r)$ is activation of indium, obtained in a large volume of the substance with a point source of fission neutrons, then the age $\tau(E_0, E')$ in slowing down from E_0 to the energy $E' = 1.46$ eV is

$$\tau(E_0, E') = \frac{1}{6} R^2 = \frac{1}{6} \cdot \frac{\int_0^\infty r^2 \mathcal{J}(r) \cdot 4\pi r^2 dr}{\int_0^\infty \mathcal{J}(r) \cdot 4\pi r^2 dr} \quad (5.72)$$

where the integral in the denominator is normalized, relating the total square translation to a single neutron which has initiated activation. Since $\tau(E_0, E_{th}) = \tau(E_0, E') + \tau(E', E_{th})$, (5.68), the age in slowing down to thermal energy is found by adding the correction computed from formula (5.68) by the known cross-sections Σ_s and Σ_{tr} near the thermal region, to the value τ measured experimentally.

Table 5.4

Neutron Age in Moderating Materials

| Moderator | Density, 10^3 kg/m^3 | τ , cm ² |
|------------------|-----------------------------------|--------------------------|
| H ₂ O | 1.0 | 27 |
| D ₂ O | 1.1 | 120 |
| Be | 1.84 | 98 |
| BeO | 2.96 | 105 |
| C | 1.6 | 350 |

Table 5.4 lists experimental values of the neutron ages in the basic moderating substances. The agreement of the age theory with experiment is the better, the heavier the moderating substance and the lower ξ , i.e., the closer the mechanism of energy losses to the model of continuous slowing-down

and the greater the number of collisions during slowing-down. It turns out, as a result, that the age theory is not at all applicable to hydrogenous media.

It is due both to the high value of ξ and to the value (rapidly increasing with slowing-down) of the cross-section of neutrons scattering by hydrogen in the energy region of about 1 MeV (see

Fig. 4.23). The latter circumstance is responsible for the fact that the greater part of neutron translation in hydrogenous media is due to the first few collisions, and the simple diffusion theory is not applicable to the case of a small number of collisions. The distribution (5.70) does not, therefore, take place in such media. However, by analogy with other moderating substances, $(1/6) R^2$ is called an age in hydrogenous media.

6. Migration length. The neutron age τ and the square diffusion length L^2 describe the neutron translation in space in slowing-down and in diffusion in the thermal region, respectively. Each of these values is the sixth part of the corresponding mean-square translation. The total square neutron translation from the origin point due to nuclear reaction to the absorption point in the thermal region is the total square partial translation, since after slowing-down diffusion is equally probable in all directions and the mean vector of translations in diffusion will be oriented with respect to the vector of translation in slowing down at the angle $\pi/2$. The one-sixth part of the total square neutron range in the substance is called the square migration length and, according to the above, it is

$$M^2 = L^2 + \tau \quad (5.73)$$

7. Slowing-down and diffusion time. Expression (5.63) establishes the relationship between the chronologic time of the slowing-down neutron and its energy. Hence, if the range of the energy change in slowing-down is known, one can find the time interval of slowing-down t_{mod}

$$t_{mod} = \int_0^{t_{th}} dt = \int_{E_{th}}^{E_0} \frac{1}{\xi \Sigma_s} \cdot \frac{dE}{vE} = \frac{2}{\xi \Sigma_s} \left(\frac{1}{v_{th}} - \frac{1}{v_0} \right) \approx \frac{2}{\xi \Sigma_s v_{th}} \quad (5.74)$$

where $v_{th} = 2.2 \cdot 10^3$ m/s and $v_0 \approx 2 \cdot 10^7$ m/s are the velocities of thermal and fission neutrons, and $\xi \Sigma_s$ is averaged over the interval of slowing-down. The second integral represents the integration intervals since, as was already mentioned, dE/dt is a negative value. Table 5.5 lists the times of slowing-down in moderating substances in the decrease of the neutron energy from $E_0 = 2$ MeV to $E_{th} = 0.025$ eV and in diffusion in the thermal region.

The total path length of the average neutron before its absorption in the thermal region is $l_a = 1/\Sigma_a$ (1.60). If v is the neutron velocity, then the mean diffusion time t_{dif} is

$$t_{dif} = \frac{l_a}{v} = \frac{1}{v \Sigma_a} \quad (5.75)$$

The residence time of the neutron in the thermal region is about 100 times longer than the slowing-down time. It means that in a

large bulk of the moderating material the number of neutrons of thermal energy is as many times higher than the number of all the other neutrons of higher energy, i.e., neutrons accumulate in the thermal region. But the total fluxes of slowing-down and thermal neutrons $n\nu$ are roughly equal (Sec. 6.8-10) since the average velocity of slowing-down neutrons is much higher than the velocity of thermal neutrons.

Table 5.5
Slowing-Down and Diffusion Times

| Moderator | $t_{mod}, \mu s$ | t_{diff}, ms |
|------------------|------------------|----------------|
| H ₂ O | 6.7 | 0.21 |
| D ₂ O | 48 | 138 |
| Be | 59 | 3.7 |
| BeO | 76 | 6.2 |
| C | 149 | 15.2 |

In thermal neutron reactors, some quantity of uranium is introduced into the moderator. Its cross-section of thermal neutron absorption is much higher than that of moderators (see Table 4.5). Because of that the residence time of the neutron in the thermal region is reduced. But it is still 10^{-3} s in graphite- and heavy

water-moderated reactors and is about 10^{-4} s in light water reactors, so that in thermal neutron reactors, the residence time of the neutron in the thermal region is also much longer than the slowing-down time. In fast neutron reactors neutron moderation does not take place and the total neutron life-time is quite short, about 10^{-7} s. Although the absorption cross-sections in the fast region are not high (Sec. 4.5-7), and before being absorbed, the neutron does not undergo many collisions with atomic nuclei of the medium, all this takes very little time due to the velocity of about 10^7 m/s.

CHAPTER SIX

NUCLEAR REACTOR

6.1. Chain Reaction

1. Mechanism of energy release. Substance transformation is accompanied by the release of free energy only in case the substance has a margin of energy. The latter means that the microparticles of a substance are in a state characterized by the rest energy greater than the rest energy of another state, the transition to which is possible. The spontaneous transition is usually hindered by an energy barrier. To get over this barrier, a particle must receive some amount of energy, i.e., excitation energy, from without. The essence of the exothermic reaction is that the energy release due to the transformation which accompanies the excitation is greater than the energy required for the excitation of the process. There are two ways to overcome the energy barrier, namely, either by the kinetic energy of the colliding particles or by the bond energy of the adding particle.

Bearing in mind the macroscopic character of the energy release it may be concluded that all the particles of the substance or at first some part of them must have some kinetic energy required for the excitation of the reaction. This is attainable only with the increase of the temperature of the medium up to temperatures at which the thermal energy is close to the energy of the barrier limiting the course of the process. In the case of molecular transformations, i.e., chemical reactions, such an increase is usually of the order of hundreds of absolute degrees. As for the nuclear reactions, the temperatures required are not lower than 10^7 °K due to a very high Coulomb barrier (2.15) of colliding particles. The thermal excitation of nuclear reactions was realized only in the synthesis of the lightest nuclei having the minimum Coulomb barriers (thermonuclear fusion).

The excitation by addition of particles occurs through the unutilized bonds of attractive forces between particles. It does not require high kinetic energies and, therefore, does not depend upon the temperature of the medium. It is the particles that are necessary for the excitation of reactions. Bearing in mind the macroscopic quantities of power generation, not the realization of an ele-

mentary act of reaction, it may be concluded that the second mechanism leads to the reaction only when the chain reaction takes place. The latter arises only when the particles, initiating the reaction appear again as products of an exoenergetic reaction.

2. Chain reaction. Chain reactions are widely distributed among the chemical reactions, in which free atoms or radicals play the role of particles with unused bonds. The chain mechanism in the nuclear transformations may be provided by neutrons for which no Coulomb barriers exist and which excite nuclei in absorption. The appearance of the necessary particle in the medium gives rise to the chain of reactions following one after another, which proceeds up to the chain termination due to the loss of the reaction carrier-particle. The basic reasons for the losses are the absorption of the particle with no emission of a secondary or the escape of the particle beyond the volume of the substance supporting the chain process. A chain reaction is called an unbranched reaction if in each elementary act only one reaction carrier is created. An unbranched chain reaction cannot cause the release of large quantities of energy.

If more than one particle arises in each elementary act or in some links of the chain, the branched chain reaction comes to existence, since one of the secondary particles continues the primary chain, and the others give rise to new chains which are also branching. However, the processes leading to the termination of the chain compete with the branching process. The specific limiting and critical phenomena are caused by this competition. The chain self-sustaining reaction becomes impossible when the number of terminations is greater than the number of the newly arising chains. Even if the reaction is induced artificially by inserting some amount of the required particles into a medium, the initial process will be damping rapidly, because the number of chains can only decrease. If the number of new chains being found exceeds the number of terminations, the chain reaction propagates rapidly over the volume of the substance even if only one particle appears in the system. The critical state is characterized by the equality of the number of the new chains to the number of terminations. The possibility of reaching the critical state is determined by the number of conditions. The fission of heavy nucleons is initiated by one neutron and more than one neutron is emitted in fission. Therefore, the fission process may create a branched chain reaction and neutrons will be the reaction carriers.

3. Multiplication factor. Since each new chain is initiated by one particle the multiplication of chains is the multiplication of particles. Therefore, the concept of the multiplication factor of particles (bearing in mind neutrons in what follows) is used in the description of the phenomena developing in branched chain reactions.

Each neutron taking part in the chain process undergoes the following conversion cycle. It is created in fission, lives in an unbound state for some time, then it is either lost or induces a new fission act, thereby producing neutrons of the second generation. Successive generations of neutrons are separated by an act of fission. The ratio of the number of neutrons of the following generation to the number of neutrons of the previous generation in the total volume of the multiplying medium is called *multiplication factor*, k . The value of the multiplication factor shows whether the total number of neutrons in the reacting volume increases or decreases or it is stable within the mean time of neutron cycle.

The critical state is characterized by the condition $k = 1$, when $k < 1$, the state of a substance is called *subcritical* and the chain reaction is rapidly damping if there initially was some number of neutrons in the medium. If no neutrons existed at the start, the chain reaction is altogether impossible. In the *supercritical* state $k > 1$, and the chain reaction increases in an avalanche manner up to the moment when, for some reasons, k becomes less than 1. Since heavy nuclei may undergo spontaneous fission (see Table 3.2), a small number of neutrons is always present in the medium involving heavy nuclides, that is, there always exists a primary neutron initiating the chain process. In addition, free neutrons appear everywhere as the products of nuclear reactions induced by cosmic-ray particles so that once the system reaches the state of $k < 1$, the spontaneous fission chain reaction immediately results.

4. Nuclear reactor. Reaching the critical state is of major interest in obtaining a controllable power source. In the critical state, the number of neutrons does not change with time. Consequently, the number of fission acts in unit time and, thereby, the energy release are constant. The absolute value of the energy release can arbitrarily be obtained here with the help of the chain-reaction control system. Near the critical state a small and simply realizable excess of k above unity is admissible (Sec. 6.10-7). The insertion of an additional amount of fissionable material into the neutron-multiplying medium gives rise to an excessive multiplication of reaction chains, i.e., it is followed by the increase of k . On the contrary, the embedding of a neutron moderator increases the number of breaks in the chains and decreases k . In addition, it is possible to use some neutron reflecting materials the translations of which near the multiplying medium decrease or increase the loss of neutrons due to leakage, which also affects the number of breaks. Manipulating these elements of the control system permits one to start a chain reaction, to reach any power level, to support a stationary regime in the critical state, and to stop the chain process. An installation in which a controllable chain reaction of fission takes

place is called a nuclear reactor. The part of the reactor, which contains fission material and in which in fact the self-sustaining chain reaction of fission proceeds is called *reactor core*.

5. Critical parameters. The reduction of the multiplication factor to unity is attained by balancing the multiplication and the loss of neutrons. In fact, there are two reasons for the loss. These are capture with no fission and leakage out of the volume of the multiplying medium. The contribution of neutron β -decay to the loss (1.65) is negligible due to the great difference between the characteristic time of a neutron cycle in the reactor ($\lesssim 10^{-3}$ s) and the life-time of a free neutron ($\approx 10^3$ s). The competition between the fission processes and neutron radioactive capture is determined, first and foremost, by the ratio of amounts of fissile material and other components in the reactor core. The neutron leakage mainly depends on the size and the geometry of the reactor core. Hence, the problem of determining the conditions corresponding to $k = 1$ is usually divided into two parts. First, the multiplication factor without leakage is determined. The absence of leakage conforms to the infinite volume of the substance, and the corresponding multiplication coefficient is called multiplication factor in the infinite medium, k_0 . Evidently, $k < k_0$, since in the finite volume, the neutron loss is surely greater than that of the infinite volume due to leakage. Therefore, if in the substance of the given composition $k_0 < 1$, the self-sustaining chain reaction is impossible in an infinite volume either in any finite volume. Thus, k_0 defines, in principle, the capacity of the medium to multiply neutrons.

There always exists a finite volume, in which the condition

$$k = k_0 w = 1 \quad (6.1)$$

is satisfied, if $k_0 > 1$. Here w is the relative part of the total number of neutrons being formed in the reactor, which are absorbed within the reactor core. In other words, w is the probability for the neutron to escape out of the finite volume. The relative part of neutrons lost due to leakage is $1 - w$. The value of w depends upon the size and geometry. It falls with the decrease of the volume of the reactor core. Indeed, since any unit volume of the substance absorbs neutrons, the number of absorption acts is proportional to the total volume, i.e., to R_{eff}^3 , where R_{eff} is the effective linear size of the material. The neutron leakage occurs through the surface of the body. Therefore, the number of neutrons which have left the finite volume is proportional to the surface area of the body, i.e., approximately, to R_{eff}^2 . Therefore, the absorption leakage ratio, proportional to $R_{eff}^3/R_{eff}^2 = R_{eff}$, falls with the decrease of the body volume and approaches zero with $R_{eff} \rightarrow 0$, providing $w \rightarrow 0$. Consequently, condition (6.1) can always be satisfied when $k_0 > 1$.

The size of the reaction core corresponding to $k = 1$ is called critical size. The corresponding volume of the reactor core is also called *critical size*. The mass of the fissionable material in the critical volume is defined as the critical mass. The determination of the critical parameters of multiplying media forms the basic problem for a nuclear reactor. Calculation of the ω -factor necessary in attaining $k = 1$ is associated with the solution of the problem of neutron diffusion in the reactor substance (Chapter Five). In the given volume with a certain ω , the critical state can be reached by the choice of the composition of the medium with an appropriate k_0 as well. The relative concentration of a fissile material within the reactor core which turns the left-hand side of (6.1) to unity, is also called critical.

6. The development of fission chain reaction in time. A change of the number of neutrons in a noncritical reactor is defined by a deviation of k from unity and by the characteristic time, τ , of the neutron cycle. If at a certain moment there are n neutrons in the reactor, then their number after one conversion cycle will be equal to kn , the cycle increment being $(kn - n) = n(k - 1)$. Consequently, the change of a neutron number in unit time is

$$\frac{dn}{dt} = \frac{n(k - 1)}{\tau} \quad (6.2)$$

The solution of this equation gives the time dependence of the number of neutrons in the form

$$n(t) = n_0 e^{\frac{k-1}{\tau} t} \quad (6.3)$$

where n_0 is the number of neutrons at the time $t = 0$. The longest period of a cycle is characteristic of thermal-neutron reactors where it comes to $\tau = 10^{-3}$ s. If one supposes that $k = 1.01$, then the number of neutrons increases $n(1)/n_0 = \exp(0.01 \cdot 1/0.001) = e^{10} \approx \approx 20,000$ times every second. The same is the increase of the number of the fission acts, i.e., the energy release in the reactor. Hence, even 0.01 excess of k above unity is inadmissible in a controllable installation. Still, this evaluation neglects delayed neutrons and is, thereby, overestimated. However, if $k - 1$ is greater than the fraction of delayed neutrons, β (see Table 3.6), the development of the chain reaction in time proceeds, in fact, following (6.3).

In media made of pure fissionable materials, the time of the neutron cycle is of the order of 10^{-8} s. With $k = 1.1$, an initial neutron creates 10^{26} neutrons after 6 μ s, i.e., one fission produces 10^{26} fission acts. This is equivalent to the fission of about 40 kg of uranium per 6 μ s. This estimate shows that the fission chain reaction rate may be extremely great. An instantaneous release of

energy is a nuclear explosion. The power of nuclear explosions is usually estimated in terms of equivalent amounts of the most widely employed chemical explosive, trinitrotoluene (TNT). The energy release due to the fission of 1 kg of uranium is equal to the energy released as a result of the explosion of 20,000 tons of TNT. The existence of the critical mass for chain reactions determines a definite limit to the fissionable material quantity, which may participate in the process because each of the parts to be joined in order to obtain the explosion must be subcritical before the beginning of the process. This restricts the possible power of nuclear explosions involving the use of uranium and plutonium by tens or hundreds of thousands of tons of TNT. There is no restriction of this type for thermonuclear explosions as the synthesis of light nuclei goes through thermal excitation which does not result in any critical phenomena.

7. Fissile materials. In a pure fissionable material, for example, in U^{235} , the chain reaction can be easily realized. Neglecting neutron moderation in inelastic scattering of neutrons by the nuclei of U^{235} , one may consider that neutrons initiate fission, having the energy of 2 MeV (Sec. 3.6-7). The number of secondary neutrons with this energy is equal to $\nu = 2.66$ (see Table 3.4). If U^{235} did not capture neutrons without fission, then k_0 in this case would be equal to ν . Radiative capture reduces the multiplication factor by the fraction of fission acts in the total number of absorption acts, i.e., by the factor $\sigma_f(\sigma_f + \sigma_\gamma)$:

$$k_0 = \nu \frac{\sigma_f}{\sigma_f + \sigma_\gamma} \quad (6.4)$$

Using the data of Table 6.2, one obtains $k_0 = 2.56$. The high value of k_0 results in a relatively small critical mass. The values of critical mass for spherical samples made of fissionable materials as well as the corresponding radii of the critical spheres are presented below.

Table 6.1
Critical Mass for Fissionable Materials

| Material | M_{cr} , kg | R_{cr} , cm |
|------------|---------------|---------------|
| U^{233} | 16 | 6 |
| U^{235} | 48 | 8.5 |
| Pu^{239} | 17 | 6 |

It should also be noted that the use of neutron reflectors reduces the critical masses and they can be 2 or 3 times lower than the masses presented in Table 6.1. The cost of pure fissionable materials is very high due to a complicated production technology. These materials are mainly used for some military purposes. For power production in nuclear reactors it is expedient to use natural uranium or the uranium of low U^{235} enrichment.

8. Natural uranium. Natural uranium has the following isotopic composition:

| Isotope | Content, % |
|---------------------|------------|
| U^{238} | 99.28 |
| U^{235} | 0.714 |
| U^{234} | 0.006 |

Natural uranium is nearly a monoisotopic element: the impurity content of the fissile U^{235} isotope is very small, and traces of U^{234} , the product of the U^{238} decay are found. Because of a very small U^{235} content, it is impossible to initiate a chain reaction directly in natural uranium metal. Table 6.2 gives the cross-sections of U^{238} and U^{235} with the mean energy of fissile neutrons being 2 MeV. Since the cross-section of U^{235} differs but slightly from that of U^{238} , and the concentration of U^{235} in natural uranium is small, the role of U^{235} in the multiplication of neutrons is of no importance. On the other hand, U^{238} is not able to support chain reaction for the following reasons. Although the cross-section of U^{238} fission for the energy of 2 MeV is considerable, it falls rapidly with the energy decrease, in fact, approaching zero when the energy tends to 1 MeV (see Fig. 4.16). A part of fission neutrons has the energy lower than 1 MeV. Therefore, only for half the total number of fission neutrons σ_f of U^{238} is as great as in Table 6.2. However, even these neutrons mainly suffer inelastic scattering in collisions with U^{238} nuclei, but do not induce fission because $\sigma_{n'}$ is much greater than σ_f .

Practically each inelastic scattering act leads to the drop of the neutron energy up to the value which is lower than the fission barrier for U^{238} . As a result, about 10% of the fission neutrons have time to fission the nuclei of U^{238} before being moderated to the energy lying below the barrier. This makes k_0 much smaller than unity and, consequently, it becomes impossible to reach the critical state in natural uranium. With energies lower than 1 MeV, the chain reaction can be maintained only by U^{235} . However, the cross-section of the radiative capture rapidly increases with the neutron energy decrease resulting from inelastic scattering. At the same time, the fission cross-section increases slowly (Sec. 4.5-7); this gives rise mainly to the absorption of neutrons in U^{238} if the con-

Table 6.2
Cross-Sections of U^{238} and U^{235}
with $E = 2$ MeV, bn

| Partial cross-section | U^{238} | U^{235} |
|-----------------------|-----------|-----------|
| σ_f | 0.57 | 1.32 |
| σ_γ | 0.03 | 0.05 |
| $\sigma_{n'}$ | 2.3 | 1.8 |
| σ_n | 4.3 | 4.2 |
| σ_t | 7.2 | 7.37 |

centration of U^{235} is small. In metallic uranium, k_0 reaches unity only for a 5% U^{235} isotope-enriched sample.

At the same time, the chain reaction can be obtained in mixtures of natural uranium or uranium of low isotope content with neutron moderators. With a sufficiently great number of moderator-atoms in the mixture, neutrons are moderated to the thermal energy faster than they are absorbed in U^{238} , and if the cross-sections, $\sigma_a = (\sigma_f + \sigma_v)$ of U^{235} and U^{238} at high energies do not differ higher than ten times, then at thermal energy this difference comes to the factor of 250 (see Table 4.4). In this case, U^{235} absorbs neutrons with the probability much greater than that of U^{238} even for such a small concentration as is in uranium of natural enrichment. Therefore, in heavy water-berillium-carbon mixtures, the critical state can be reached with the use of natural uranium.

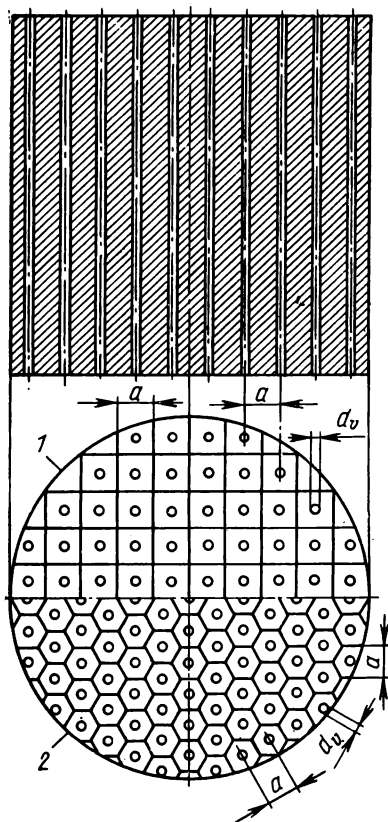


Fig. 6.1. Longitudinal and transverse cross-section of the core of a heterogeneous reactor. Orthogonal (1) and hexagonal (2) lattices

9. Homogeneous and heterogeneous reactor. If the moderator and uranium make up a homogeneous mixture, for example, such as the solution of uranium salt in water, the reactor is called homogeneous. When uranium is distributed in the moderator in the form of separate slugs, the reactor is called heterogeneous. For convenient handling of uranium and for the simplicity of heat removal from the reactor, the uranium slugs are prepared in the form of cylindrical rods or assemblies

of rods or plates arranged in a certain order in the moderator bulk (Fig. 6.1). A regular system of uranium rods forms a *lattice* of the heterogeneous reactor core. The distance between the axes of the rods, i.e., the lattice spacing a and the diameter of the uranium rod, d_U , are the basic parameters of the lattice. These quantities define the relation between the volumes of uranium

and the moderator and, thus, determine k_0 of the substance of the reactor core. Usually, the axes of the uranium rods are located either in the corners of the squares (then the lattice is called a square one) or in the corners of rectilinear triangles (then the lattice is triangular or hexagonal). An uranium rod or an assembly of rods together with the adjacent moderator constitute a *unit cell* of the reactor core. In the case of the square lattice, the cross-section of the unit cell is a square with a side a . For the triangular lattice, it is a rectilinear hexagon with a distance a between the parallel sides. That is why, the lattice is called hexagonal. The whole volume of the heterogeneous reactor core is composed of identical unit cells so that the balance of the energy release and of the neutron absorption in the moderator and uranium of a single cell is equal to the balance of the entire reactor core.

The detailed structure of the unit cell is shown in Fig. 6.2. The central part of the cell free of the moderator is called a *fuel channel*. The uranium rod or an assembly of rods are mounted along the axis of the fuel channel. More than 90% total fission energy is released in the uranium bulk: this is the energy of fission fragments, the energy of β -particles and about half the energy of

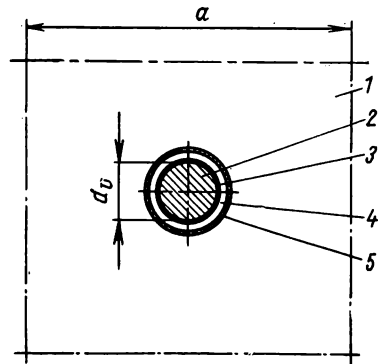


Fig. 6.2. Unit cell

1—moderator; 2—uranium; 3—pressurized uranium blanket; 4—coolant; 5—coolant pipe

γ -ray quanta (see Table 3.3). Therefore, the uranium rods are called fuel elements. For heat removal, the flow of the coolant liquid or gas — is directed along the surface of the fuel element. If the coolant must be separated from the moderator, it is directed along the corresponding pipe. If the same material acts both as a moderator and a coolant, or if the coolant is allowed to enter the moderator, no pipe is needed. The uranium rod, as a rule, has cladding, which prevents the coolant from the chemical interaction with the uranium slug material; it inhibits erosion of the material in the coolant flow and does not either permit the fission fragments to enter the coolant. When the fission fragments get into the coolant, its radioactivity increases to a considerable degree, which is not desirable. The pipe materials, uranium and the materials of other structural elements, are called structural materials of the core.

6.2. Multiplication Factor for Infinite Medium

1. **k_0 Factor.** The multiplication factor in an infinite medium, k_0 , defines the possibility of obtaining the self-sustaining chain reaction in a finite volume of the substance. Only on condition that $k_0 > 1$, the critical state is attainable. Though, in the limiting case, k_0 is simply equal to the number of secondary fission neutrons, ν which is much greater than unity (see Table 3.4), in real media, however, k_0 is often far from being greater than unity. Even for pure fissionable materials, $k_0 < \nu$ (6.4), because fissile nuclei do not only undergo fission, but capture neutrons without fission.

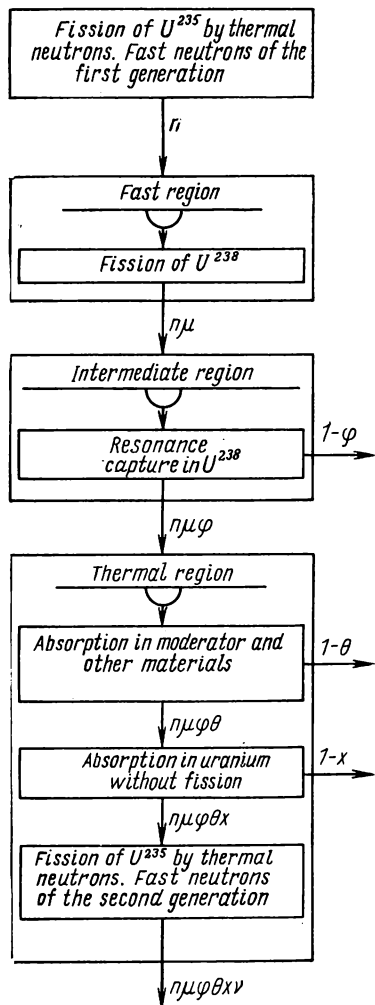


Fig. 6.3. Neutron cycle in a thermal reactor

The cores of reactors, especially of thermal ones, usually contain a low fraction of a fissionable material. When the natural or enriched uranium is used, U^{238} is always present together with U^{235} . A diluter is added to the pure fissionable material. Uranium and plutonium may be applied in the form of chemical components with oxygen, carbon and nitrogen. In addition, the cores contain structural materials and a coolant-material, the greatest part of the thermal reactor core bulk being occupied by the moderator. All these materials absorb neutrons, which leads to the decrease of k_0 . The calculation of k_0 for a concrete multiplication is based on the consideration of the neutron cycle and on account of all possible processes which give rise to the change of the number of neutrons of a single generation. The longest is

that of the thermal neutron reactor. We shall consider this cycle for systems where the fuel, i.e., a material providing the energy release, is uranium.

2. Neutron cycle. Figure 6.3 shows a scheme of a neutron cycle in a thermal reactor. In the consideration of the cycle it is expedient to begin with U^{235} nuclear fission induced by thermal neutrons and the production of n fast neutrons of the next generation. Since about 50% of these neutrons have the energy higher than the U^{238} fission barrier, they may initiate the fission of U^{238} nuclei. Each fission act requires the loss of one fast neutron but, on the average, 2.8 fast neutrons are emitted in fission (see Table 3.4); thus, the process results in neutron multiplication. The value of the multiplication depends upon the composition of the medium and upon the mode of distribution of the reactor core materials over the volume of the core. The μ factor which shows how much the number of U^{235} fission neutrons is increased due to an additional fission of U^{238} is called the fast-neutron multiplication factor.

Fast and intermediate neutrons are absorbed but weakly by atomic nuclei, except for the absorption by the low-lying resonances of intermediate- and high-mass number nuclei (Sec. 4.5-8). In spite of the fact that the resonance level width Γ is much lower than the mean energy loss in slowing-down, ξE , and the majority of slowing-down neutrons never has the energy equal to the energy of resonances, nevertheless, the resonance absorption appears to be considerable. This is explained by the high values of the resonance capture cross-section as well as by the decrease of ξE in slowing-down (see Fig. 5.3) which defines the increase of the flux Φ for low energies (5.24).

Since the chain reaction is hindered by resonance capture, materials with resonance capture are not used in reactor cores, the only exception being U^{238} which inevitably gets into the nuclear reactor together with U^{235} . Therefore, speaking of neutron absorption in slowing-down one, first of all, bears in mind resonance capture in U^{238} . As compared with this process, the absorption of neutrons with non-resonance energies is very low. During the reactor operation some other materials involving resonance capture appear in the system. Pu^{239} is extracted from U^{238} (3.57), then follows Pu^{240} , which is the threshold nuclide, and, therefore, the absorption by the resonances of Pu^{240} is accompanied by the radiative capture alone, as in the case of U^{238} . The resonance capture is typical of many fission products; however, the major contribution is usually made by U^{238} .

The fraction of neutrons which were not absorbed in slowing-down is allowed for by the factor ϕ , the resonance escape probability. Uranium-235 is also capable of resonance absorption which more often results in fission but not in radiative capture. Consequently, in the presence of U^{235} , neutron multiplication takes place. However, in thermal reactor involving natural or slightly enriched uranium, the number of such fission acts is small compared to the

number of fissions in the thermal region, and they may be neglected.

It should be noted that although the resonance capture in U^{238} hinders the development of the chain reaction, it is not an absolutely useless loss of neutrons, since the capture of an arbitrary-energy neutron by the U^{238} nucleus always results in the production of fissionable Pu^{239} (3.57). However, such conversion of non-fissionable U^{238} into fissile material is possible only as a result of the self-sustaining chain reaction. But for this reaction to proceed the increase of k_0 is necessary and, thus, in particular, the resonance capture must be decreased.

In the absence of leakage, the neutrons moderated to thermal energy are absorbed by the atomic nuclei of the medium. Part of neutrons are absorbed due to the radiative capture process, the other one is captured with U^{235} fission. In homogeneous media both fractions are simply expressed through the cross-sections. In heterogeneous media a singularity arises in the neutron flux distribution over the unit cell, because of which the expression for the neutron absorption within the uranium slug differs from that beyond the slug. The fraction of neutrons absorbed by the substance of the uranium slug is defined by the thermal utilization factor, θ . In absorption, the fraction of neutrons inducing U^{235} fission is denoted by x in Fig. 6.3. This fraction alone induces the birth of neutrons of the next generation

3. Four-factor formula. In each fission act ν -fast neutrons are released. As a result, upon completion of a neutron cycle, n neutrons of the preceding generation transform into $n\mu\phi\theta x\nu$ neutrons of the next generation. By definition, the neutron multiplication factor (Sec. 6.1-3) is

$$k_0 = \mu\phi\theta x\nu \quad (6.5)$$

The factor x is expressed through the cross-sections and is not used independently. Instead of x , the quantity

$$\eta = \nu x \quad (6.6)$$

is introduced, which is the number of secondary neutrons per thermal neutron absorbed in the fuel material. Taking account of (6.6), the expression for k_0 of the thermal reactor takes the form

$$k_0 = \mu\phi\theta\eta \quad (6.7)$$

and is called a formula of four factors.

6.3. Number of Neutrons per Absorption Act

1. **Fission fraction.** Supposing that the fuel material consists of U^{235} and U^{238} only, x in (6.6) is expressed as follows. If N_5 and N_8 are concentrations of U^{235} and U^{238} in the multiplying medium, then the corresponding macroscopic cross-sections (1.55) may be expressed through these parameters as may the reaction rates (1.56) for a given flux. Consequently, the number of fission and radiative capture acts in unit time in unit volume with the flux Φ is equal to $\Phi\Sigma_{f5}$, $\Phi\Sigma_{\gamma5}$, $\Phi\Sigma_{\gamma8}$, respectively. Here, the subscripts f and γ correspond to the fission and capture processes, respectively, and subscripts 5 and 8 correspond to U^{235} and U^{238} . Then, the ratio of the fission acts to the total number of acts of the neutron absorption by uranium alone may be written as:

$$x = \frac{\Phi\Sigma_{f5}}{\Phi\Sigma_{f5} + \Phi\Sigma_{\gamma5} + \Phi\Sigma_{\gamma8}} = \frac{\Sigma_{f5}}{\Sigma_{f5} + \Sigma_{\gamma5} + \Sigma_{\gamma8}} = \frac{\sigma_{f5}}{\sigma_{f5} + \sigma_{\gamma5} + \sigma_{\gamma8} \frac{N_8}{N_5}} \quad (6.8)$$

Since the numerator and denominator contain the same factor Φ , the flux, expression (6.8), does not depend upon the flux, and the fission fraction x is constant over the multiplying medium volume. Moreover, x depends upon the *ratio* of concentrations of the uranium isotopes, not on their absolute values, i.e., it is determined by the isotopic composition of uranium used, not by the composition of the multiplying medium as a whole. The absolute values, N_8 and N_5 , which are maximum in pure uranium metal and are decreasing with the dilution of uranium by other materials are sure to depend on the composition of the medium. However, the dilution of uranium of a given isotopic composition does not affect N_8/N_5 . In natural uranium, $N_5/(N_8 + N_5) = 0.00714$ (Sec. 6.1-8).

2. **Number of neutrons per absorption act.** As some part of absorption acts may not result in fission, the average number of secondary neutrons per absorption act, η , is the number of neutrons emitted in a fission act, ν , reduced by the factor of the fraction of the fission acts, x :

$$\eta = \nu x = \nu \frac{\Sigma_{f5}}{\Sigma_{f5} + \Sigma_{\gamma5} + \Sigma_{\gamma8}} = \nu \frac{\sigma_{f5}}{\sigma_{f5} + \sigma_{\gamma5} + \sigma_{\gamma8} \frac{N_8}{N_5}} \quad (6.9)$$

For natural uranium in the thermal region, $\eta = 1.34$ (Table 6.3), which is considerably greater than unity. Such a value of η permits one to obtain $k_0 > 1$ for a number of natural uranium-containing multiplying media involving neutron moderation. If in some medium the condition $k_0 > 1$ is not achievable, expression (6.9) shows most drastic way of increasing k_0 , that is the increase

of the relative concentration of U^{235} in uranium. In the ultimate case of pure U^{235} , $N_8 = 0$ and

$$\eta_5 = \nu_5 \frac{\sigma_{f5}}{\sigma_{f5} + \sigma_{\gamma 5}} = \frac{\nu_5}{1 + \alpha_5} \quad (6.10)$$

where

$$\alpha = \frac{\sigma_{\gamma}}{\sigma_f} \quad (6.11)$$

and subscript 5 corresponds to U^{235} . The numbers of secondary neutrons per absorbed thermal neutron in pure fissile materials and in natural uranium are presented in Table 6.3 together with the values of α . From these data it follows that the enrichment of uranium by U^{235} isotope leads to the increase of η up to 1.5 times.

Table 6.3
Values of ν , η and α in Thermal Region

| Fissionable material | ν | η | α |
|----------------------|-------|--------|----------|
| U^{233} | 2.507 | 2.292 | 0.0885 |
| U^{235} | 2.442 | 2.079 | 0.1694 |
| Pu^{239} | 2.881 | 2.122 | 0.3659 |
| Natural uranium | — | 1.34 | — |

Such an increase of η is not usually required. The enrichment of natural uranium, increasing the U^{235} content 2 or 3 times is sufficient to reach $k_0 > 1$. It should be borne in mind that the values of η for pure fissionable materials are substantially lower than the values of ν , especially for Pu^{239} and U^{235} , which is caused by an essential contribution of the radiative capture to the total number of absorption acts in the thermal region (see Table 4.4).

If some other fissionable material besides U^{235} is contained in the fuel, e.g., accumulated Pu^{239} , the number of secondary neutrons per act of capture is described by the expression

$$\eta = \frac{\nu_5 \Sigma_{f5} + \nu_9 \Sigma_{f9}}{\Sigma_{a5} + \Sigma_{a9} + \Sigma_{\gamma 8}} \quad (6.12)$$

instead of (6.9) where subscript 9 corresponds to Pu^{239} , and $\Sigma_a = \Sigma_f + \Sigma_{\gamma}$ for each fissile material. Equation (6.12) follows from the same arguments on which Eqs (6.8) and (6.9) were based.

3. Uranium slug. In a heterogeneous reactor, the uranium slug apart from the fuel material involves other materials the alloying

additions to uranium metal, atoms chemically bonded with uranium, and fission products. All these substances absorb neutrons and decrease η of the uranium slug as a whole. In a homogeneous multiplying medium, this neutron loss component is taken into account in calculating the thermal utilization factor since all the materials are distributed evenly over the volume of this medium. All the neutron loss in the heterogeneous reactor uranium slug is allowed for by factor η . If $\Sigma_{a\ oth}$ is the sum of the macroscopic cross-sections of all other materials of the uranium slug composition, then, by analogy with (6.12), η for the heterogeneous reactor is:

$$\eta_{het} = \frac{\nu_5 \Sigma_{f5} + \nu_9 \Sigma_{f9}}{\Sigma_{a5} + \Sigma_{a9} + \Sigma_{\gamma8} + \Sigma_{a\ oth}} \quad (6.13)$$

Here the account is taken of the presence of Pu^{239} in uranium. In the absence of plutonium, all the macroscopic cross-sections with the subscript 9 are equal to zero.

6.4. Thermal Utilization Factor

1. Homogeneous medium. The thermal utilization factor defines the fraction of thermal neutrons absorbed by uranium or, generally, by the fuel material. The rest of the neutrons are absorbed by the moderator and by the other materials of the reactor core. If the macroscopic absorption cross-section of uranium is $\Sigma_{aU} = \Sigma_{f5} + \Sigma_{\gamma5} + \Sigma_{\gamma8} = \Sigma_{a5} + \Sigma_{\gamma8}$ and $\Sigma_{a\ mod}$ is that of the moderator, then in the absence of any other substances, θ_{hom} (of the homogeneous multiplying medium) is equal to

$$\theta_{hom} = \frac{\Sigma_{aU}}{\Sigma_{aU} + \Sigma_{a\ mod}} = \frac{1}{1 + q_{mod}} \quad (6.14)$$

where $q_{mod} = \Sigma_{a\ mod}/\Sigma_{aU}$. Expression (6.14) is derived by analogy with (6.8). If some other materials, apart from uranium and the moderator, characterized by the total macroscopic absorption cross-section, $\Sigma_{a\ oth}$, are present in the reactor core bulk then

$$\theta_{hom} = \frac{\Sigma_{aU}}{\Sigma_{aU} + \Sigma_{a\ mod} + \Sigma_{a\ oth}} = \frac{1}{1 + q_{mod} + q_{oth}} \quad (6.15)$$

If, apart from uranium, other fuel materials are contained in the medium such as plutonium, thorium, etc., one puts $\Sigma_{a5} + \Sigma_{a9} + \Sigma_{\gamma8} + \dots$ for Σ_{aU} . In general, in the homogeneous medium the balance of thermal neutrons absorbed by various materials is determined only by the ratio of the macroscopic cross-sections.

Since the numerator in (6.14) and (6.15) is the same as the denominator in (6.9) and (6.12), it is clear that in the homoge-

neous medium, $(\eta\theta)_{hom}$, may be immediately obtained in the form of (6.13), where $\Sigma_{a\ oth}$ is implied to be the total macroscopic absorption cross-section of all the substances of the core except the fuel but involving the moderator. Nevertheless, the concept of thermal utilization factor is also applicable in the homogeneous medium because in the analysis of the multiplying properties of the heterogeneous medium this concept was not specially concerned with the heterogeneity of the system.

2. Heterogeneous medium. In the heterogeneous medium uranium, the structural materials, and the coolant are separated in the reactor core volume (Sec. 6.1-9). The neutron cross-sections, primarily, the absorption cross-sections of all these materials, may strongly differ. Sufficiently high values of absorption cross-sections are typical of natural, and mostly of enriched uranium. The moderators are made of light substances characterized by the weakest absorption (see Table 4.5). Spatial distribution of neutrons over the volumes filled with different components are always different, especially, when the dimensions of the volumes are compared with or exceed l_{tr} . The number of neutrons is always smaller in the regions with greater absorption, and greater for the regions with weak absorption. The number of absorbed neutrons is not only defined by the macroscopic cross-section of the material but by the real neutron flux. As the flux in uranium differs from the flux in the moderator, the thermal utilization factor for the heterogeneous medium, θ_{het} , which defines the balance of neutrons absorbed by uranium and the moderator, must be dependent upon the macroscopic cross-section ratio as well as upon the ratio of the neutron fluxes.

3. Distribution of neutrons over unit cell. The whole volume of the heterogeneous reactor core may be regarded as consisting of unit cells (Sec. 6.1-9). The analysis of the peculiarities of neutron distribution characteristics is reduced to the consideration of a unit cell since they are identical for all the cells. The additional materials occupy a small volume. Thus, one may consider that the unit cell is only formed by the cylindrical uranium slug in the centre and by the moderator surrounding the slug. The neutron flux along the cell axis varies in the moderator and uranium in the same manner. The radial distribution is shown in Fig. 6.4, where a point of the uranium rod axis is put for the origin and the basic lattice parameters are denoted as d_U and a . The spatial dependence on the neutron flux, $\Phi(r)$, is usually obtained from the solution of the diffusion equation for the unit cell [24, 26]. But the qualitative pattern of $\Phi(r)$ may be revealed by using the following arguments.

The number of neutrons in the thermal region is determined by the rate of their production, i.e., by the slowing-down density, q_{th} , and, on the other hand, by the absorption rate, $\Phi\Sigma_a$. In the dyna-

mic equilibrium state ($\frac{\partial \Phi}{\partial t} = 0$), and in the case the diffusion flow is neglected or really absent ($\Delta \Phi = 0$), $q_{th} = \Phi \Sigma_a$ (5.43), and $\Phi = q_{th}/\Sigma_a$; in other words, the thermal neutron flux decreases with the increase of Σ_a for the given source, q_{th} . Consequently, the neutron flux in uranium must be lower than the flux in the moderator for this reason alone. But, in addition, uranium does not, practically, moderate neutrons and so there are no sources of thermal neutrons in its volume. The sources are distributed in the moderator. Thermal neutrons appear in the uranium slug only due to diffusion from the moderator. Therefore, the number of neutrons in uranium is not simply smaller than in the moderator: the neutron flux falls continuously as it approaches the centre of the slug because the longer the path length of neutrons in the absorbing medium, the less the number of the remaining neutrons. Thus, the neutron flux minimum is observed at the centre of the uranium slug and the maximum is expected for the points of the moderator most distant from the adjacent absorbing rods i.e., for $r = \pm a/2$ in Fig. 6.4. By averaging the thermal neutron flux over the volume of uranium, $\bar{\Phi}_U$, and the moderator, $\bar{\Phi}_{mod}$, as follows from the above consideration, one always obtains the inequality

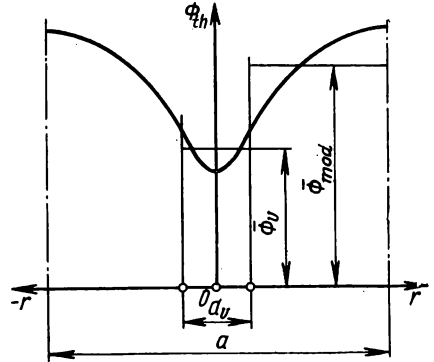


Fig. 6.4. Thermal neutron distribution in a unit cell

$$\bar{\Phi}_U < \bar{\Phi}_{mod} \quad (6.16)$$

4. **θ_{het} coefficient.** The thermal utilization factor is determined to be the ratio of the number of the acts of thermal neutron absorption in the uranium slug to the total number of absorption acts in the unit cell, i.e.,

$$\begin{aligned} \theta_{het} &= \frac{\int_{V_U} \Phi_U \Sigma_{aU} dV}{\int_{V_U} \Phi_U \Sigma_{aU} dV + \int_{V_{mod}} \Phi_{mod} \Sigma_{a mod} dV} = \\ &= \frac{\Sigma_{aU} \bar{\Phi}_U V_U}{\Sigma_{aU} \bar{\Phi}_U V_U + \Sigma_{a mod} \bar{\Phi}_{mod} V_{mod}} = \frac{1}{1 + q_{mod}} \end{aligned} \quad (6.17)$$

Here, Σ_{aU} and $\Sigma_{a\text{mod}}$ independent of special coordinates are taken out of the integral signs, the integration being performed over V_U and V_{mod} which are the volumes of uranium and the moderator, respectively. Also the integral mean value theorem is applied, so that $\bar{\Phi}_U$ and $\bar{\Phi}_{\text{mod}}$ are the quantities in inequality (6.16). Expression (6.17) has been reduced to the form of (6.14). However, now q_{mod} differs from the same quantity in the case of the homogeneous reactor. The difference lies in the appearance of the two factors multiplying $\Sigma_{a\text{mod}}/\Sigma_{aU}$, viz., V_{mod}/V_U , $\bar{\Phi}_{\text{mod}}/\bar{\Phi}_U$. The former contains no new information, because $\Sigma_{a\text{mod}}$ and Σ_{aU} of the homogeneous medium differ from those of the heterogeneous medium. If one considers the homogeneous and the heterogeneous media composed of equal proportions of the moderator and uranium, then, $\Sigma_{a\text{mod}}V_{\text{mod}}/\Sigma_{aU}V_U$ of the heterogeneous medium is simply equal to $\Sigma_{a\text{mod}}/\Sigma_{aU}$ of the homogeneous medium since $\Sigma_{a\text{mod}}V_{\text{mod}}/\Sigma_{aU}V_U = \sigma_{a\text{mod}}N_{\text{mod}}V_{\text{mod}}/\sigma_{aU}N_UV_U = (\sigma_{a\text{mod}}/\sigma_{aU}) \cdot b$ and $\Sigma_{a\text{mod}}/\Sigma_{aU} = \sigma_{a\text{mod}}N_{\text{mod}}/\sigma_{aU}N_U = (\sigma_{a\text{mod}}/\sigma_{aU}) \cdot b$, where b is the ratio of the atomic number of the moderator to that of uranium, which is the same for the volume of the unit cell and for the unit volume of the homogeneous medium. As for the factor $\bar{\Phi}_{\text{mod}}/\bar{\Phi}_U$, it defines the θ_{het} dependence upon the neutron flux distributions mentioned above. Since in the heterogeneous medium (6.16) is always valid,

$$\theta_{\text{het}} < \theta_{\text{hom}} \quad (6.18)$$

which decreases k_0 in the heterogeneous medium. However, it appears that the effect decreasing θ_{het} at the same time leads to a much greater increase of φ_{het} compared to φ_{hom} . Therefore, the maximum of the product $(\varphi\theta)_{\text{het}} > (\varphi\theta)_{\text{hom}}$, and k_0 for the heterogeneous medium may be greater than for the homogeneous one. The calculation of θ_{het} is reduced to obtaining an expression for neutron distribution over the cell and to the evaluation of $\bar{\Phi}_{\text{mod}}/\bar{\Phi}_U$ [24, 26]. The fraction of absorption acts in other materials which are present in the volume of the cell can also be calculated if the neutron flux distribution is known.

6.5. Resonance Escape Probability

1. φ probability. The probability of absorption of neutrons of the energy exactly corresponding to some certain resonance of U^{233} is much greater than the cross-section of thermal neutron absorption by uranium. But the larger the uranium slug absorption cross-section, the greater the difference between the neutron fluxes in uranium and the moderator, and the smaller the fraction of neutrons absorbed by uranium with respect to the homogeneous me-

dium. Since φ , in contrast to θ , is the fraction of the resonance neutrons "non-absorbed" by uranium, φ of the heterogeneous medium proves to be greater than that of the homogeneous medium. For the above reason, this increase of φ exceeds the decrease of θ .

The resonance escape probability is expressed by (5.35) where the integral in the exponent is usually transformed into

$$\int_{E_{th}}^{E_0} \frac{\Sigma_a}{\Sigma_a + \Sigma_s} \cdot \frac{dE}{\xi E} = \frac{N_U}{\xi \Sigma_s} \int_{E_{th}}^{E_0} \sigma_{aU} \frac{\Sigma_s}{\Sigma_a + \Sigma_s} \cdot \frac{dE}{E} = \frac{N_U \cdot I_{a\text{eff}}}{\xi \Sigma_s} \quad (6.19)$$

Here $I_{a\text{eff}}$ is the effective resonance absorption integral for uranium in the given multiplying medium, $I_{a\text{eff}}$ has the meaning of a cross-section. Formula (5.35) corresponds to the homogeneous mixture of uranium and the moderator, where $\Sigma_a = \Sigma_{aU} + \Sigma_{a\text{mod}}$, and since the moderator absorption cross-section is negligible for the intermediate energy region, $\Sigma_a \approx \Sigma_{aU} = N_U \sigma_{aU}$. As for the scattering cross-section, it is equal to $\Sigma_s = \Sigma_{sU} + \Sigma_{s\text{mod}} \approx \Sigma_{s\text{mod}}$ when the concentration of uranium is small; $\xi \Sigma_s$ is the moderating power of the moderator (Sec. 5.1-5). In (6.19), all the quantities independent of energy, Σ_s including, are taken out of the integral sign. Taking into account the performed transformations, one obtains probability φ as

$$\varphi = e^{-\frac{N_U I_{a\text{eff}}}{\xi \Sigma_s}} \quad (6.20)$$

i.e., it is determined by the relation between the effective macroscopic absorption cross-section of uranium, $\Sigma_{aU}^I = N_U I_{a\text{eff}}$ and by the moderating power of the moderator.

2. Resonance integral. The number of neutrons being moderated in the unit energy interval near the energy E (in unit time in unit volume) and absorbed by the nuclei of the given sort with the $\sigma_a(E)$ cross-section is equal to $\Phi(E) \cdot \Sigma_a(E)$. Hence, the number of absorption acts, ν_a , for the entire energy range of the moderated neutrons is

$$\begin{aligned} \nu_a &= \int_{E_{th}}^{E_0} \Phi \Sigma_a dE = \int_{E_{th}}^{E_0} \frac{Q}{\xi \Sigma_s E} \Sigma_a dE = \frac{Q}{\xi \Sigma_s} N_a \times \\ &\quad \times \int_{E_{th}}^{E_0} \sigma_a \frac{dE}{E} = \frac{Q}{\xi \Sigma_s} N_a I_a = Q \frac{\Sigma_a^I}{\xi \Sigma_s} \quad (6.21) \end{aligned}$$

where it is supposed that the absorption is so small that the flux, Φ , may be written in the form of (5.24). The concentration of

atoms of the moderator in question is N_a , and the expression

$$I_a = \int_{E_{th}}^{E_0} \sigma_a(E) \frac{dE}{E} \quad (6.22)$$

is called resonance-absorption integral. Evidently, I_a is of the dimension of the microscopic cross-section, $N_a I_a = \Sigma_a'$ is a macroscopic cross-section of the absorber.

Resonance integral, (6.22), is, in essence, the absorption cross-section for a given substance averaged over the energy dependence of the Fermi spectrum (5.24), i.e., integrated with weight $1/E$. The neutron absorption is not only proportional to the cross-section but to the flux as well and the flux of the slowing-down neutrons is just proportional to $1/E$. Even if $\sigma_a(E)$ is considered to be constant within the entire range of moderation, the contribution to absorption will mainly be due to low energy neutrons the flux of which is high. Thus, the flux depends on energy even for a constant cross-section. Therefore, I_a characterizes the absorbing power with respect to slowing-down neutrons as a whole, just as $\sigma_{a\ th}$ is the absorbing power of the nuclei of the substance for the thermal energy region. True, the absorption of each of the neutrons decreases their number so that subsequently the same nuclei interact with a flux of lower intensity and the number of absorption acts reduces compared to the limiting number. This effect in the thermal energy region was already discussed (Sec. 6.4-3). Finally, the number of absorption acts is always equal to the product of the macroscopic cross-section and the flux which, with a given source, is in its turn determined by the macroscopic cross-section: the larger the cross-section, the weaker the flux. It should not be inferred from the above that the number of absorption acts decreases with the increase of the macroscopic cross-section. It is certain to increase but not in proportion to the macroscopic cross-section. The neutron absorber always prevents itself from absorbing neutrons most effectively. The greatest possible absorption per absorbing atom at a given energy is defined by the microscopic cross-section, σ_a . This absorption takes place when the macroscopic cross-section $\Sigma_a = N_a \sigma_a$ approaches zero, i.e., in the case of the maximum flux in the medium containing the absorber. The quantity I_a has the same meaning, but with respect to the slowing-down neutrons of all energies. The resonance integral describes neutron absorption within the Fermi spectrum nondistorted by the absorption (5.24). Evidently, the neutron flux is distorted in real media, especially in those involving strong resonance absorption. This leads to some decrease of the total number of absorption acts as compared to the utmost possible number. This decrease is usually

attributed to the absorption cross-section but not to the neutron flux which is considered to be constant. Corrected for the effect of the flux decrease, the resonance integral is called effective resonance integral, $I_{a\text{ eff}}$ (6.19), (6.26).

As follows from (6.22), the resonance integral, I_a , can be calculated for any given substance and expressed in barns if the $\sigma_a(E)$ dependence is known. The absorption cross-sections are measured for each energy value E , with the use of the flux not reduced by absorption in the sample under investigation. That is, these are real cross-sections. If the absorption in the sample turns out to be considerable, the observed cross-section decreases, and the final experimental result needs to be corrected in order to restore the real value of the cross-section. The resonance integrals are, naturally, small for substances with weak absorption. They are great if there are absorption resonances, giving the major contribution to the values of I_a which are called resonance integrals. The values of resonance integrals for some nuclides are presented below.

| Nuclide | I_a , bn |
|-----------------------------|------------|
| Ag ¹⁰² | 1160 |
| In ¹¹⁵ | 2640 |
| Au ¹⁹⁷ | 1558 |
| Th ²³² | 83 |
| U ²³⁸ | 280 |

Since the analytical form of $\sigma_v(E)$ is known within each resonance (4.41), one may express I_a in terms of the resonance parameters, if the latter are known. The resonance integral for a single resonance, I'_a , is

$$I'_a = \int \sigma_v(E) \frac{dE}{E} = \frac{\pi\Gamma}{2E_r} \sigma_{v0} \quad (6.23)$$

where E_r is the resonance energy, Γ is the total level width, and σ_{v0} is the maximum radiative capture cross-section (4.42). The integration in deriving (6.23) is carried out over all possible values of E , but in fact the integral relates to a sufficiently narrow energy region of the resonance peak. For a set of non-overlapping resonances the resonance integral is the sum of I'_a over all i resonances

$$I_a = \sum_i \frac{\pi\Gamma_i}{2E_{ri}} \cdot \sigma_{v0i} \quad (6.24)$$

Expression (6.21) was obtained in order to reveal the physical meaning of the resonance integral; it corresponds to the case of a very weak absorption. If v_a is known, by definition the probability

$$\varphi = \frac{Q - v_a}{Q} = 1 - \frac{N_a I_a}{\xi \Sigma_s} \quad (6.25)$$

which is in agreement with general expression (6.20). In the case of weak absorption, $l_{a\text{ eff}} = l_a$, and $N_a I_a \ll \xi \Sigma_s$, since the macroscopic cross-section of the absorber, $\Sigma_a^I = N_a I_a$, is close in this case to zero. Therefore, the exponent in (6.20) is expressed by the first two terms of the series expansion, which coincide with (6.25). General formula (6.20) describes ϕ for any absorption.

3. Homogeneous medium. In the thermal region, the real neutron flux is the less, the greater the macroscopic absorption cross-section of the medium (Sec. 6.4-3). Within the intermediate energy region, neutrons possess the given energy only till the next collision after which they are either absorbed or scattered inelastically

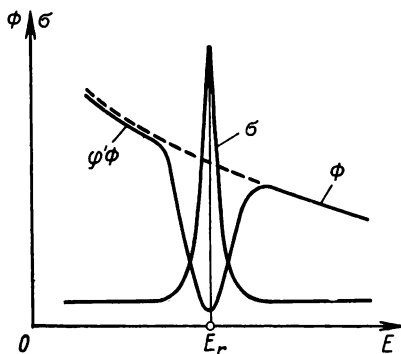


Fig. 6.5. Neutron flux with nearly resonance energy

with the decrease of their energy. Hence, the absolute value of the flux for the intermediate region is defined by the value of the total macroscopic cross-section, $\Sigma_t = \Sigma_a + \Sigma_s$, not by Σ_a . The flux of the slowing-down neutrons, (5.24), is inversely proportional to energy only when $\Sigma_t \approx \Sigma_s$ and is constant within the entire range of moderation. If for a certain energy the cross-section increases, the flux immediately falls (Fig. 6.5) just by the factor of the cross-section relation, $\Sigma_s/(\Sigma_s + \Sigma_a)$, where Σ_s in the numerator is, in

fact, the total cross-section for the non-resonance energies. But there, $\Sigma_a \ll \Sigma_s$, and the value of the above factor is elucidated by the following arguments. For a given total number of the slowing-down neutrons, the flux for any energy E is determined by the path length $l_s = 1/\Sigma_s$ (1.59), which the average neutron passes before being scattered. If at some energy, the cross-section sharply increases, decreased is the path of the average neutron before collision, $l_t = 1/(\Sigma_s + \Sigma_a) \ll l_s$. This means that the flux at any point of the medium is formed by a smaller number of the slowing-down neutrons. Neutrons of non-resonance energies reached a given point from all the distances up to l_s . For the case of resonance energies, the distance cannot exceed $l_t \ll l_s$. If the average-neutron path is now $l_t/l_s = \Sigma_s/(\Sigma_s + \Sigma_a)$ times smaller, the same factor decreases the number of the slowing-down neutrons which reaches the given point from adjacent regions, i.e., the flux is $\Sigma_s/(\Sigma_s + \Sigma_a)$ times decreased. It is this factor of the flux decrease that appears in expression (6.19) for the homogeneous-medium effective reso-

nance integral:

$$I_{a\text{ eff}} = \int_{E_{th}}^{E_0} \sigma_a \left(\frac{\Sigma_s}{\Sigma_s + \Sigma_a} \right) \frac{dE}{E} = \int_{E_{th}}^{E_0} \sigma_{a\text{ eff}} \frac{dE}{E} \quad (6.26)$$

As was mentioned above, the affect of the absorption decrease due to the flux decrease is related to the cross-section which for a given energy is taken to be $\sigma_{a\text{ eff}} = \sigma_a \left[\frac{\Sigma_s}{\Sigma_s + \Sigma_a} \right]$. Averaging the effective cross-section $\sigma_{a\text{ eff}}$ over the non-distorted moderation spectrum one obtains the effective resonance integral, $I_{a\text{ eff}}$. Evidently, the value $I_{a\text{ eff}}$ is not only determined by the absorption cross-section of the absorbing material $\sigma_a(E)$, but also by the ratio of the macroscopic cross-sections of scattering, Σ_s , and of absorption, Σ_a of a medium as a whole. If $\Sigma_a \ll \Sigma_s$, then $I_{a\text{ eff}} \approx I_a$ but with $\Sigma_a \gg \Sigma_s$ $I_{a\text{ eff}}$ is much smaller than I_a .

It follows from these arguments that the flux decrease at some energy is defined by the total cross-section and not only by the absorption cross-section. The flux also falls when only the scattering cross-section increases. On the contrary, if the scattering cross-section decreases, which is, for example, observed at the energy somewhat lower than the resonance one in the case of the interference of resonance and potential scattering, then the flux increases as compared to the flux which corresponds to the non-distorted moderation spectrum. After the energy has passed the resonance, the cross-section is restored to Σ_s and the neutron flux is restored correspondingly. However, this flux is the φ' th part of the flux in the case of sub-resonance energies (φ' being the resonance escape probability for the given resonance) (see Fig. 6.5). Except for the lowest level resonances, the φ' values are close to unity because the neutron energy loss due to a collision with a moderator nucleus, ξE , increases and the resonance absorption cross-section decreases with the increase of the energy E .

4. Experimental data. The measurement of the effective resonance integrals for homogeneous multiplying media has shown that their values do not depend upon the nature of the moderator, but are only defined by the ratio Σ_s/N_U where $\Sigma_s = N_U\sigma_{sU} + N_{mod}\sigma_{smod}$, i.e., by the barn number of the scattering cross-section per uranium atom. Within the range of the cross-sections lying between 10 (pure metallic uranium without moderator) and 1000 barns per uranium atom, the experimental data are presented by the expression

$$I_{a\text{ eff}}^{\text{hom}} = 3.8 \left(\frac{\Sigma_s}{N_U} \right)^{0.42} \quad (6.27)$$

where $I_{a\text{eff}}^{\text{hom}}$ is in barns, if Σ_s/N_U also given in barns. For Σ_s/N_U greater than 1000 barns, the experimental data are presented in a graphical form [27]. The intrinsic resonance integral of uranium $I_{aU} = 280$ barns (Sec. 6.5-2) while for pure metallic uranium $I_{a\text{eff}} \approx 10$ barns, which is obtained after substituting $\Sigma_s/N_U = \sigma_{sU} = 10$ barns into (6.27), that is, the effective resonance integral of uranium in the ultimate case of uranium without multiplying medium is 28 times smaller than the real resonance integral, I_{aU} . Such a strong decrease of $I_{a\text{eff}U}$ with the increase of uranium concentration in the multiplying medium essentially affects the probability value ϕ (6.20). Here, ϕ certainly decreases as a whole, since ϕ directly depends on the uranium concentration, N_U . However, the decrease of $I_{a\text{eff}}$ which accompanies the increase of the uranium content slows down the decrease of the probability ϕ and the exponent in (6.20) proves to be dependent upon N_U/Σ_s to the power less than one as $(N_U/\Sigma_s)^{0.58}$, within the validity of (6.27).

The measurement of the effective resonance integrals in homogeneous media is based on the detection of the neutron flux attenuation at the resonance energies. If uranium is the absorber, the flux in the bulk of a homogeneous mixture of uranium and moderator is attenuated only for the energies corresponding to the uranium resonances. If a thin uranium plate covered with cadmium to avoid the uranium activation by neutrons is introduced into this medium, its activation will be proportional to the attenuated flux or to the effective resonance integral, $I_{a\text{eff}U}$. A plate made of some other material with resonance capture, e.g., Au, In, covered with cadmium will be activated in the same medium by an unattenuated flux of slowing-down neutrons since different materials have different resonance energies. As a result, the activity of the standard \mathcal{J}_{st} appears to be proportional to its actual resonance integral, I_{ast} , which is supposed to be known. Since uranium and the standard specimens are irradiated by the neutrons of the same source, the following relation is valid between the induced activities and the resonance integrals: $\mathcal{J}_U/\mathcal{J}_{st} = I_{a\text{eff}U}/I_{ast}$. Hence, the effective resonance integral for uranium in the medium with a given uranium-to-moderator ratio is determined from the value of the measured activities.

5. Heterogeneous medium. The following expression analogous to (6.20) holds for the resonance escape probability in the heterogeneous medium ϕ_{het}

$$\phi_{het} = e^{-\frac{N_U V_U I_{a\text{eff}}^{\text{het}}}{\xi \Sigma_s V_{mod}}} \quad (6.28)$$

This expression also depends upon the uranium-to-moderator ratio in the multiplying medium presented as N_U/Σ_s in (6.20) and $N_U V_U/\Sigma_s V_{mod}$ in this case (Sec. 6.4-4), and upon the effective re-

sonance integral. The heterogeneous effective resonance absorption integral $I_{a\text{eff}}^{\text{het}}$ bears the same meaning of microscopic absorption cross-section responsible for the neutron absorption over the whole moderation range but corrected for the neutron flux attenuation on account of absorption. The interpretation of the physical meaning of $I_{a\text{eff}}^{\text{het}}$ cannot be as simple as in the case of the homogeneous medium because the characteristics of the energy distribution of the neutron flux in the heterogeneous medium are superposed by the characteristics of the spatial distribution over the volume of the unit cell. The moderator has no uranium in the heterogeneous medium and, therefore, the flux of delayed neutrons in the moderator volume is not attenuated at any energies. In the uranium slug, the flux of neutrons of the energy which does not coincide with the resonance energies is not attenuated either. But the resonance-neutron fluxes are subject to strong attenuation. In the central regions of uranium slugs this attenuation corresponds to attenuation in the homogeneous medium consisting of pure uranium where value of the homogeneous effective resonance integral is extremely low. This characteristic increases φ_{het} compared to φ_{hom} . With the same uranium-to-moderator ratio, the effective resonance integral appears to be considerably lower in the heterogeneous medium than in the homogeneous one, approaching its minimum value in the medium with no moderator. This is achieved with the uranium rod diameters of about 1 cm.

The resonance neutron distribution over the volume of the unit cell is rather complicated. The matter is that the effective resonance integral must reflect all the distribution characteristics for all intermediate energies, the cross-sections at the resonance maxima are not the same and in the limits of each resonance the absorption cross-section continuously changes from very large value at the resonance energy E_r , practically to zero beyond the resonance band. The character of the flux attenuation over the rod diameter is determined by the absolute value of the absorption cross-section at a given energy. The greatest contribution to the resonance absorption is, however, made by the low-lying U^{238} capture resonances and also at energies close to E_r of each resonance band where the capture cross-section is the greatest. Therefore, it is the distribution of strongly absorbed neutrons that first of all determines the nature of the heterogeneous effective resonance integral. Fig. 6.6 shows the scheme of the distribution of neutrons with some resonance energy E_r . As was already mentioned earlier (Sec. 6.4-3), the moderator is the source of neutrons with the intermediate or thermal energies. Resonance neutrons produced in the moderator penetrate into the bulk of the uranium slug. But because of their strong absorption they vanish in the very first

tenths of a millimetre, i.e., in the thin surface layer. There would be no resonance neutrons inside the uranium slug at all if uranium did not moderate neutrons completely. Since above-resonance neutrons freely pass through the uranium slug, and the drop of the energy, $\xi_U E$, during the collision of the neutron with the uranium nucleus is comparable to the resonance width, some small number of neutrons acquire the energy E_r in the uranium slug directly. It produces a uniform resonance neutron flux in the centre of the slug.

In comparison with the non-attenuated above-resonance flux, this equilibrium flux with the energy E_r is attenuated to the same

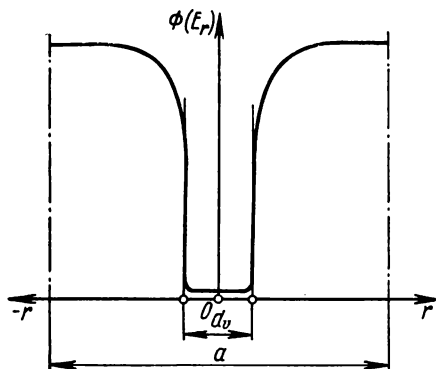


Fig. 6.6. Resonance neutron distribution in unit cell

extent as the flux with this energy is in an infinite bulk of uranium, i.e., in the homogeneous medium with no moderator. In this case, the neutron absorption is described by the minimum value of the effective homogeneous resonance integral (6.27). If the same quantity of uranium and moderator as that in the unit cell of the heterogeneous medium was uniformly mixed, the resonance absorption integral would be much greater as it would be determined by the relation N_U/Σ_s

of the mixture but not of pure uranium. The neutron absorption by the surface layers of the uranium slug takes place in the flux coming from the moderator and not attenuated by absorption and, hence, with the ultimate value of the resonance integral, i.e., it is more effective than even in the homogeneous medium. This surface portion of the total absorption may, however, be reduced by increasing the uranium rod diameter since the surface-to-volume ratio decreases with the increase of the body volume. Thus, the increase of the uranium rod diameter $I_{a\text{eff}}^{\text{het}}$ decreases and, consequently, ϕ_{het} increases.

It should, of course, be borne in mind that the increase of the uranium rod diameter simultaneously results in some decrease of θ (Sec. 6.4-4). And since the purpose is usually to obtain as high value of k_0 as possible in which ϕ and θ are the products, there must be an optimum diameter d_U and an optimum lattice spacing a . If near the optimum, both ϕ and θ change as a function of d_U with almost the same velocity, the condition of the maximum value of the product $\phi\theta$ is $\phi = 0$. The value k_0 is, however, strongly in-

fluenced by the factor μ as well (Sec. 6.6), which also depends on the uranium rod diameter.

6. Heterogeneous resonance integral. As far as the mode of the resonance neutron distribution over the unit cell volume is concerned, the heterogeneous effective resonance integral is usually taken to be made up of two parts. One of them describes the neutron absorption over the bulk of the uranium slug with the minimum value of the homogeneous resonance integral, and the other, the surface absorption some fraction of which depends upon the ratio of the uranium rod surface to the volume, i.e., to its mass. The values of the heterogeneous integrals are obtained experimentally by studying the mode of the distribution of the resonance neutron absorption over the uranium rod radius.

Each neutron absorption in U^{238} is accompanied by the production of the radioactive U^{239} , so that the U^{239} radioactivity distribution over the bulk of the slug is the resonance capture distribution if the uranium slug was protected by cadmium coating against thermal neutron capture. The experimental results are interpreted in conformity with the above in the form of relations of the type

$$I_{a\text{ eff}}^{\text{het}} = 8.0 + 27.5 \frac{S}{M} \quad (6.29)$$

where $I_{a\text{ eff}}^{\text{het}}$ is given in barns if the lateral surface area of the uranium slug S is taken in square centimetres, and its mass M , in grams. Expression (6.29) relates to metallic uranium and the first term in the sum is an approximate value of $I_{a\text{ eff}}^{\text{hom}} U$ in pure uranium. At the same time, the simplest interpretation of $I_{a\text{ eff}}^{\text{het}}$ given above is not comprehensive or absolutely correct. The delayed neutron distributions over the unit cell volume are complicated, and the ratio of the number of absorptions to the intensity of the neutron source, i.e., the ratio of the attenuated flux to the non-attenuated one, is less definite than for the homogeneous medium since its derivation is complicated by space diffusion of resonance neutrons. Therefore, experimental formulae of the type of (6.29) are valid in narrow ranges of the lattice parameter changes. In wider ranges, data on $I_{a\text{ eff}}^{\text{het}}$ are applicable, also expressed through the sum of the volume and surface terms, but with the surface term proportional to $(S/M)^{1/2}$. In this case, both the volume and the surface terms have no clear physical interpretation and are mere experimental components of $I_{a\text{ eff}}^{\text{het}}$ [27].

7. Blocking effect. The small contribution of the inner part of the uranium rod to resonance absorption is explained by the fact that the moderator is a resonance neutron source and the overwhelming majority of neutrons penetrate into uranium due to diffusion. A very large absorption cross-section at the resonance

energy results in the flux absorpton in the thin surface layer so that neutrons do not reach the inner regions. Otherwise stated, screening the inner volume of uranium by its surface layer is the reason why it does not take part in resonance capture. Therefore, the increase of φ in the heterogeneous medium in comparison with the homogeneous one is due to the surface blocking effect.

If the uranium rod material is not pure uranium but a mixture of it with the moderator, e. g., UO_2 , then Σ_s/N_U inside the rod increases and $I_{a\text{ eff}}^{\text{hom}}$ is increased correspondingly (6.27), being the volume component of $I_{a\text{ eff}}^{\text{het}}$ (6.29). Hence, the presence of the moderator in the uranium slug increases $I_{a\text{ eff}}^{\text{het}}$ and decreases φ_{het} as compared with the case of a slug of pure metallic uranium. The presence of the moderator in the slug means the occurrence of resonance neutron sources distributed over its volume, i.e., additional absorption of neutrons by uranium. Surface blocking is, in part, reduced in this case since resonance neutrons absorbed by uranium do not only traverse the slug surface but are also produced in it.

6.6. Fast Multiplication Factor

1. **Homogeneous medium.** Some of fission-produced neutrons have an energy higher than the energy of the U^{238} fission threshold. However, after several collisions with the nuclei of the moderator atoms the energy of all neutrons becomes lower than the threshold and the fission of U^{238} proves to be impossible. Therefore, multiplication of fast neutrons due to U^{238} fission is practically observed only when a high probability of the first collision of the fast neutron with the U^{238} nucleus exists. Since in homogeneous multiplying media the number of uranium atoms is low compared to the number of the moderator atoms (see Table 6.4), the probability in question is near zero, fission of U^{238} does not practically take place and the fast multiplication factor in the homogeneous medium is $\mu_{\text{hom}} = 1$.

2. **Heterogeneous medium.** The maximum value of the fast multiplication factor μ_0 is to be expected in an infinite medium (consisting of U^{238} or, which is practically the same, of natural uranium) where each collision of the neutron with the nucleus is collision with the U^{238} nucleus. Even in such a medium, however, μ is not very high due to the efficient slowing-down of fast neutrons in inelastic scattering by the same U^{238} nuclei (Sec. 6.1-8). The ultimate value is $\mu_0 = 1.28$.

In the heterogeneous medium, fission neutrons are produced in a uranium slug. Before entering the moderator, they pass a considerable distance through pure uranium. It means that the probability of the first and even of several first collisions with U^{238} nuclei

in the heterogeneous medium is sufficiently high. The value of the multiplication factor μ in a certain heterogeneous medium is determined both by the partial cross-sections of U^{238} with above-threshold energies and, first and foremost, by the mean path length of the neutron in the uranium slug. Since elastic scattering of fast neutrons by uranium is not spherically symmetrical (Sec. 4.5-7), in calculating the factor μ , one does not make use of the uranium scattering cross-section but of the transport cross-section (5.39) corrected for the scattering symmetry. This cross-section is responsible for the deflection of the trajectory of the motion of the neutron after scattering by the angle $\pi/2$. And under such assumption, one calculates the mean path across the uranium slug. Just as in Table 6.2, the remaining partial cross-sections should, however, be averaged over the fission-neutron spectrum with above-threshold energies, i.e., with $E > 1$ MeV. Calculations [24] show that in cylindrical uranium slugs of the diameter $d_U = 1-4$ cm, the fast neutron multiplication factor in the heterogeneous medium μ_{het} is roughly

$$\mu_{het} \approx 1 + 0.01d_U \quad (6.30)$$

where d_U is given in centimetres. In a graphite reactor, for instance, $d_U = 3$ cm, $\mu \approx 1.03$ and makes a significant contribution to k_0 .

3. Close-spaced lattices. Heterogeneous multiplying media with light water as moderator have small lattice spacings. Such lattices are called close-spaced. Water is a perfect moderator, hence, the smallest volume of water, as compared to the volumes of any other moderators, is required to moderate neutrons. On the other hand, water, among other moderators, absorbs thermal neutrons most strongly. Therefore, to obtain the desired thermal utilization factor, less water should be used than would be required under the conditions of most efficient moderation. In the run, k_0 reaches its maximum at a very low uranium-water volume relation, which results in a close-spaced lattice. In close-spaced lattices, one observes an increase of the coefficient μ due to the increase of the efficient path travelled by the fast neutron in uranium till the first collision with the nucleus of the hydrogen atom. The fast neutron, which has been produced in the bulk of some uranium rod and has left it, will, with great probability, traverse a thin layer of the moderator without collision and will enter the next uranium slug, then the next one and so on. This penetration of fast neutrons into the neighbouring slugs is favoured by the low value of the hydrogen scattering cross-section in the fast region (see Fig. 4.23). The high moderating power of water is determined by the abnormally large hydrogen scattering cross-section in the intermediate energy region where $\sigma_s = 20$ barns. However, even with $E = 1$ MeV, $\sigma_s = 4$ barns and the moderating power of water in the fast region is much

lower than in the intermediate one. Thin water layers are sufficiently transparent for fast neutrons. In the run, the coefficients of μ in the uranium-water lattices appear to be much higher than the values of μ of the heterogeneous media with other moderators although the diameters of the uranium rods are usually smaller here. The coefficient $\mu = 1.06$ for the water-to-uranium volume ratio $(V_{mod}/V_U) = 2$ and $d_U = 1.5$ cm, and, with $V_{mod} = V_U$, it even reaches the value 1.1.

6.7. Optimum Parameters of Multiplying Media

1. **Homogeneous medium.** On account of rather strong absorption of intermediate neutrons by the resonances of U^{238} , the maximum values of k_0 in homogeneous multiplying media are achieved with a very low uranium content in the mixture. Table 6.4 gives

Table 6.4

Optimum Parameters of Homogeneous Media (natural uranium)

| Moderator | N_{mod}/N_U | k_0 |
|------------------|---------------|-------|
| H ₂ O | 2.5 | 0.84 |
| D ₂ O | 167 | 1.14 |
| Be | 193 | 0.8 |
| C | 452 | 0.85 |

the corresponding data for natural uranium. It gives optimum ratios of molecular concentrations, when the moderator is a chemical compound, or optimum ratios of the atoms of the moderator and uranium. If the moderation of thermal neutrons by the moderator is very low, the reduction of the resonance capture is achieved by diluting uranium. The reduction of the concentration of N_U increases φ (6.20) but the decrease of N_U simultaneously results in the

increase of $I_{a\text{eff}}^{hom}$ (6.27) which hinders the increase of φ . As a result, the ratio of the atomic concentrations of the moderator and uranium corresponding to the maximum of k_0 proves to be very high. And it is the higher, the lower ξ of the moderator since ξ decreases with the decrease of φ (6.20). The increase of the N_{mod}/N_U ratio results in the increase of φ and in the decrease of θ since φ is the non-absorption probability, and θ is, on the contrary, the probability of absorption of neutrons by uranium. The optimum concentration ratio, therefore, shifts to the lower values of N_{mod}/N_U with the increase of the absorption of thermal neutrons by the moderator. This ratio is lower in the case of berillium than in that of heavy water since the D₂O molecule has two atoms of deuterium, and it is quite low in the case of light water which, among moderating materials, is distinguished by its rather strong absorption of thermal neutrons.

Most interesting are the values k_0 of homogeneous media of optimum compositions. It is only in the case of heavy water that $k_0 > 1$ and, hence, in the homogeneous mixture of natural uranium, it is possible to achieve the critical state only with D_2O . Homogeneous mixtures of all the other moderators with natural uranium have $k_0 < 1$, so that the self-sustaining chain reaction does not prove feasible in such media. It may be achieved only in mixtures with enriched uranium. However, for berillium, and, which is most significant, for carbon (graphite) and natural uranium $k_0 > 1$ may be achieved in a heterogeneous multiplying medium.

2. **Heterogeneous medium.** The increase of φ due to the blocking effect in the heterogeneous medium as compared to the homogeneous one and the increase of the coefficient μ cause a significant increase of k_0 in spite of some decrease of θ . The very high values of the optimum concentration ratios N_{mod}/N_U in homogeneous media are determined by strong resonance absorption. The blocking effect increases φ and decreases θ , and both these tendencies result in the decrease of the optimum ratio N_{mod}/N_U . The latter fact is also a favourable physical characteristic of the heterogeneous multiplying medium. Table 6.5 lists the optimum parameters of heterogeneous media with natural uranium. Here, N_{mod}/N_U is an optimum ratio of the numbers of molecules or atoms of the moderator to the number of uranium atoms in the unit cell, i.e., as usual, the moderator-to-uranium ratio in the multiplying medium, V_{mod}/V_U is the corresponding moderator-to-uranium volume ratio, and k_0 is the maximum factor of multiplication in an infinite medium. The last two columns of Table 6.5 present the optimum diameters of uranium rods, d_U , and the lattice spacings, a .

Table 6.5
Optimum Parameters of Heterogeneous Media
(natural uranium)

| Moderator | N_{mod}/N_U | V_{mod}/V_U | k_0 | d_U , cm | a , cm |
|-------------|---------------|---------------|----------|---------------|----------|
| Light water | 1.4 | 2 | ≤ 1 | 1.5 | 2.5 |
| Heavy water | 20 | 30 | 1.2 | 3 | 15 |
| Graphite | 80 | 50 | 1.08 | 3 | 20 |

Although in a uranium-graphite lattice, k_0 exceeds but slightly unity, it is, however, quite sufficient for the critical state to be reached. Since graphite is a more available material than heavy water, the majority of reactors operating with natural uranium are uranium-graphite reactors. In its optimum version, the natural

uranium-berillium lattice also has k_0 somewhat higher than unity. The high cost of berillium, however, makes it non-competitive as compared to graphite since berillium practically has no advantages over graphite in the value of k_0 . Berillium as a moderator is only used in special-purpose reactors with highly enriched uranium, in which the cores are small and the leakage of neutrons is high (fast neutrons included, which are more successfully moderated by berillium than by light water, light water being transparent enough for fast neutrons). Heavy water, though expensive, is, however, used in heterogeneous reactors since its incomparable advantage over all other moderators is that, with its high moderating power, it absorbs but slightly thermal neutrons.

In the heterogeneous natural uranium-light water system, k_0 approaches unity. The natural concentration of U^{235} , 0.7%, is considered to be just an ultimate value at which $k_0 = 1$ in the optimum uranium-water lattice. It is possible that in a sufficiently large volume the critical state may be achieved. But it cannot be of any practical use. With uranium enriched only up to 1%, $k_0 = 1.12$, and the critical state is readily achievable. Here again, most significant is a very low relative concentration of water and uranium in the heterogeneous multiplying medium, which is, first and foremost, due to the high moderating power of water. Uranium-light water thermal reactors contain the lowest quantity of additional moderating material and this is their main advantage over other thermal reactors. The cores of such reactors are most compact. But unfortunately, they operate only on enriched uranium.

6.8. Reactor Critical State

1. **Critical size.** The critical state is characterized by that the neutron multiplication factor, k , is equal to unity (6.1). In a medium of a finite volume, on condition that $k_0 > 1$, the critical state is achieved by changing the relation between leakage and neutron absorption. The relation between the numbers of neutrons absorbed and those which have left the multiplying medium depends upon the character of spatial neutron distributions taking place in the process of diffusion. Neutron distributions are, in their turn, determined by the geometrical form and size of the body in which the chain reaction occurs, and by the value of k as well. Achieving the critical state in media with the given k_0 is, hence, reduced to finding such a size of a body of a certain geometrical form to which the neutron distributions ensuring the required fraction of neutrons ω (6.1) remaining in the volume of the multiplying medium conform. This size is critical, and its expression through the diffu-

sion constants and the multiplication factor k_0 follows from the solution of the diffusion equations.

2. Diffusion equations. In the general case of the multiplying medium, the thermal neutron flux Φ and the slowing-down density q are time-dependent. An exception is the critical state in which the neutron flux remains constant. Hence, the condition equivalent to $k = 1$ is

$$\frac{\partial \Phi}{\partial t} = 0 \quad (6.31)$$

It stands to reason that the slowing-down density is not time-dependent in this case either. It means that the consideration of the stationary diffusion equations in the multiplying medium implies in advance the geometrical size of the body to be critical.

The diffusion equations of slowing-down (5.69) and thermal (5.43) neutrons in the assumption of (6.31) are of the form:

$$\Delta q(\mathbf{r}, \tau) = \frac{\partial q(\mathbf{r}, \tau)}{\partial \tau} \quad (6.32)$$

$$D\Delta\Phi(\mathbf{r}) - \Sigma_a\Phi(\mathbf{r}) + \varphi q(\mathbf{r}, \tau_{th}) = 0 \quad (6.33)$$

The former equation corresponds, as before, to the case with no absorption in slowing-down. And the resonance capture is allowed for by the factor φ in writing down the term of thermal neutron sources in equation (6.33). Since the function Φ is not presented in equation (6.32), the set of equations (6.32), (6.33), in fact, described neutron diffusion for one cycle. To close the cycles following one another, it is necessary to consider the birth of fast neutrons in absorption of thermal ones. The number of thermal neutrons absorbed in unit time in unit volume at the point with the coordinates \mathbf{r} is $\Sigma_a\Phi$. The portion θ is absorbed by uranium and gives η fast neutrons per each absorption. Upon multiplying μ times in the fast region on account of U^{238} fissioning at the point \mathbf{r} , moderation of $\Sigma_a\Phi\theta\eta\mu$ neutrons of the new generation is initiated. The energy of these neutrons is E_0 and, hence, their age is $\tau = 0$ (5.68). The number of the neutrons of the second generation beginning their slowing-down is the initial condition along the coordinate of the slowing-down density function q which, taking into account (6.7), takes the following form:

$$q(\mathbf{r}, 0) = \frac{k_0}{\varphi} \Sigma_a\Phi(\mathbf{r}) \quad (6.34)$$

In conjunction with (6.34), the set of equations (6.32), (6.33) describes continuous cycle and diffusion of neutrons in the critical reactor. Its solutions are the functions of distribution of thermal, $\Phi(\mathbf{r})$, and slowing-down, $q(\mathbf{r}, \tau)$, neutrons over the reactor volume.

Analytical solutions of partial differential equations (6.32), (6.33), are usually obtained by the method of separable variables which results in some ordinary differential equations with known solutions. This method, however, assumes that the solution as a function of one variable is independent of the remaining variables. In the particular case, for instance, the τ coordinate distribution of the slowing-down density should not depend upon any space coordinate (x, y, z) , and distributions of neutrons along each space coordinate should not depend either upon the remaining space coordinates or upon the coordinate τ . These requirements are, in fact, met only in a limited number of physical problems.

For space variables to be separated, the geometrical form of the body should meet certain requirements of symmetry, and the heterogeneity of the medium is only permissible in the direction of one space coordinate. Along other space coordinates, the medium must be homogeneous and either infinite (then Φ and q at once prove to be dependent upon one space coordinate only) or finite but bordering on vacuum where the boundary conditions are zero (Sec. 5.3-6). With the zero boundary conditions, solutions in finite volumes are represented by periodical functions vanishing at the boundaries and being of some physical sense only within the volume of the body. In the case involved, in addition to the space coordinates, the energy coordinate τ is to be separated. It is possible only if the intensity of the fast neutron sources at each point of the body volume is proportional to the number of slowing-down neutrons of any energy. The energy spectrum of slowing-down neutrons is the same over the entire volume in spite of diffusion in slowing-down. This picture is observed in a homogeneous multiplying medium bordering on vacuum only, i.e., in a homogeneous reactor without a neutron reflector. If a body possessing some other diffusion or multiplying properties borders on the multiplying medium, then there are no analytical solutions of equations (6.32), (6.33). Solutions are obtained by approximate methods [24, 26]. The definition of the critical size is most easily illustrated with the help of the known analytical solution.

3. Homogeneous reactor without reflector. The function Φ does not enter into equation (6.32). The solution of the set of (6.32), (6.33) is to successively find solutions of (6.32) and (6.33), and to connect them with the help of condition (6.34). In the method of separable variables the function of many variables is represented by the product of functions of one variable

$$q(\mathbf{r}, \tau) = R(\mathbf{r}) T(\tau) \quad (6.35)$$

with $R(\mathbf{r})$ also implying the product of several functions in, say, the Cartesian coordinates

$$R(\mathbf{r}) = X(x) \cdot Y(y) \cdot Z(z) \quad (6.36)$$

Substituting (6.35) into (6.32) gives

$$T\Delta R = R \frac{dT}{d\tau} \quad (6.37)$$

or

$$\frac{\Delta R}{R} = \frac{1}{T} \cdot \frac{dT}{d\tau} = -\kappa^2 \quad (6.38)$$

Since R does not depend upon τ , and T does not depend upon the space coordinates, expression $\Delta R/R$ may be equal to $(1/T) \times (dT/d\tau)$ only in the case if each of these expressions with any \mathbf{r} and τ , always gives the same constant number denoted by $-\kappa^2$. The form of solutions of the ordinary differential equations, derived from (6.38), depends upon the algebraic sign in front of the constant κ^2 (Sec. 5.3-8). The conditions of the present physical problem are only met by solutions corresponding to the minus sign in front of κ^2 in (6.38).

Expression (6.38) comprises two differential equations

$$\Delta R + \kappa^2 R = 0 \quad (6.39)$$

and

$$\frac{dT}{d\tau} = -\kappa^2 T \quad (6.40)$$

The solution of the former will be given below and the solution of the latter is

$$T(\tau) = T_0 e^{-\kappa^2 \tau} \quad (6.41)$$

where T_0 is an integration constant determined by the initial conditions along the coordinate τ and equal to $T_0 = T(0)$. The necessity of the minus sign in (6.38) follows from (6.41). All fast neutrons are supposed to be produced with the energy E_0 and, therefore, in the process of slowing-down, the number of neutrons and, hence, the slowing-down density cannot increase. The number of slowing-down neutrons can only decrease, which is in agreement with (6.41). Leakage out of the finite volume is the cause of the decrease of the number of neutrons in slowing-down without absorption.

It follows from (6.34) that the intensity of the fast neutron sources, $q(\mathbf{r}, 0)$, is proportional to the thermal neutron flux, $\Phi(\mathbf{r})$, which in the case of separation of variables means the proportionality is preserved during the entire process of slowing-down over the whole volume. Since the intensity of the thermal neutron sources is expressed through $q(\mathbf{r}, \tau_{th})$, it must be proportional to $\Phi(\mathbf{r})$. The third term in equation (6.33), with (6.35), (6.41) and (6.34) taken into account, is, actually,

$$\varphi q(\mathbf{r}, \tau_{th}) = \varphi R(\mathbf{r}) T_0 e^{-\kappa^2 \tau_{th}} = k_0 \Sigma_a e^{-\kappa^2 \tau_{th}} \cdot \Phi(\mathbf{r}) \quad (6.42)$$

because $R(\mathbf{r})T_0 = q(\mathbf{r}, 0)$, and substituting in into (6.33) yields the following equation

$$D \Delta \Phi(\mathbf{r}) - \Sigma_a \Phi(\mathbf{r}) + k_0 \Sigma_a e^{-\kappa^2 \tau_{th}} \cdot \Phi(\mathbf{r}) = 0 \quad (6.43)$$

It turns out that in the critical reactor, $k_0 e^{-\kappa^2 \tau_{th}} > 1$ (see below), and equation (6.43) with respect to $\Phi(\mathbf{r})$ appears to be an equation of the same type as (6.39) with respect to the spatial part of $q(\mathbf{r}, \tau)$. Furthermore, if (6.34) is valid throughout the whole volume of the reactor, then the function $\Phi(\mathbf{r})$ merely satisfies equation (6.39), i.e., an equation with the same numerical value of κ^2 that is in (6.39):

$$\Delta \Phi + \kappa^2 \Phi = 0 \quad (6.44)$$

This can be proved by directly substituting $R(\mathbf{r})$, expressed with the help of (6.35) and (6.34) through $\Phi(\mathbf{r})$, into equation (6.39). The conclusion drawn attests that the spatial distributions of thermal and slowing-down neutrons in the homogeneous reactor without reflector are the same. They are determined by equation (6.44) or (6.39) and by the size of the body in which the chain reaction is taking place. The size is not yet known but the constant κ^2 is expressed through it in choosing the solutions which satisfy equation (6.44) and meet the zero boundary conditions. Thus, the constant κ^2 in equation (6.44) is in its implicit form the critical size of the reactor without reflector. It is evident that if the constant κ^2 is expressed through the coefficient of the initial equations, then the critical size proves to be directly related to the diffusion constants and to the value k_0 , i.e., dependence will be established between the properties of the multiplying medium and the critical size. It will just be the solution of the problem of determining the critical size of the reactor without reflector.

4. **Critical equation.** If equation (6.43) is presented in the form of (6.44), i.e., as

$$\Delta \Phi + \left[\frac{k_0}{L^2} e^{-\kappa^2 \tau_{th}} - \frac{1}{L^2} \right] \Phi = 0 \quad (6.45)$$

where $L^2 = D/\Sigma_a$ is the square of thermal neutron diffusion length in the multiplying medium, and if the coefficients of Φ in (6.44) and (6.45) are equated, then, upon transformations one obtains the relation

$$\frac{k_0 e^{-\kappa^2 \tau}}{1 + \kappa^2 L^2} = 1 \quad (6.46)$$

Here and henceforth, τ omits its subscript "th" since τ will always imply the age of thermal neutrons. Equation (6.46) relates the material characteristics of the medium, k_0 , τ and L^2 to the critical

size of the reactor without reflector, implicitly represented by the parameter κ^2 , and is termed critical equation. The parameter κ^2 as solution of the critical equation is called the material buckling of the multiplying medium (κ_m^2).

Equation (6.46) is, in its sense, identical to relation (6.1) and expresses the multiplication factor of the reactor, k , through the constants of the medium and through the critical size. It is easy to make certain that the coefficients of k_0 in (6.46) are some portions of intermediate, w_{in} , and thermal, w_{th} , neutrons which have escaped leakage in the process of diffusion. In the infinite medium, by the definition of k_0 , $k_0\Phi\Sigma_a$ new thermal neutrons are produced per each $\Phi\Sigma_a$ neutrons absorbed in the thermal region upon the conversion cycle. But in the finite medium, the term of the thermal neutron sources differs from this value by the factor $e^{-\kappa^2\tau}$ (6.42) and the only cause of this difference is leakage, i.e.,

$$w_{in} = e^{-\kappa^2\tau} \quad (6.47)$$

The fraction of thermal neutrons which have escaped leakage, w_{th} , is the ratio of the number of neutrons absorbed over the whole volume of the multiplying medium to the total number of thermal neutrons which have been absorbed in the medium and have escaped from the medium in diffusion. In the vicinity of any point of the reactor, \mathbf{r} , in unit volume in unit time, $\Phi\Sigma_a$ neutrons are absorbed. The decrease due to diffusion out of the same volume in unit time is $\text{div } \mathbf{j} = -D\Delta\Phi$ (Sec. 5.3-5). In the critical reactor without reflector, neutron distribution may only be such that the total second derivative $\Delta\Phi$ is proportional to the function Φ itself at any point (6.44), or $\Delta\Phi = -\kappa^2\Phi$. Consequently, the relation between absorption and the sum of absorption and leakage in arbitrary unit volume of the reactor without reflector is as follows:

$$w_{th} = \frac{\Phi\Sigma_a}{\Phi\Sigma_a + D\kappa^2\Phi} = \frac{1}{1 + \kappa^2 L^2} \quad (6.48)$$

and since it does not depend upon the space coordinates, it is a fraction of absorptions over the entire volume of the reactor as a whole, i.e., the probability of escaping leakage during diffusion in the thermal region. Thus, critical equation (6.46) may be written as

$$k_0 w_{in} w_{th} = k_0 w = k = 1 \quad (6.49)$$

where $w = w_{in} w_{th}$ is the total probability of escaping leakage out of the finite volume during the whole neutron cycle, as determined by (6.1).

5. **Geometrical buckling.** The parameter κ^2 expressed through the geometrical dimensions of the critical reactor is termed geo-

metrical buckling (κ_g^2). The geometrical buckling is considered to be a value which depends only upon the dimensions of the reactor. In a medium with given properties or with given value of the material buckling κ_m^2 (6.46) the critical state is achieved when the geometrical buckling coincides with the material one, $\kappa_g^2 = \kappa_m^2$, i.e., the size of the body coincides with the critical size. If $\kappa_g^2 < \kappa_m^2$ (with κ_m^2 being the value of κ^2 reducing (6.46) to unity), then $k > 1$, the reactor is in its above-critical state and its dimensions are larger than the critical size. If $\kappa_g^2 > \kappa_m^2$, then $k < 1$, and the

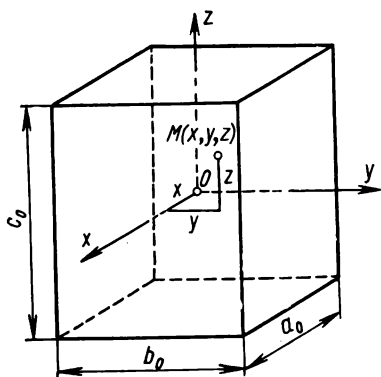


Fig. 6.7. Cartesian coordinates in a parallelepiped

distribution of neutrons in the parallelepiped should be symmetrical with respect to the same planes. This symmetry most directly reveals its mathematical expression if the problem on neutron diffusion is solved in coordinates having the same symmetry. Then, in solving equation (6.44), the method of separable variables proves applicable. The Cartesian coordinates possess the symmetry of the parallelepiped. In Fig. 6.7, the origin is in the centre of the parallelepiped, with a_0 , b_0 , and c_0 being the geometrical dimensions of the parallelepiped along the axes x , y , and z , respectively. In the Cartesian coordinates, equation (6.44) is of the form

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} + \kappa^2 \Phi = 0 \quad (6.50)$$

The solution of this equation should be obtained under zero boundary conditions (Sec. 5.3-6) which, with the origin chosen, are

state is subcritical. The geometrical bucklings are expressed in different ways in bodies of different geometrical forms. They are calculated below for the parallelepiped, cylinder and sphere. The cores of nuclear reactors usually have some of the above simple geometrical forms or a form nearly the same as one of them.

6. Parallelepiped. The rectangular-parallelepiped is a body symmetrical with respect to the planes passing through its centre, parallel to the faces. Since the mode of neutron distributions in bodies of finite volumes is due to leakage through the boundary surfaces,

reduced to the following:

$$\left. \begin{aligned} \Phi\left(\pm \frac{a}{2}, y, z\right) &= 0 \\ \Phi\left(x, \pm \frac{b}{2}, z\right) &= 0 \\ \Phi\left(x, y, \pm \frac{c}{2}\right) &= 0 \end{aligned} \right\} \quad (6.51)$$

where a , b and c are the extrapolated boundaries of the parallelepiped; $a = a_0 + 2 \cdot 0.71 l_{tr}$, etc. The correction (5.48) is added twice on the side of each parallel face. To separate the variables means to present $\Phi(x, y, z)$ in the form of (6.36):

$$\Phi(x, y, z) = X(x) \cdot Y(y) \cdot Z(z) \quad (6.52)$$

Substituting (6.52) into (6.50) and dividing by Φ give

$$\frac{1}{X} \cdot \frac{d^2 X}{dx^2} + \frac{1}{Y} \cdot \frac{d^2 Y}{dy^2} + \frac{1}{Z} \cdot \frac{d^2 Z}{dz^2} + \kappa^2 = 0 \quad (6.53)$$

Since each of the functions X , Y and Z depends upon its independent variable, equation (6.53) can only be satisfied by such functions whose second derivatives are proportional to the values of the functions themselves with all the values of the independent variables, that is, expressions $(1/X) \cdot (d^2 X/dx^2)$ are constants. All the coordinates in the critical reactor are equal. The constants should, hence, be of the same sign, and since κ^2 in (6.53) is greater than zero, the constants must be negative. Denoting $(1/X) \times (d^2 X/dx^2) = -\alpha^2$, etc., one obtains

$$-\alpha^2 - \beta^2 - \gamma^2 + \kappa^2 = 0 \quad (6.54)$$

and for the functions X , Y , Z one obtains equations similar to the following equation:

$$\frac{d^2 X}{dx^2} + \alpha^2 X = 0 \quad (6.55)$$

the solution of which (Sec. 5.3-8) is

$$X(x) = B \sin \alpha x + C \cos \alpha x \quad (6.56)$$

where B and C are integration constants. The function $\sin \alpha x$ does not satisfy the symmetry condition of the problem involved and the constant B must, therefore, be equal to zero. One more unknown constant is defined from the boundary conditions which for the function $X(x)$ are of the form:

$$X\left(\pm \frac{a}{2}\right) = C \cos\left(\pm \alpha \frac{a}{2}\right) = C \cos \alpha \frac{a}{2} = 0 \quad (6.57)$$

The cosine vanishes when the argument is equal to the odd number $\pi/2$

$$\alpha_i \frac{a}{2} = i \frac{\pi}{2} \quad (6.58)$$

where $i = 1, 3, 5, \dots$ Hence

$$\alpha_i = i \frac{\pi}{a} \quad (6.59)$$

Consequently, there are countless numbers of α_i and the relevant functions $C_i \cos \alpha_i x$, satisfying the conditions of the problem. The square of the least of all α_i , i.e., α_1^2 , is termed geometrical buckling, or, to be more precise, a component of the geometrical buckling along the x -axis. In the strictly critical reactor, all the coefficients C_i appear to vanish, except C_1 [26], so that the solution along the coordinate x is the function

$$X(x) = C \cdot \cos \frac{\pi}{a} x \quad (6.60)$$

The solutions along the other coordinates are analogous and upon substituting X, Y, Z into (6.52) the function of distribution of the thermal neutron flux in the reactor in the form of a parallelepiped takes the following form:

$$\Phi(xyz) = C \cdot \cos \frac{\pi}{a} x \cdot \cos \frac{\pi}{b} y \cdot \cos \frac{\pi}{c} z \quad (6.61)$$

Now, the connection between the buckling κ^2 and the critical size of the reactor follows from relation (6.54):

$$\kappa_g^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 \quad (6.62)$$

Equation (6.62) is the geometrical buckling of the parallelepiped reactor.

The agreement of the form of the function $X(x)$ with the zero conditions on two boundary surfaces of the parallelepiped, $x = +a/2$ and $x = -a/2$ led to finding only one undefined constant from the general expression (6.56). However, only two constants, $B = 0$ and $\alpha = \pi/a$ were actually found from two boundary conditions. Using the symmetry condition is also using one boundary condition, since the requirement of the symmetry of the problem immediately results in the symmetry of conditions at two boundaries from which only one constant can subsequently be determined. More significant is the fact that one integration constant it left altogether undefined (6.61). This is principally important

and stems from the fact that from the very beginning the problem was posed for the strictly critical reactor.

Solution in a volume with the known dimensions a , b , c would result in a fully defined solution. From the very beginning, however, there was imposed the condition (6.31) that made the dimensions of the volume of the multiplying medium a sought-for quantity. The dimensions were found and expressed through the constants of the medium with the help of relations (6.62) and (6.46). But there were no conditions enough to define one integration constant. It was actually due to the restriction of (6.31) for one more independent variable, that of time, which will not appear in further equations. The undefined constant C in (6.61) should, therefore, be considered to be the consequence of the fact that the initial time condition is not made use of. There may be any neutron flux in the critical reactor. It depends upon what it was at some initial instant of time when the reactor became critical, i.e., when $d\Phi/dt = 0$. It corresponds to the physical essence of the critical state. In the critical state, after each cycle on the average the neutron reproduces itself infinitely long, and the total number of cycled neutrons depends upon their number in the medium at the instant when the critical state was achieved. From this, it follows that the power of the critical reactor may also be arbitrary although in each particular case it is equal to some value and remains time-invariable as long as the critical state is preserved. The constant C in (6.61) is a neutron flux in the centre of the parallelepiped. Distribution of neutrons over the volume is determined by relationship (6.61). And the ultimate number of neutrons at each point depends upon the initial condition for the flux in the centre of the reactor.

7. Cylinder. A cylinder has an axial symmetry. The same symmetry is inherent in the cylindrical coordinates and variables are separated in these coordinates while obtaining solutions of equation (6.44) in the cylinder. Fig. 6.8 shows a cylinder of the height H_0 and the radius of the basis R_0 . The origin of cylindrical coordinates is in the centre of the cylinder. Since the neutron flux does not depend upon the coordinate φ , the flux derivative with respect to φ is zero and equation (6.44) in cylindrical coordinates takes the

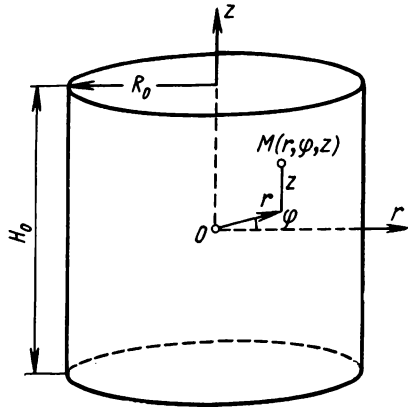


Fig. 6.8. Cylindrical coordinates in a cylinder

form:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial z^2} + \kappa^2 \Phi = 0 \quad (6.63)$$

where the function Φ may be considered to be a function of only two space coordinates $\Phi(r, z)$. The boundary condition is reduction of the flux to zero at the points of the surface of the cylinder:

$$\left. \begin{aligned} \Phi(R, z) &= 0 \\ \Phi\left(r, \pm \frac{H}{2}\right) &= 0 \end{aligned} \right\} \quad (6.64)$$

where $R = R_0 + 0.71l_{tr}$, and $H = H_0 + 2 \cdot 0.71l_{tr}$ (5.49). If the variables are separated, then the function Φ is presented in the form of the product

$$\Phi(r, z) = \mathcal{R}(r) \cdot Z(z) \quad (6.65)$$

Substituting this into (6.63) results in two ordinary differential equations:

$$\frac{d^2 \mathcal{R}}{dr^2} + \frac{1}{r} \cdot \frac{d\mathcal{R}}{dr} + \alpha^2 \mathcal{R} = 0 \quad (6.66)$$

$$\frac{d^2 Z}{dz^2} + \beta^2 Z = 0 \quad (6.67)$$

on condition that there is a connection between the constants

$$\alpha^2 + \beta^2 = \kappa^2 \quad (6.68)$$

If equation (6.66) is divided by α^2 , it will be reduced to the zero-order Bessel equation for the function of the argument $x = \alpha r$

$$\frac{d^2 \mathcal{R}}{dx^2} + \frac{1}{x} \cdot \frac{d\mathcal{R}}{dx} + \mathcal{R} = 0 \quad (6.69)$$

The general solution of equation (6.69) with the plus sign before the third term is the sum of two zero-order linearly independent Bessel functions of the first and second kind, $J_0(x)$ and $Y_0(x)$. Turning to the argument r , one obtains

$$\mathcal{R}(r) = C J_0(\alpha r) + B Y_0(\alpha r) \quad (6.70)$$

As $r \rightarrow 0$, $Y_0(r) \rightarrow \infty$ and in conformity with the requirement of the finiteness of the solutions of equation (6.44) (Sec. 5.3-6) the constant $B = 0$. From the boundary condition on the lateral surface of the cylinder

$$J_0(\alpha R) = 0 \quad (6.71)$$

it follows that $\alpha_i R = \zeta_i$, where ζ_i is the roots of the Bessel function J_0 . The function J_0 has an infinite number of roots, so that (6.71) defines countless numbers of α_i and their corresponding

solutions of $C_i J_0(\alpha_i \cdot r)$. However, as in the case of the parallelepiped, in the strictly critical reactor, all the C_i are zero, except C_1 to which corresponds

$$\alpha = \frac{2.405}{R} \quad (6.72)$$

where $\xi_1 = 2.405$ is the first root of the Bessel function, $J_0(2.405) = 0$. The lowest of the numbers, α_i , i.e., (6.72) squared is the radial part of the geometrical buckling in the cylinder.

Equation (6.67) does not at all differ from equations for the parallelepiped the solutions of which were obtained in the previous section. Consequently, the full solution for the cylinder is obtained in the form

$$\Phi(r, z) = C J_0\left(\frac{2.405}{R} \cdot r\right) \cdot \cos\left(\frac{\pi}{H} z\right) \quad (6.73)$$

and the expression for the geometrical buckling is:

$$\kappa_R^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 \quad (6.74)$$

Solution (6.73), as it is, contains the arbitrary function C which is the neutron flux in the centre of the cylinder, since $J_0(0) = 1$, and which is determined by the initial condition in time when the reactor becomes critical.

8. Sphere. Neutron distribution over the volume of a homogeneous sphere has spherical symmetry, so that the variables in equation (6.44) are separated in spherical coordinates (Fig. 6.9). Moreover, spherical symmetry means that the flux is independent of the coordinates φ and θ and equation (6.44) in spherical coordinates takes the form:

$$\frac{d^2\Phi}{dr^2} + \frac{2}{r} \cdot \frac{d\Phi}{dr} + \kappa^2\Phi = 0 \quad (6.75)$$

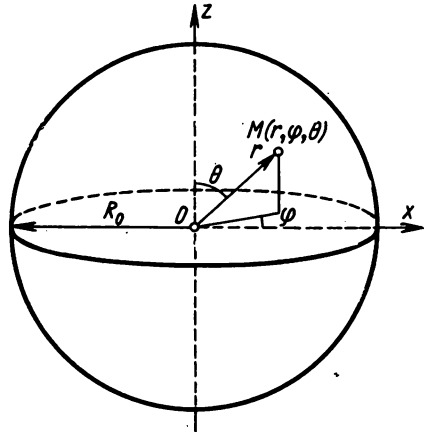


Fig. 6.9. Spherical coordinates in a sphere

where Φ is the function of the coordinate r , $\Phi(r)$ alone. The boundary condition of the function $\Phi(r)$ is

$$\Phi(R) = 0 \quad (6.76)$$

where $R = R_0 + 0.71l_{tr}$ (5.49), and R_0 is the geometrical radius of the sphere. The solution of equation (6.75) was obtained in

Sec. 5.3-8 (5.55). Turning to the function of the flux Φ in (5.55), one obtains the general solution in the form

$$\Phi(r) = C \frac{\sin \kappa r}{r} + B \frac{\cos \kappa r}{r} \quad (6.77)$$

Since with $r = 0$ the second term goes into infinity, $B = 0$. From the condition at the boundary of the sphere, the relation between κ and R in the critical sphere is determined:

$$\sin \kappa R = 0 \quad (6.78)$$

Relation (6.78) vanishes if the sine argument is multiple to π , i.e., $\kappa_i R = i\pi$ where $i = 1, 2, 3 \dots$. The function $(C_i \sin \kappa_i r)/r$ corresponds to each value of κ_i but, as in the previous cases, in the critical reactor, only C_1 is not zero, for which $\kappa = \pi/R$. In the run, the solution of equation (6.75) for the critical sphere is obtained in the following form, where the constant number π/R is added to the denominator for the arbitrary constant C to have the sense of the neutron flux in the centre of the sphere, by analogy with the previous cases,

$$\Phi(r) = C \frac{\sin\left(\frac{\pi}{R} r\right)}{\frac{\pi}{R} r} \quad (6.79)$$

The geometrical buckling of the spherical reactor is

$$\kappa_g^2 = \left(\frac{\pi}{R}\right)^2 \quad (6.80)$$

9. Minimum critical volume. As follows from (6.62), the critical state of a reactor in the form of the parallelepiped can be achieved not only with a single set of the values of its faces a, b, c . The value of each face may vary in certain limits but relation (6.62) should be fulfilled. Variation of the values a, b, c brings about the change in the volume of the parallelepiped, and with the given composition of the multiplying medium, it is reasonable to have a reactor of the smallest critical volume. The volume of the parallelepiped is $V = abc$. Since between the arguments a, b, c of the function of the volume V there is relation condition (6.62) ensuring the critical state of the parallelepiped reactor, when solving the problem of the conditional extremum of the function V , one may find those values of a, b, c which correspond to the minimum of the volume in the critical state. The condition for the minimum of the volume states that

$$a = b = c \quad (6.81)$$

i.e., of all the critical parallelepipeds, the cube is of the smallest volume. It is quite clear from the physical point of view. With the given volume, the cube has the smallest surface of all the parallelepipeds. Neutrons escape through the surface and, hence, with the given volume, the cube has the greatest w in (6.49). It is natural that with k_0 fixed, reduction of $k_0 w$ to unity is achieved with different volumes in different parallelepipeds, the cube having the minimum volume. In the critical state, $\kappa_g^2 = \kappa_m^2$. If the composition of the medium is given, it means that κ_m^2 is given (6.46). With (6.62) and (6.81) taken into account, the minimum volume of the critical parallelepiped is expressed through the material buckling, as follows:

$$V_p = \left(\frac{\pi \sqrt{3}}{\kappa_m} \right)^3 = \frac{161}{\kappa_m^3} \quad (6.82)$$

The same is with critical cylinders, of which one is of the smallest volume. Under relation conditions (6.74), the conditional extremum of the cylinder volume function $V = \pi R^2 H$, is determined by the following relation:

$$\frac{H}{D} = \frac{\pi}{2.405 \sqrt{2}} = 0.924 \quad (6.83)$$

where $D = 2R$ is the diameter of the cylinder. The volume of this cylinder is

$$V_c = \frac{148.3}{\kappa_m^3} \quad (6.84)$$

The cylinder with the relation of dimensions (6.83) is not the cylinder with the smallest surface, which has $H = D$. Relation (6.83) defines the cylinder with the lowest leakage, and the leakage depends both upon the size of the surface of the body and upon the character of the corresponding neutron distributions which are not the same along the radius and the height of the cylinder (6.73). A lower number of neutrons escapes through unit lateral surface of the cylinder than through unit surface of its bases or through unit surface of the parallelepiped. The cylinder with the lowest leakage is, therefore, somewhat flattened in its height, as compared to the cylinder with the smallest surface. However, the relation $H = D$ differs but little from (6.83). Therefore, the cylinder with the lowest surface, i.e., with $H = D$, is often roughly taken to be the cylinder with the lowest leakage.

The critical volume of the sphere is unambiguously expressed through the material buckling

$$V_{sph} = \frac{4}{3} \pi \left(\frac{\pi}{\kappa_m} \right)^3 = \frac{130}{\kappa_m^3} \quad (6.85)$$

The comparison of (6.82), (6.84) and (6.85) shows that of all geometrical forms the sphere has the smallest critical volume. With the volume given, the sphere has the smallest surface of all geometrical bodies. Neutron distribution over the volume of the sphere is, in addition, such that the flux gradient (5.41) at some point of the surface equal to leakage from unit surface of the sphere is also the lowest. Both these facts result in the smallest critical volume of the sphere. At the same time, a particular choice of the geometry of the nuclear reactor core is not only determined by physical but by some other considerations as well, by technological, in particular. The liquid moderator tank and the pressure vessel are most often made in the form of a cylinder. The critical volume of the cylinder exceeds the critical volume of the sphere by as little as 14%. The majority of reactor cores are, therefore, of cylindrical forms or nearly cylindrical. If the requirement of the minimum critical volume is the main one, then the cylinder is given its optimum form in accordance with (6.83).

10. Slowing-down neutrons. Space distribution of slowing-down neutrons in the reactor without reflector is the same as space distribution of thermal neutrons since the equation for the spatial part of the slowing-down density (6.39) and for the thermal neutron flux (6.44) is the same. Distributions of slowing-down neutrons can, therefore, differ from distributions of thermal neutrons (6.61), (6.73), (6.79) only by the flux value in the centre of the reactor. The flux value in the centre may be arbitrary (Sec. 6.8-6) but the relation between the fluxes of thermal and intermediate neutrons is strictly definite.

The flux of slowing-down neutrons, $\Phi(E)$, in unit energy range near the energy E is related to the slowing-down density by relation (5.25). Absorption in the process of slowing-down and leakage cut of the finite volume of the reactor are respectively allowed for by the factors φ (5.35) and w_{abs} (6.47). With the energy E , the flux is, therefore, equal to

$$\Phi(E) = \frac{q_0}{\xi \Sigma_s E} \varphi(E) e^{-\kappa_g^2 \tau(E)} \quad (6.86)$$

where q_0 is the slowing-down density with no absorption or leakage, equal, in this case, to the slowing-down density, with $\tau = 0$ (6.34):

$$q_0 = \frac{k_0}{\varphi} \Sigma_a \Phi_{th} \quad (6.87)$$

κ_g^2 being the geometrical buckling of the reactor (6.62), (6.74) or (6.80), and $\tau(E)$ the age of neutrons with the energy E (5.68). The total flux of slowing-down neutrons, i.e., the number of neutrons traversing unit area in unit time with any intermediate energy E ,

is equal to the integral of expression (6.86) over the whole energy range from the fission energy, $E_0 = 2 \cdot 10^6$ eV up to the thermal energy $E_{th} = 0.025$ eV. Leakage of neutrons from large-volume reactors is low and in (6.86) one may roughly assume $w_{abs} = 1$ at any energy. The highest number of resonance capture is observed at the energy of the first resonance level of U^{238} , 6.7 eV. Therefore, in the greater part of the slowing-down range, $\varphi(E)$ may also be roughly assumed to be equal to unity and the difference between q and q_0 , with the energy lower than 6.7 eV, may be neglected. With the above simplifications, the ratio of the total flux of slowing-down neutrons Φ_{mod} to the flux of thermal neutrons Φ_{th} is

$$\frac{\Phi_{mod}}{\Phi_{th}} = \frac{k_0 \Sigma_a}{\varphi \xi \Sigma_s} \int_{E_{th}}^{E_0} \frac{dE}{E} = 18.2 \frac{k_0 \Sigma_a}{\varphi \xi \Sigma_s} = 18.2 \frac{k_0}{\varphi \xi} \cdot \frac{\sigma_{aU}}{\sigma_{smod}} \frac{N_U}{N_{mod}} \quad (6.88)$$

where, over the above-mentioned energy range, the integral is equal to 18.2 (Sec. 5.1-4), the macroscopic cross-section of thermal neutron absorption is, with high accuracy, the macroscopic cross-section of uranium, and the macroscopic scattering cross-section of slowing-down neutrons is approximately the macroscopic cross-section of the moderator. In the homogeneous heavy-water reactor with the optimum ratio N_{mod}/N_U (See Table 6.4) the ratio Φ_{mod}/Φ_{th} is about 0.2. In the heterogeneous graphite reactor with the optimum lattice (see Table 6.5) and $\varphi = 0.9$, the flux ratio is 2.8. It means that the fluxes of thermal neutrons and of neutrons of all other energies are roughly equal in thermal reactors. Concentrations of thermal neutrons, $n = \Phi/v$ are, accordingly, much higher than those of non-thermal neutrons (Sec. 5.4-7) since the velocity of the motion of thermal neutrons is much lower than the averaged velocity of slowing-down neutrons.

11. Heterogeneous reactor. The present section is, strictly speaking, concerned with the homogeneous reactor. But if the heterogeneous reactor is large and consists of a great number of unit cells, the theory of the critical volume of the homogeneous reactor appears to be applicable to the heterogeneous reactor as well. In the limits of each unit cell, the neutron flux has its minimum in the centre of the uranium slug (see Figs 6.4 and 6.6). But the entire process of neutron distribution throughout the volume of the reactor moderator proves to be the same as in the homogeneous reactor since it obeys the same boundary conditions. If the factor of multiplication in an infinite medium, k_0 , is found taking into account the characteristics of neutron distributions in the unit cell, as is pointed out in Sections 6.4 through 6.6, the critical size is determined from critical equation (6.46) and from relations for geometrical bucklings (6.62), (6.74) or (6.80). The neutron age, τ ,

and the diffusion square length, L^2 , entering into critical equation (6.46) should also be calculated proceeding from the particular structure of the reactor core [24].

12. Age and diffusion length of neutrons in multiplying medium.

The age of neutrons, τ , (5.68) is determined by the relation of the diffusion coefficient and the moderating power of the substance of the reactor core. Although in thermal reactors there is usually much more moderator than uranium (see Table 6.4), the dilution of the moderator by uranium somewhat lowers the moderating power of the medium and, hence, prolongs the age of neutrons in comparison with their age in the pure moderator. The increase of the age is not, however, great. As a first approximation, the age of neutrons in the reactor is equal to the age in the pure moderator especially, the more so that one more effect is associated with uranium, that causes the decrease of the neutron age. Colliding with uranium nuclei, fast neutrons with high probability undergo inelastic scattering (see Table 6.2) that brings about high mean energy loss (Sec. 4.5-7) which is much greater than the mean energy loss in elastic collision with the moderator nucleus. If inelastic scattering of the fast neutron produced does take place, then the neutron begins slowing down from a lower energy value than the fission energy and, hence, it has a shorter age. This effect is, however, noticeable only in heterogeneous reactors, where the probability of the first collision of the fission neutron with the U^{238} nucleus is relatively high, and, first and foremost, in light-water reactors with closely spaced lattices. In closely spaced lattices, the age of neutrons may substantially differ from the age in the pure moderator.

The diffusion square length of thermal neutrons is inversely proportional to the macroscopic absorption cross-section of the medium (5.59). Since Σ_a of the uranium moderator mixture is always much higher than the analogous value of the pure moderator, $\Sigma_{a\ mod}^0$ the diffusion length in the multiplying medium is, correspondingly, much smaller than the diffusion length in the moderator. It can, however, be expressed through the moderator diffusion length, L_{mod} and through the thermal utilization factor, θ . Actually,

$$\begin{aligned} L^2 &= \frac{D}{\Sigma_a} \cdot \frac{\Sigma_{a\ mod}^0}{\Sigma_{a\ mod}^0} = \frac{D}{\Sigma_{a\ mod}^0} \cdot \frac{\Sigma_{a\ mod}^0}{\Sigma_a} = \\ &= L_{mod}^2 \frac{\Sigma_{a\ mod}^0}{\Sigma_{aU} + \Sigma_{a\ mod}} \approx L_{mod}^2 (1 - \theta) \quad (6.89) \end{aligned}$$

The diffusion coefficient, D , of the pure moderator and of its mixture with uranium is practically the same because it is, in the long run, expressed through scattering cross-sections (5.42) and (5.37).

Since usually uranium atoms are few in comparison with the number of the moderator atoms, $\Sigma_{a\text{ mod}}^0 = N_{0\text{ mod}}\sigma_{a\text{ mod}}$ of the pure moderator is approximately equal to $\Sigma_{a\text{ mod}} = N_{\text{mod}}\sigma_{a\text{ mod}}$ in the mixture, with $N_{0\text{ mod}}$ and N_{mod} being the numbers of the moderator atoms in unit volume of the pure moderator and the mixture. The factor $\Sigma_{a\text{ mod}}^0/(\Sigma_{a\text{ U}} + \Sigma_{a\text{ mod}}) \approx \Sigma_{a\text{ mod}}/(\Sigma_{a\text{ U}} + \Sigma_{a\text{ mod}})$ is, therefore, the fraction of thermal neutrons absorbed by the moderator. And if θ is the fraction of thermal neutrons absorbed by uranium (6.14), and if absorption of neutrons by other substances in the core is neglected, then L^2 of the mixture will just be expressed through L_{mod}^2 and θ as in (6.89). In a heterogeneous medium, L^2 is also presented by relation (6.89) if θ is calculated taking into account the distribution of the thermal neutron flux over the volume of the unit cell (Sec. 6.4-2). Since in solving critical equation (6.46), k_0 and, hence, all its components are supposed to be known, L^2 in the critical equation is obtained too, with the help of relation (6.89) and of the known value of the diffusion length of the pure moderator (see Table 5.3).

The critical size of the reactor with low k_0 is always large because the factor ω in (6.49) must roughly be unity. It, in turn, means that the value κ^2 in the critical equation is small. In this case, the exponent in (6.46) can be expanded and use may only be made of the terms with the power of the value κ not higher than the second:

$$\frac{k_0 e^{-\kappa^2 \tau}}{1 + \kappa^2 L^2} \approx \frac{k_0}{(1 + \kappa^2 L^2)(1 + \kappa^2 \tau)} \approx \frac{k_0}{1 + \kappa^2 M^2} = 1$$

or

$$\kappa^2 = \frac{k_0 - 1}{M^2} \quad (6.90)$$

where M^2 is the migration square length of neutrons (5.73) in the multiplying medium. Expression (6.90) is also a critical equation but it is, however, valid if k_0 differs but little from unity.

6.9. Neutron Reflector

1. Reflector effect. Practically, no reactors exist without neutron reflectors. At least at its bottom, the reactor core resting on the supporting plate has a reflector since all materials reflect neutrons. It is, however, clear that the efficiency of neutron reflection is the higher, the higher the albedo of the moderator (Sec. 5.3-10). To reduce leakage of neutrons, reflectors are placed near the body in whose volume neutrons produced by some source diffuse. Part of neutrons, which upon reaching the surface of the body left the volume of the body in the process of diffusion, come back since in

the reflector neutrons also take part in chaotic diffusion. The reflected neutrons are once more reflected from the first volume, etc. This, first and foremost, results in the increase of the neutron flux at the boundary of the body and the reflector. The increase of the flux at the boundary points with respect to the flux inside the body means lowering of the boundary flux gradient or of the diffusions current (5.41) equal to neutron leakage from the unit area of the body surface in unit time.

If not an arbitrary medium with some external neutron source but a multiplying medium in its critical state is implied, then the reduction of leakage by providing the core with a reflector increases ω in (6.49) and converts the reactor into the above-critical state. For the reactor to again become critical, with k_0 being constant, ω should, in turn, be lowered by decreasing the volume of the core. Thus, the critical size of the reactor with a reflector is always smaller than that of the reactor without reflector. It is practically advantageous. The reactor can only release energy while being in its critical state. When for some natural reasons (Chapter Seven) the reactor becomes subcritical its energy release ceases and part of the fissile material of the reactor remains unutilized. It goes without saying that it is quite reasonable to make this part as small as possible. Reflectors serve this purpose to some extent.

2. Reflector savings. The critical dimensions of reactors without reflectors are determined by expressions (6.62), (6.74) and (6.80). If the critical size is reckoned from the centre of the reactor core and is denoted by R_0 in the case of the reactor without a reflector, and by R with a reflector, then $R_0 > R$, and the difference

$$\delta = R_0 - R \quad (6.91)$$

is called reflector savings. The volume of the core of the reactor with a reflector is actually smaller than the volume of the reactor core without reflector by the value of the volume of the external layer of thickness δ . The reflector is placed immediately beyond the core, so that part of the core volume of the reactor without a reflector is replaced by the reflector material.

3. Neutron distribution in the reactor with a reflector. In the reactor with a reflector, the neutron flux vanishes at the boundary of the reflector with vacuum, with $r = R_{ref}$ (Fig. 6.10). In the figure, Φ_{th} and Φ_{mod} are the fluxes of thermal and slowing-down neutrons, respectively. Space distribution of each of them is presented with respect to its value in the centre of Φ_0 and the dotted line represents the flux in the reactor without reflector, vanishing at the above-mentioned boundary with vacuum, with $r = R_0$. The size of the reflector is denoted by Δ . The thickness of the layer, δ , replaced by the reflector is determined by the neutron flux formed

at the boundary of the new core and the reflector. Roughly speaking, this boundary should run inside the former core just where the new boundary flux coincides with the former flux inside the core volume of the reactor without a reflector. In fact, the addition of the reflector in no way changes the properties of the medium of the core in which the neutron balance, is, as previously, presented by equation (6.44) or (6.39), with κ^2 being the buckling determined by the medium constants (6.46). The function of neutron distribution over the core volume should, therefore, also remain the same. The only exception is the boundary areas of the new core where deflection from the main distribution function may appear because of the neutron exchange with the reflector. The constants of the two

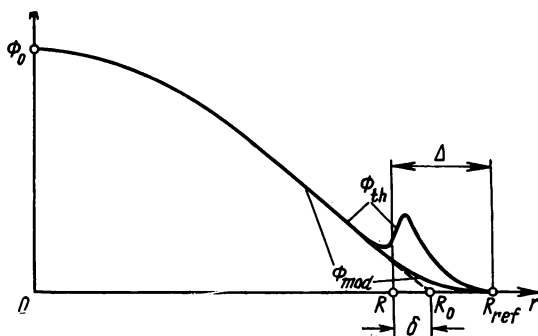


Fig. 6.10. Neutron flux distribution in a reactor with a reflector

media, those of the core and of the reflector, are different. Consequently, neutron distributions in each of them are also different and diffusion near the boundary varies the relevant distributions.

Most significant peculiarity is observed in the distribution of thermal neutrons. Their flux is determined by the relation between the production rate, q_{th} and the absorption rate, $\Phi\Sigma_a$ (Sec. 6.4-3). The slowing-down density, q_{th} is usually almost the same in the boundary areas of the core and the reflector. But the macroscopic absorption cross-section, Σ_a , differs greatly because neutron reflectors in thermal reactors are moderators with a very low absorption. The thermal neutron flux in the reflector may, therefore, prove to be even higher near the boundary with the core than it is in the boundary areas of the core, as shown in Fig. 6.10. The absolute value of this effect, depends, of course, upon the particular relation of q_{th} and Σ_a in the core and the reflector. If the reflector is water whose moderating power is very high and if, besides, a considerable portion of intermediate neutrons escape from the reactor core, the increase of the thermal-neutron flux in the reflector may be substantial. If the reflector is graphite, usually no increase of the

thermal neutron flux is observed. In the reflector near the core however, some deceleration of attenuation of the thermal flux function may occur. This effect is especially pronounced in the reflectors of intermediate-neutron reactors whose cores contain little moderator, so that not only Σ_a 's but q'_{th} s as well differ greatly in the reflector and the core, q'_{th} s in the latter being just relatively low.

The distribution of thermal neutrons over the volume of the reactor with a reflector such as shown in Fig. 6.10, means that there is no leakage of thermal neutrons from the core and the core even receives them from outside on account of the moderation of intermediate neutrons by the material of the reflector. This stems from the change of the curve slope of the flux distribution at the boundary of the core and the reflector and, as a result, the diffusion current vector (5.41) at point $r = R$ is directed to the centre of the reactor.

The absorption of slowing-down neutrons in the core is low and the distribution of the intermediate neutron flux averaged over all the energies of the intermediate neutron flux does not suffer any essential changes at the boundary of the core. However, no fission takes place in the reflector and as a consequence there are no fast-neutron sources in it. Slowing-down neutrons appear only in the reflector as a result of diffusion in the core, and since diffusion causes the loss of energy, with a greater distance from the core, fluxes of high-energy neutrons become progressively lower in comparison with fluxes of low-energy (or thermal) neutrons. Thus, in the reactor with a reflector, energy distribution of neutrons proves to be dependent upon space coordinates not only in the reflector but in the periphery of the core as well.

4. Determination of the critical size. With the properties of the core and reflector materials given and the reflector thickness Δ , upon which the neutron reflection efficiency depends, fixed (Sec. 5.3-10), the reactor has a certain critical size. Solving the stationary diffusion equations (6.32) and (6.33) written for the core and the reflector, respectively, and obtaining (throughout the volume of the reactor) space and energy distributions of neutrons, satisfying the boundary conditions, also give the values of the critical size of the core. The main difficulty is that analytical solutions of the diffusion equations for the reactor with a reflector are not known. Analytical solutions of partial differential equations are usually obtained when these equations are reduced to ordinary differential ones whose solutions are expressed in terms of elementary functions. And this is achieved by separating independent variables. The applicability of the method of separable variables is, however, obvious only in the case when the one-independent-variable distribution of the sought-for function does not depend upon any other distribution, i.e., when the sought-for function is the pro-

duct of several other functions dependent only upon one variable (6.35) and (6.36). Neutron distributions in the reactor with a reflector do not possess this property. As pointed out in Sec. 6.9-3, energy distribution of slowing-down neutrons depends upon the space coordinates. Besides, if the reactor is from all sides surrounded by the reflector, the space distributions by one of the coordinates depend upon other space coordinates except for the case of the spherically symmetrical reactor. As a result, the method of separable variables does not prove applicable and solutions of the diffusion equations in the reactor with a reflector are found by approximate methods.

The most widely used method is the multigroup method [26, 24]. Following this method, all neutrons are divided into energy groups. Diffusion with no energy change is considered in the limits of each group. Here, the neutron flux depends only upon the space coordinates. If the neutron reflector is arranged in the direction of one of the space coordinates, e.g., along the lateral surface only or along only the bases of the cylinder, the space variables are separated and the problem, with the above-mentioned simplification, has an analytical solution. In each group, the solutions must satisfy the boundary conditions and must agree with the solutions of the neighbouring energy groups. If there are many energy groups, the equations are solved by numerical methods, by reducing the system of differential equations to the algebraic one. The accuracy of the method is the higher, the greater the number of energy groups over the entire range of neutron slowing-down. However, if the number of the groups is great the amount of calculations required to solve the system of algebraic equations is so large that it cannot be performed in the reasonable time without making use of high-speed electronic computers.

In the simplest case, slowing-down is not at all taken into account and fission neutrons are considered to be produced thermal. Such consideration of the reactor is called a single-group consideration since all neutrons are combined into one thermal group. More accurate is the two-group approach where, in addition to the thermal group, diffusion of all slowing-down neutrons as neutrons of some mean energy is separately considered. Fig. 6.10, in fact, presents the result of the two-group consideration of the reactor with a reflector.

5. Reflector materials. The reflectors of thermal and intermediate nuclear reactors are made of moderating materials. These reflectors weakly absorb neutrons and promote their moderation in the reactor. In graphite and heavy-water reactors graphite is used as a reflector as it is the most available material of appropriate diffusion properties. The value of the reflector savings, δ (6.91) depends upon the thickness of the reflector, Δ , since the reflector

albedo depends upon it (Sec. 5.3-10). The reflector savings, however, reach their ultimate value, practically, with $\Delta \approx 1.5M$, where M is the migration length (5.73), which is about 90 cm in the case of graphite (see Tables 5.3 and 5.4). Consequently, it is not reasonable to make graphite reflectors of the thickness exceeding 90 cm. Here, the reflector savings are approximately equal to the neutron diffusion length in graphite, i.e., about 50 cm.

In light water, the value $1.5M$ is about 10 cm. In light-water reactors, used as a moderator is a water layer of 10 cm or more between the core and the reactor vessel. This layer is itself a reflector ensuring the ultimately possible reflector savings. Beyond the vessel of the light-water reactor, therefore, the graphite reflector is of no use and these reactors have no reflector as an individual unit. The cores of the reactors with a water moderator scatter neutrons mainly of high energy, not thermal ones. The reflector savings are, therefore, determined by the return of intermediate neutrons partially moderated. These savings are as high as 7 cm in the absolute value.

The intermediate-neutron reactors contain little moderator, and neutrons are absorbed by the material before they become thermal. The cores of these reactors scatter fast and intermediate neutrons. The best reflector of such neutrons is berillium which has the lowest value of l_{tr} in the fast region (5.36) because of the high concentration of atoms and a sufficiently high value of σ_{tr} [23]. For the same reason, berillium is the best moderator for intermediate reactors of low critical sizes, i.e., reactors of high concentration of fissile material in the core. Light water is inferior to berillium since, because of the decrease of the hydrogen scattering cross-section with energies higher than 0.1 MeV (see Fig. 4.23), water more readily than berillium transmits fast neutrons through its volume.

The fast-neutron reactors must not contain materials moderating neutrons because moderation decreases the conversion ratio, the high value of which is the main physical advantage of the fast reactor over reactors of other types. The reflector material of fast-neutron reactors is heavy U^{238} or Th^{232} which are at the same time fertile isotopes for conversion of new fissile materials (Sec. 3.6-2). Such materials used as reflectors, of course, reduce the critical size of the core but their main function is to accumulate new fissile materials produced by absorbing neutrons scattered by the core (3.56) and (3.57). They are not, therefore, called neutron reflectors but breeding blankets.

6. Neutron-flux flattening. In the reactor without a reflector, the neutron flux vanishes at the boundary of the core. The ratio of the maximum flux in the centre to the flux averaged over the whole core is called the non-uniformity factor, χ , $\chi > 1$. Since the fission and energy release rate is proportional to the thermal neutron

flux, the power produced in the reactor core is χ times lower than the maximum possible power attainable in the ultimate case when the neutron flux is constant through the whole volume of the core. As was pointed out in Sec. 6.9-1, the use of reflectors increases the flux in the periphery of the core, decreases the non-uniformity factor, χ , and, consequently, increases the mean power density of the core, which is, naturally, advantageous. This gain is not, however, great in reactors with low k_0 and, hence, of a large critical size.

A more radical way of flattening the flux in the volume of the core is to decrease k_0 in the central part of the core down to $k_0 = 1$. It goes without saying, that the critical size of the core with such a central part should be larger than it was before the

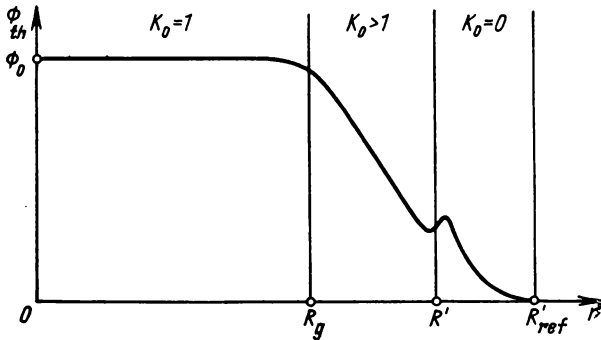


Fig. 6.11. Neutron flux distribution in a reactor with a flattened core

formation of the core with $k_0 = 1$. But within the core, the flux remains constant. There is no leakage from the central part of the core, $w = 1/k_0 = 1$, and equation (6.43) for thermal neutrons is transformed into $\Delta\Phi = 0$ the solution of which is $\Phi = \text{const}$ under the same condition of no leakage. In this case, the flux in the core appears to be such as shown in Fig. 6.11. The critical size R' is larger than R in Fig. 6.10 but the non-uniformity factor, χ is nearer to unity in Fig. 6.11 than it is in Fig. 6.10 and the power taken from unit volume of the core is correspondingly higher.

The physical sense of the gain in the value of the neutron flux by decreasing k_0 in the central part of the core is that, at first, such a decrease actually reduces the flux in the centre of the reactor down to its value with $r = R_g$. But since the flux is now constant in the range from $r = 0$ to $r = R_g$, it can be increased in the entire core so that it will not exceed the ultimately permissible value, Φ_0 , anywhere. It should be kept in mind that the absolute value of the flux in the critical reactor may be any (Sec. 6.8-6) while its limiting value is determined from the per-

missible heat loads on the fuel element which must not fail during the reactor operation. The flux flattening over the whole volume of the central part of the core is technologically difficult to realize. It is easier to obtain some partial flattening along the radius of the cylindrical core by increasing absorption, constant in height, of thermal neutrons in its central part [24].

6.10. Non-Critical Reactor

1. Small deviations from criticality. The increase of the reactor size above its critical values leads to the time increase of the neutron flux. In the simplest case of the reactor without a reflector, the new function of the flux vanishes at a larger distance from the centre of the reactor. This means the decrease of the flux gradient over the whole volume and, consequently, the reduction of leakage in comparison with the critical reactor. The reduction of leakage is the increase of $D\Delta\Phi$, and the neutron balance (5.43) at any point of the reactor is satisfied only taking into account the term $d\Phi/dt$ in the right-hand side. On condition that the excess of the size over the critical one is not high in comparison with the critical size itself, a noticeable increase of the neutron flux takes place in the time greatly exceeding the time required for the average cycled neutron to reach any point of the reactor, and the character of neutron distribution over the reactor volume is, as usual, determined by diffusion and the boundary conditions. The time change of the flux at each point of the volume follows the same law. The development of the process in time in the critical state may, consequently, be considered with respect to the total number of neutrons in the reactor core. The same is true of the non-critical state resulting from the slight change of k_0 with the constant size of the reactor core.

2. Delayed neutrons. The rate of the increase of the number of neutrons with time is determined by the value of the excess of the multiplication factor k above unity and by the time of the neutron cycle, τ , (6.3). As follows from examples of Sec. 6.1-6, because of the very low values of the cycle time of the free neutron the slightest excess of k above unity is accompanied by a very rapid increase of the number of neutrons if all neutrons are produced as prompt ones. However, part of neutrons are released in β -decay of fission products with a considerable time delay. Although the fraction of delayed neutrons is not high (see Table 3.6), it can make an essential contribution to the cycle time averaged over all neutrons. As a matter of fact, in the strictly critical reactor, the β -th part of fission neutrons has the cycle time determined by the mean lifetime of fission products which are neutron precursors. This time is equal to τ_d (3.60). The part $(1 - \beta)$ has the cycle time τ_p deter-

mined by the prompt neutron cycle. Taking into account the yields of each component, the mean cycle time is, therefore,

$$\tau = (1 - \beta) \tau_p + \beta \tau_d \approx \beta \tau_d \quad (6.92)$$

U^{235} fission products have $\beta \tau_d = 0.085$ s, which is much higher than τ_p even in the typically thermal reactor where the cycle time of the free neutron is especially high (up to 10^{-3} s). The increase of the mean cycle time due to the fact that the delayed neutrons long after fissioning remain in their bound state in those nuclei by which they are then emitted during β -decay, is just the reason for the deceleration of the increase of the number of neutrons with time in the above-critical state of the reactor. True, all this holds only on condition that the reactor is maintained under such operating conditions that it is not altogether critical without some addition of delayed neutrons. The time of the mean neutron cycle (6.92) relates to the strictly critical reactor. With k exceeding unity, the effective fraction of delayed neutrons at a given time instant is reduced and τ approaches τ_p . However, within some easily controllable interval of the variation of k , τ remains much greater than τ_p . This is used to control the chain reaction with the help of delayed neutrons.

3. Equations of the reactor kinetics. When analysing in detail the behaviour of neutrons in time, one should take into account all the six groups of delayed neutrons (see Table 3.6), each with its partial yield and lifetime. However, to make the most significant features of the process clear, it is sufficient to make use of one group of delayed neutrons having the total yield β and the averaged lifetime τ_d (3.60). Since delayed neutrons are emitted in β -decay of fission products, their production in free form is governed by the radioactive decay law (3.1). The decay of the precursor atom results in emitting one neutron. The number of emitted neutrons is equal to the number of decays which, in turn, is proportional to the number of accumulated radioactive atoms N . The rate of accumulation of precursor atoms, Q , (3.9) is determined by the number of neutrons that have passed into the bound state and depends upon the total number of free neutrons at the given moment. In the long run, the number of fissions and, hence, the number of precursor atoms produced are proportional to the number of free neutrons. If n is the total number of free neutrons in the reactor at a certain moment of time and the neutron multiplication factor is k , then the kinetic equations are of the form

$$\left\{ \begin{array}{l} \frac{dn}{dt} = \frac{kn(1 - \beta) - n}{\tau_p} + \lambda N \\ \frac{dN}{dt} = \frac{kn\beta}{\tau_p} - \lambda N \end{array} \right. \quad (6.93)$$

$$\quad (6.94)$$

The first term in the right-hand side of (6.93) is the increase of the number of prompt neutrons in unit time. It is written as in (6.2) but taking into account that, having made the cycle, the number of free neutrons becomes $kn(1 - \beta)$, not kn , because the β th part of all fission neutrons transfers into the bound state. The term λN is the number of delayed neutrons which have arrived in unit time in free form. Here, λ is the constant of the radioactive decay of precursor atoms equal to $\lambda = 1/\tau_d$ (3.5). Equation (6.94) is the balance in unit time of precursor atoms at any instant of time in accordance with (3.9). The gain of the number of radioactive atoms is the number of neutrons which have transferred into their bound state in unit time and the decrease is the number of decays in unit time.

4. **Reactivity.** The value

$$\rho = \frac{k - 1}{k} \quad (6.95)$$

is called the *reactivity* of the nuclear reactor. The concept of reactivity is widely used in describing the non-critical states of reactors. Since k usually differs little from unity, $\rho \approx k - 1$, i.e., the reactivity shows the increase of k above unity. In the critical reactor $\rho = 0$, in the above-critical one the reactivity is positive, and in the sub-critical it is negative. If some phenomenon results in the decrease of the multiplication factor, then it is said to give rise to negative reactivity. If, as a result of some effect, k is increased, the effect results in positive reactivity. Taking into account expression (6.95) for reactivity, the kinetic equations are obtained in the following form:

$$\left\{ \begin{array}{l} \frac{dn}{dt} = \frac{\rho - \beta}{\tau_p'} n + \lambda N \end{array} \right. \quad (6.96)$$

$$\left\{ \begin{array}{l} \frac{dN}{dt} = \frac{\beta}{\tau_p'} n - \lambda N \end{array} \right. \quad (6.97)$$

where $\tau_p' = \tau_p/k \approx \tau_p$ since $k \approx 1$.

5. **Reactor periods.** The solutions of the system of linear differential equations (6.96), (6.97) are the expressions of the form

$$n(t) = A_1 e^{\frac{t}{T_1}} + B_1 e^{\frac{t}{T_2}} \quad (6.98)$$

$$N(t) = A_2 e^{\frac{t}{T_1}} + B_2 e^{\frac{t}{T_2}} \quad (6.99)$$

where two of the constants, A and B , are arbitrary integration constants defined from the initial conditions, the other two are proportional to them and expressed through them and through the coefficients of equations (6.96), (6.97), and the periods T of the

functions $n(t)$ and $N(t)$ are found as solutions of the characteristic equation of the system of (6.96), (6.97):

$$\begin{vmatrix} \frac{\rho - \beta}{\tau'_p} - \frac{1}{T}, & \lambda \\ \frac{\rho}{\tau'_p}, & -\lambda - \frac{1}{T} \end{vmatrix} = 0 \quad (6.100)$$

Upon transformations, equation (6.100) usually reduces to the form

$$\rho = \frac{\tau'_p}{T} + \frac{\beta}{1 + \lambda T} \quad (6.101)$$

and is a relation between the reactor reactivity and the reactor periods, i.e., the periods of increasing or decreasing the number of neutrons in time. If all the six groups of delayed neutrons are present, instead of one equation (6.97) there will be six equations. Each of them will be written with respect to its N_i with the corresponding λ_i and β_i (see Table 3.6), the term λN in (6.96) will be transformed into the sum of all $\lambda_i N_i$, and the second term in the right-hand side of (6.101) will also become the sum of six terms of the form $\beta_i/(1 + \lambda_i T)$. Then, (6.101) will appear to be the seventh order algebraic equation with respect to the period T , and (6.98) and (6.99) will be represented by the sums of seven exponents with different periods. In this case, (6.101) is a square equation and its two roots, T_1 and T_2 , are simply found. If the U^{235} constants ($\beta = 0.0065$; $\tau_d = 13$ s, $\lambda = 1/\tau_d = 0.077$ s⁻¹) are taken into account, then, on condition that the reactivity ρ is not quite near β , $\rho < 0.005$ or $\rho > 0.0085$, the periods, with precision as high as 3%, are

$$T_1 \approx -\frac{\tau'_p}{\beta - \rho + \lambda \tau'_p} \approx -\frac{\tau'_p}{\beta - \rho} \quad (6.102)$$

$$T_2 \approx \frac{\beta - \rho + \lambda \tau'_p}{\lambda \rho} \approx \frac{\beta - \rho}{\lambda \rho} \quad (6.103)$$

where $\lambda \tau'_p = 7.7 \cdot 10^{-5}$ is always considered low in comparison with $|\beta - \rho|$.

Figure 6.12 shows relation (6.101) over the whole range of the change of the constants ρ and T , including $\rho \approx \beta$. Two periods correspond to each value of reactivity. If $\rho > 0$, one of the periods is positive, determines the time-increasing function and is called a stable reactor period, the second period is negative and is called a transient reactor period. With six groups of delayed neutrons, there are six negative transient periods since ρ as a function of T has six discontinuities at each value of $T = -\tau_i$ analogous to the

discontinuity at $T = \tau_d = -13$ s in Fig. 6.12 and, correspondingly, six average branches instead of one. If $\rho < 0$, both periods are negative and, consequently, the neutron flux (6.98) only decreases with time. The period, longer in its absolute value, is stable, the shorter period is transient. The physical sense of the stable and the transient periods can be found out if one considers some specific example.

6. **Reactivity jump.** The simplest example is as follows. The reactor operates at a constant pressure for a sufficiently long period of time, the number of radioactive neutron precursor atoms reaches

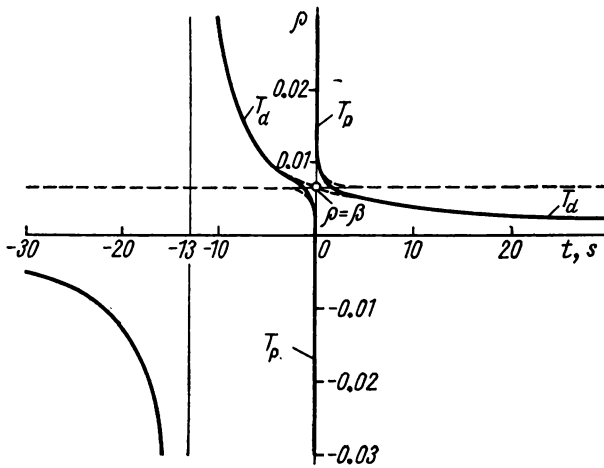


Fig. 6.12. Relation of reactivity to reactor period

the state of dynamical equilibrium, $dN/dt = 0$, and at the instant $t = 0$ reactivity discontinuously changes from $\rho = 0$ to some value $\rho > 0$ or $\rho < 0$ after which it remains invariable. If the constants in (6.98), (6.99) are calculated with the same accuracy as the periods (6.102), (6.103), then, under the initial conditions of $n(0) = n_0$ and $N(0) = N_0$ the neutron number function $n(t)$ assumes the following form:

$$\frac{n(t)}{n_0} = -\frac{\rho}{\beta - \rho} e^{-\frac{\beta - \rho}{\tau_p} t} + \frac{\beta}{\beta - \rho} e^{\frac{\lambda \rho}{\beta - \rho} t} \quad (6.104)$$

As the approximation involved, the constant A_2 in (6.99) is negligible and

$$N(t) = N_0 e^{\frac{\lambda \rho}{\beta - \rho} t} \quad (6.105)$$

With the specific value of ρ satisfying the conditions of obtaining (6.102), (6.103), e.g., with $\rho = +0.0025$, (6.104) is equal to

$$\frac{n(t)}{n_0} = -0.625e^{-4t} + 1.625e^{-\frac{t}{21}} \quad (6.106)$$

and with $\rho = -0.0025$,

$$\frac{n(t)}{n_0} = 0.28e^{-9t} + 0.72e^{-\frac{t}{47}} \quad (6.107)$$

Relations (6.106) and (6.107) are shown in Fig. 6.13. The positive reactivity jump is accompanied by a fast momentary increase of

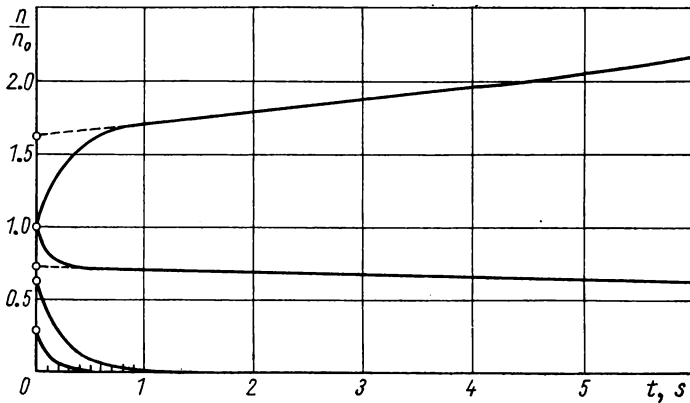


Fig. 6.13. Time change of the flux, with $\rho = \pm 0.0025$

the number of neutrons due to the presence of prompt neutrons in the reactor. It is only after the damping of the transient that a relatively low increase associated with delayed neutrons begins. The same phenomenon is also observed with the negative reactivity jump.

Transients arise because all free neutrons in the reactor have a double origin. The greater part of neutrons are prompt, having very short cycles. Others are delayed, having rather long cycles in comparison with prompt neutrons. In the long-critical reactor, the number of delayed neutrons transferring into their bound state is equal to the number of neutrons released during β -decay. The fraction of free neutrons of delayed origin is, therefore, equal to β , i.e., to the fraction of delayed fission neutrons. It is significant that without this addition, the reactor would be sub-critical and it may, therefore, be considered that without delayed neutrons the reactor would have negative reactivity equal to $-\beta$. This negative reactivity would always become lower in the absolute value when the

fraction of delayed neutrons is increased but the criticality remains. Such a reduction of the fraction of delayed neutrons in their free state is the transient process with discontinuous increase of reactivity.

The increase of reactivity effects first of all the most active part of neutrons, prompt neutrons, which having gained some additional reactivity $\rho (< \beta)$, have a lower negative reactivity equal to $(-\beta + \rho)$. To achieve the critical state, they now require a smaller fraction of delayed neutrons $(\beta - \rho)$. The number of delayed neutrons remains constant for a short time after the reactivity jump, but the number of prompt neutrons increases so much that the previous number of free delayed neutrons is the smaller fraction of the total number of neutrons, *viz.*, $\beta - \rho$. The fraction of delayed neutrons is reduced $\beta/(\beta - \rho)$ times, with their total number remaining invariable. Hence, the number of prompt neutrons, *i.e.*, practically the total number of neutrons in the reactor, increases roughly the same number of times because the fraction of delayed neutrons is low. If the number of delayed neutrons did not subsequently change, the reactor would become critical after the transient process but prompt neutrons alone would have a lower reactivity deficiency. However, as the radioactive precursors of the delayed neutrons accumulate, which is observed in the increased total flux of neutrons the number of the released delayed neutrons also starts growing. After the transient, therefore, the increase of the number of neutrons continues but with a greater period defined by the accumulation of precursor atoms. The new relation between the numbers of free neutrons of delayed and prompt origins is, all the time, preserved due to the fast multiplication of prompt neutrons, and the fraction of delayed neutrons in their free state is not now equal to β but to $\beta - \rho$. It means that the averaged neutron-cycle time (6.92) also becomes less than in the critical reactor and is $\tau = [1 - (\beta - \rho)]\tau_p + (\beta - \rho)\tau_d \approx (\beta - \rho)\tau_d$ when $\rho \ll \beta$, but when ρ approaches β , it tends to τ_p .

By (6.3), the reactor period is $\tau/(k - 1) \approx \tau/\rho$, which in the case involved gives $(\beta - \rho)\tau_d/\rho = (\beta - \rho)/\lambda\rho$ in accordance with (6.103) obtained from precise mathematical relations. The same is the case of the transient in the negative reactivity jump. In the sub-critical reactor, however, the fraction of delayed neutrons, on the contrary, decreases and the averaged neutron-cycle time τ increases in comparison with τ in the strictly critical reactor. Therefore, with the reactivity jump being the same in the absolute value, the stable period is longer in the sub-critical state than in the above-critical one, 47 s and 21 s in examples (6.107) and (6.106).

The transient establishes a new relation between the number of prompt and delayed neutrons, which is determined by the value of reactivity, ρ . Therefore, the increase or decrease of the number of

neutrons in the transient after the reactivity jump is not determined by the algebraic sign of the new value of reactivity, ρ , but by its increment sign. As a result, the time behaviour of the number of neutrons in the reactor with different subsequent jumps of reactivity is analogous to that shown in Fig. 6.14. One can get an idea of the change rate of the number of neutrons in the transient if the time derivative of the function $n(t)/n_0$ (6.104) is calculated and is taken with $t = 0$. It is equal to $\rho/\tau_p' + \lambda\rho\beta/(\beta - \rho)^2 \approx \rho/\tau_p'$, i.e., at the initial time instant after the reactivity jump, the neutron flux in the reactor increases as if there were no delayed neutrons

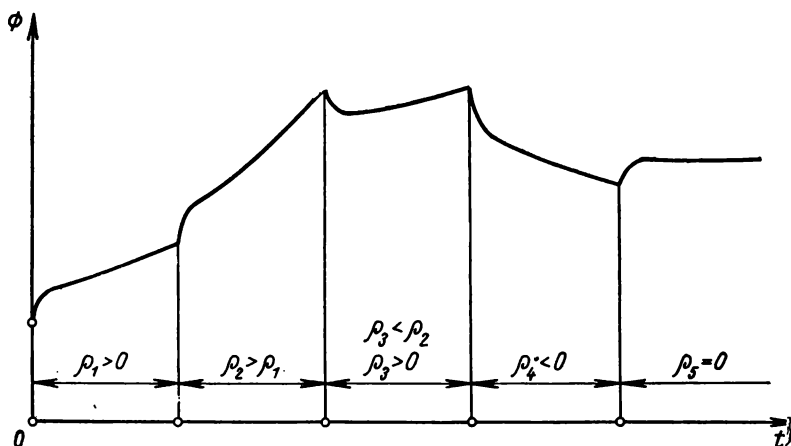


Fig. 6.14. Transients with different reactivity jumps

at all and the reactor gained the same increment of reactivity. This follows from the comparison with the analogous derivative of (6.3). In the presence of delayed neutrons, however, the increase of the number of prompt neutrons rapidly decays if, without delayed neutrons, the reactor remains subcritical as before. Subsequently, the number of prompt neutrons increases in strict proportion with the number of delayed ones released during β -decay of fission products.

7. Prompt criticality. It follows from the previous arguments that, with $\rho = \beta$, the reactor becomes critical with prompt neutrons alone. It also stems from relation (6.101). Very short periods, $T \approx 0$, correspond to high reactivities determined by the first term of (6.101) or by the branches of the curve in Fig. 6.12 referred to prompt neutrons and denoted by the subscripts T_p . If $\lambda T \ll 1$, then (6.101) is transformed into

$$\rho_p = \rho - \beta = \frac{\tau_p'}{T} \quad (6.108)$$

and is the relation between the reactivity and the period with no delayed neutrons (6.3). If there were no delayed neutrons at all, then (6.108) would hold with any T , which is shown by the dotted line in Fig. 6.12. In the strictly critical reactor, the period turns to infinity. It would be observed with $\rho_p = 0$, i.e., with $\rho = \beta$ in the terms of reactivity of the reactor with delayed neutrons. It means that, with $\rho = \beta$, the reactor is critical without any addition of delayed neutrons. This state of the reactor is called prompt criticality.

The curve branch denoted by T_d in Fig. 6.12 would intersect the axis of ordinates just with $\rho = \beta$ in the absence of the first term in expression (6.101). This value of reactivity separates the positive values of T from the negative ones along the curve T_d . With $\rho < \beta$, positive, i.e., stable, periods are determined by delayed neutrons and transient ones, by prompt neutrons. With $\rho > \beta$, on the contrary, stable periods are determined by prompt neutrons and transient ones, by delayed neutrons. In fact, expression (6.101) contains both terms, so that the above said relates only to those values of ρ which are not very close to β . Near $\rho = \beta$, both prompt and delayed neutrons have a certain effect both upon the stable and the transient periods of the reactor (see Fig. 6.12). Nevertheless, it is near $\rho = \beta$ that the stable period of the reactor becomes short and the reactivity value $\rho = \beta$ is assumed to be ultimately permissible in controlling the delayed neutron reactor. When ρ becomes larger than β , the prompt neutron effect increases and the stable period is soon determined only by the time of the prompt-neutron cycle.

With no delayed neutrons, to control the fission self-sustaining chain reaction would be extremely difficult, if at all possible. The stable reactor period is 1.5 s, with delayed neutrons taken into account, when $\rho = \beta = 6.5 \cdot 10^{-3}$ and with constants used in Sec. 6.10-6. With prompt neutrons alone, this period corresponds to the reactivity $\rho_p = \tau_p' / T = 6.7 \cdot 10^{-4}$, with $\tau_p' = 10^{-3}$ s, i.e., the range of the reactivity values with the going-up periods as low as 1.5 s appears to be 10 times as narrow as in the case of delayed neutrons. And this is in the reactor with the longest item of the free-neutron cycle. In light-water reactors, the neutron cycle time is about 10^{-4} s and the reactivity interval with safe going-up periods without delayed neutrons would be 100 times as long as in the presence of delayed neutrons. This interval would be negligible in the fast-neutron reactor where the neutron-cycle time is 10^{-7} s. To control this reactor without delayed neutrons would be quite complicated.

The above period of 1.5 s, with $\rho = \beta$, is the stable period of the reactor. However, just after the reactivity jump, due to the transient process the number of neutrons increases faster than with

$T = 1.5$ s. Therefore, with $\rho = \beta$, during the first 1.5 s, the number of neutrons does not increase e times but about 10 times, which corresponds to the effective averaged period of about 0.7 s. Thus, with the prompt criticality, the rate of the increase of the number of neutrons in time is high, especially during the first fractions of a second. The latter fact is characteristic of any reactivity value (Sec. 6.10-6 and Fig. 6.13).

8. Reactivity control. The reactivity of the nuclear reactor varies by shifting in the core the elements of the chain reaction control, such as control rods of cylindrical or some other form, whose material contains substances strongly absorbing neutrons (boron, cadmium and others). One such rod, when completely immersed into the core, introduces negative reactivity or, in other words, binds the reactivity of the reactor in some thousandths, which is less than $\beta = 6.5 \cdot 10^{-3}$. The value of the bound reactivity depends both upon the material and size of the rod surface and upon its location in the core, since the number of neutrons absorbed in the material of the rod depends upon the neutron flux which is at a minimum in the peripheries of the core. The withdrawal of the rod from the core results in the reactivity release, and since the rod always moves along its axis, the reactivity increment is characterized by the change of the position of the rod end in the core. With the rod being completely immersed, the highest possible reactivity is bound but the shifting of the rod along the given fraction of its full length, i.e., along one hundredth results in the least change of the reactor reactivity as the rod end is moved about the region of the lowest neutron flux (see Fig. 6.15 which shows the dependence of the neutron flux Φ as a function of the core height, h).

If the rod is half immersed, it binds half of the possible reactivity but now shifting the rod along the same fraction of its length results in the maximum reactivity release. In the latter case, the value of the released reactivity exceeds twice the *mean* reactivity bound by the same fraction of the rod length. If one assumes for the sake of certainty that the total reactivity bound by the rod is equal to $5 \cdot 10^{-3}$, then reactivity release by shifting the rod along

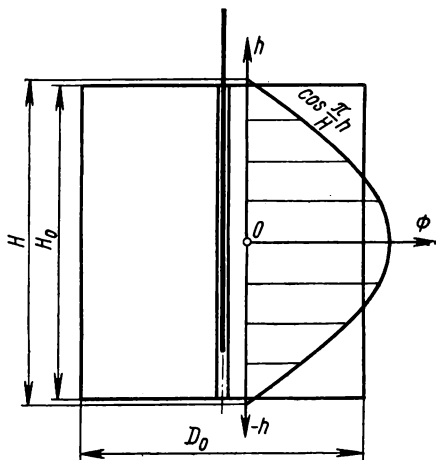


Fig. 6.15. Core control rod

the one hundredth of its length does not exceed 10^{-4} . The height of the reactor core is usually more than one metre and the position of the control rod end is fixed with the precision much higher than a centimetre. It appears, as a result, that in the reactivity range of zero to β , the reactivity of the reactor can be controlled with the precision up to 10^{-5} and the stable periods corresponding to such low reactivities are measured in hours. In the absence of delayed neutrons, to control the reactivity with the precision up to 10^{-5} would obviously be insufficient.

9. Units of reactivity. First, reactivity is measured in the absolute units following from the definition of the reactivity (6.95). For U^{235} , $\rho = \beta = 0.0065$ in these units. Second, reactivity is measured in per cent, consequently the same $\rho = 0.65\%$. Third, reactivity is measured in inverse hours. The latter unit is used for low reactivities in measuring the reactor periods. The inverse hour is such reactivity to which corresponds the stable reactor period of one hour. Since all the time constants in (6.101) are expressed in seconds, the reactivity of the reactor in inverse hours is represented by the following expression:

$$\rho = \frac{\frac{\tau'_p}{T} + \frac{\beta}{1 + \lambda T}}{\frac{\tau'_p}{3600} + \frac{\beta}{1 + 3600\lambda}} \quad (6.109)$$

where T is the reactor period observed. If the reactor period is about 1 h, the first component in formula (6.101) makes contribution to the reactivity as low as 1%, and taking into account that $\lambda = 1/\tau_d$,

$$\rho \approx \frac{\beta \tau_d}{\tau_d + T} \approx \frac{\beta \tau_d}{T} \quad (6.110)$$

i.e., the reactivity is proportional to the reactor period. Substituting $T = 3600$ s into (6.101) gives the reactivity in the absolute units, taken to be 1 inverse hour. For the constants used above, $1 \text{ inv. h} = 2.37 \cdot 10^{-5}$. And finally, fourth, the reactivity is measured in the β -units or in dollars and cents. The reactivity equal to β is taken as one dollar and cents are the hundredth fractions of this reactivity. Since $\rho = \beta$ is the limiting value of the reactivity of the controlled delayed neutron reactor, it is clear why this reactivity value is taken to be the unit especially as the absolute value of this unit depends upon the type of nuclear fuel. Thus, β of Pu^{239} is three times as low as β of U^{235} (see Table 3.6), and the reactivity expressed in the absolute units does not always show how close it is to the limiting value. Reactivity in cents is always expressed in the fractions of its limiting value and this representation of reactivity is universal.

CHAPTER SEVEN

PHYSICAL PROCESSES IN ACTUAL REACTORS

7.1. Retention of Criticality in Time

1. **Reactivity reduction.** An infinitely critical reactor is a mathematical abstraction. Actually, the reactor processes worsen the multiplication properties of the medium and without restoration of reactivity the reactor would be unable to operate even for a short time. The conversion of the neutrons in the reactor includes the process of fission. Each act of fission means the loss of one atom by the fissile material and, hence, lowering of η (6.9), i.e., k_0 (6.7). Fissionable atoms are partly recovered on the account of the absorption of excess neutrons by the nuclei of U^{238} with the formation of Pu^{239} (3.57). However, the accumulation of new fissile material does not usually compensate for the loss of fissionable atoms and reactivity falls. Furthermore, each act of fission is accompanied by the appearance of two new atoms the nuclei of which, as with any other nuclei, absorb neutrons. The build-up of the fission products also affects reactivity (6.13) and (6.15). Finally, a simple temperature increase in the core of the reactor is usually accompanied by a reduction in the reactivity while the cores of the power reactors must be heated to the highest possible temperature as the efficiency of a heat engine is determined in the long run by the difference of temperatures between the heat source and the condenser, i.e., the surrounding medium.

2. **Control system.** A nuclear reactor can operate at a prescribed power for a long period only if it has initially a sufficient reactivity margin. The release of bound reactivity as it lowers due to natural reasons continuously keeps the reactor in a critical state. The critical reactivity margin is created by constructing the core of the reactor with the dimensions considerably exceeding the critical ones. For the reactor not to become supercritical, k_0 of the multiplying medium is simultaneously artificially reduced. This is attained by the introduction of neutron absorbers which may be subsequently withdrawn from the core. Just as with the elements providing the control of a chain reaction (Sec. 6.10-8), the absorbing materials are components of the material of the rods of this or that cross-section which move along the corresponding channels in the core. For control, one, two or several rods are sufficient but

to balance the initial excess reactivity their number may reach a hundred. These rods are termed *shim* rods. Control and shim rods may be of the same design, several shim rods may act as control ones, but they differ in their function. *Control* rods are intended for continuous maintaining of the reactor in its critical state, for shutdown, for starting, and change-over from one power level to another. All these operations require small changes of reactivity. Shim rods are gradually withdrawn from the core, thereby ensuring the reactor criticality throughout its operation.

In the case of an unforeseen catastrophic development of the chain reaction or any other emergency conditions connected with the release of energy from the core, each reactor is provided with means for stopping the chain reaction in the form of *emergency shut-off* rods which are dropped into the core. The shut-off rods are made of neutron absorbing material, they are dropped under the influence of gravity into the central part of the core where the neutron flux is the greatest and, hence, negative reactivity introduced by the rod is the greatest. Usually control and shut-off rods are two or three in number. Unlike control rods, shut-off rods are to bind the greatest possible reactivity. Some of the shim rods can be used as shut-off rods.

Control rods are sometimes made of fissionable or scattering but not of absorbing materials. However, in thermal reactors, they are usually neutron-absorbers. There are no effective fast-neutron absorbers (Sec. 4.5-7). Such absorbers as cadmium, hafnium and others are good thermal-neutron absorbers since the first resonance is close to the thermal energy region (Sec. 4.5-9) and outside it they do not differ from other thermal neutron absorbing materials. Boron is the only exception since its absorption cross-section decreases much more slowly with energy than with the above-mentioned substances, following the law $1/v$ (see Fig. 4.17). Therefore, being a weak absorber boron is, nevertheless, slightly better than the other materials. It is only boron enriched with B^{10} isotope that can be used as an absorber in fast-neutron reactors. Besides boron, fissionable materials can also be used for control rods in fast-neutron reactors. A shim rod made of a fissionable material has the same function as a neutron absorbing rod, that is, it increases the reactivity of the reactor in case it falls due to natural reasons. Unlike an absorber, however, a shim rod is outside the core at the start-up of the reactor and then it is introduced into the core. Of all scattering materials nickel is used in fast-neutron reactors since it has a slightly larger fast-neutron scattering cross-section than the others. Scattering rods are inserted throughout the periphery of the core and their insertion into a corresponding channel decreases neutron leakage from the core, thus increasing reactivity. In some special cases movable parts of neutron reflectors may be used

for control, which, while moving, affect the neutron leakage from the core. Control, shim and shut-off rods together with the equipment providing their proper functioning constitute the reactor control and safety system (CSS). The efficiency calculation of control rods is given in [24].

3. The reactor life-time. When the reactor is no longer reactive, i.e., when shim rods have taken the extreme final position, a chain reaction stops. It is only after uranium is refueled in the core that it starts again. A run during which a reactor can operate without refueling is called the *reactor life-time*. Naturally it is desirable to have the power reactor life-time as long as possible since the cost of the energy produced depends on the amount of energy produced at one uranium charge. The reactor life-time is, however, limited by a certain minimum of the critical mass value. A part of fissile material making up the critical mass at the end of the reactor life-time does not decay since the chain reaction stops. It is to be unloaded and may be used as fuel again only after a proper recovery of uranium, in case it is justifiable.

4. Fission product build-up. Natural uranium reactors have a very low value of initial reactivity margin usually determining the reactor life-time. Enriched-uranium reactors may have a higher value of reactivity margin. There is, however, one more limitation on the duration of the reactor life-time associated with the fuel element material response to fission product build-up. Every act of nuclear fission results in the formation of two atoms instead of one, the sum volume of which is twice as large as that of the original atom, as all atoms are of about the same volume (Sec. 1.1-5). Furthermore, the location of atoms in the uranium crystal lattice is random since new atoms cannot be located in uranium crystal lattice sites. Finally, since a considerable part of fission products are essentially gases (Sec. 3.6-6), the build-up of fission products gives rise to internal over stresses and gas pressure increase resulting in cracks, swellings and deformation of fuel elements. Uranium elements are to be unloaded as soon as reactivity margin is exhausted as the life-time of uranium elements is much shorter than that of the main components of the reactor. This, however, proves impossible in case they are deformed. Besides, the cladding of the fuel elements is no more hermetic and radioactive gases penetrate into the coolant. It is to be concluded that the life-time of uranium lumps in a nuclear reactor depends upon their stability to the destructive effects of the built-up fission products. Correspondingly, it is the stability of the fuel elements to the particular effects that primarily determines the reactor life-time. As to the initial reactivity margin, it should be of such value as to have been completely exhausted by the time the uranium lumps are burnt out. Otherwise, fissionable material that is left in the reactor by the

end of the reactor life-time would have to be unloaded, which is unprofitable.

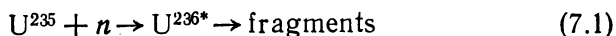
5. The depth of burnup. Although the build-up of fission products can be determined by their quantity in grams per ton of uranium, the direct calculation of fission product mass is, however, extremely complicated. The net fission energy released in the core is always known. Hence, since 1 g of uranium is known to result in about 1 MW·day of energy (358) and in the formation of 1 g of fission products, one can estimate that the value of thermal energy in MW·day is approximately equivalent to that of fission products in grams. The net mass of uranium in the reactor being also known, the quantity of fission products is expressed in MW·day/t, that is, the quantity of MW per day per ton of uranium.

A material is characterized by a certain limiting value of fission product build-up, that is, by the possible allowable depth of burnup of fissile atoms. Say, the depth of burnup for uranium metal is 3000-3500 MW·day/t, as for its compounds it may be much higher. Uranium oxide, for instance, is a porous material. It can, therefore, build up much more fission products than uranium metal, up to 20 000 MW·day/t without any apparent destruction of the fuel elements and, perhaps, even more, up to 100 000 MW·day/t. A ton of natural uranium contains about 7 kg of U^{235} (Sec. 6.1-8). The depth of burnup of 3500 MW·day/t is equivalent to 3.5 kg of fissile atoms. However, since fission products result from Pu^{239} as well that also participates in the process of fission, the amount of consumed U^{235} is not equivalent to that of fission products. The higher the possible allowable depth of burnup, the longer the duration of the reactor life-time and the more efficient the nuclear power plant. The enriched uranium that is much more expensive than the natural one has a much higher burnup. The value of the minimum critical mass by the end of the life-time is less in case uranium metal itself acts as fuel, not its compounds, say, with oxygen. It is to be concluded that many factors affect the efficiency of a nuclear fuel application.

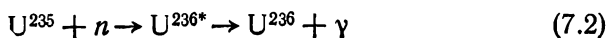
7.2. Nuclear Fuel Conversion

1. Fuel burnup. With time, U^{235} concentration decreases due to the two processes:

fission



and radioactive capture



U^{236} nucleus has an even number of neutrons and is unfissile (Sec. 3.6-2). Therefore, the processes of radioactive capture (7.2) and the process of fission (7.1) decrease the number of fissile atoms. By analogy with thermoengineering, fissile materials are called fuel or nuclear fuel and the conversion of fuel into fission products or unfissile material, fuel burnup. The process of fission (7.1) results in the release of energy and in the formation of secondary neutrons maintaining the chain reaction. Radioactive capture (7.2) is nothing more than a useless loss of both neutrons and fissile material. U^{236} is α -radioactive, its half-life is $T = 2.4 \cdot 10^7$ years, so that it builds up in uranium with the burnup of U^{235} . It is noteworthy that neutron absorption by U^{236} nuclei that follows does not contribute to the build-up of fissile atoms for, having uneven number of neutrons, U^{237} rapidly disintegrates to form long-lived Np^{237} with an even number of neutrons. The same is true of neutron absorption by Np^{237} . The resulting Np^{238} has a short half-life and disintegrates into unfissile Pu^{238} . It is only the fourth neutron absorption that results in fissile Pu^{239} . However, since the absorption cross-section of thermal neutrons by U^{236} nuclei is small, it is of no practical interest.

The production branching ratio between the number of fission acts of U^{236*} compound nucleus through channels (7.1) or (7.2) is determined by the values of the corresponding cross-sections of U^{235} and is designated as factor α (6.11). The greater α , the greater the useless loss of U^{235} and the smaller the number of secondary neutrons (6.10) produced in the process of one neutron absorption by U^{235} . U^{235} (as well as any other fissile nuclides) has the greatest value of α in the lower part of the intermediate energy range, which decreases with the increase of neutron energy (Fig. 7.1, and Secs 4.5-7, 4.5-8, 4.5-9). In the thermal energy region, however, (See Table 6.3) the value of α is smaller than in the intermediate one. The lowest values of α are observed to be characteristic of the fast region, which is made use of in fast neutron reactors to increase the production of new fissile materials.

2. Plutonium build-up. Each act of fission releases more neutrons than are necessary to maintain a chain reaction since along with fission there occurs radioactive capture of neutrons by fissile atoms. It takes more than one $(1 + \alpha)$ neutron on the average to excite a subsequent fission. As for the rest of neutrons, they either get absorbed by other materials contained in the core or get scattered outside it. The fraction of neutrons escaping from the core is not large, the greatest part of the remainder gets absorbed by U^{238} with the formation of the Pu^{239} (3.57). Intermediates of U^{239} and Np^{239} are of no practical importance as their half-life is short. The equilibrium number of these atoms in the core is equal to $N_n = Q/\lambda$

(3.10), where Q is the rate of neutron absorption by U^{238} (3.8) and λ is the constant of radioactive decay. λ being great, the equilibrium number of atoms is correspondingly small. Only long-lived products of neutron reactions build up in the core. Pu^{239} nucleus is a fissile nuclide since it has an uneven number of neutrons. As in the process of nuclear reactor operation the quantity of Pu^{239} increases, it takes a greater part in maintaining a chain reaction. But, like U^{235} , Pu^{239} can absorb neutrons with and without fission with the formation of Pu^{240} . The latter has an even number of neu-

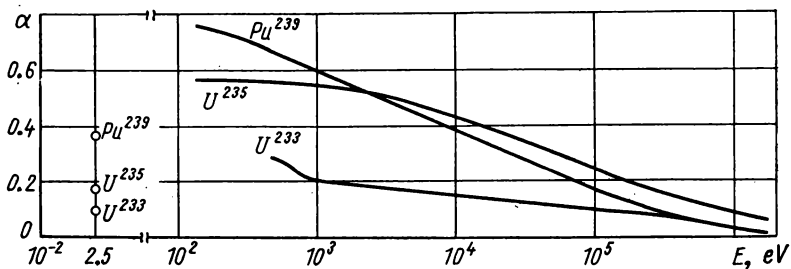
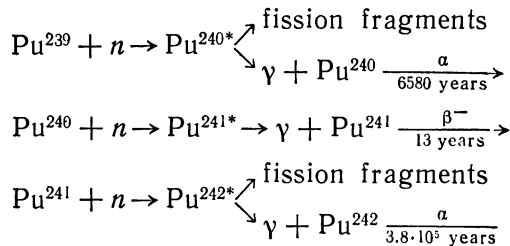


Fig. 7.1. $\alpha = \sigma_\gamma / \sigma_f$ to energy relation. (The points are the values of α at reference heat energy)

trons in the nucleus and is subjected to fission only by fast neutrons, its radioactive capture resulting in fissile Pu^{241} nuclide, etc.



Plutonium isotopes up to mass number 242 have long half-lives and build up in the process of reactor operation. The rate of their build-up and that of U^{235} burnup are governed by the value of the neutron flux, by the corresponding cross-sections and contributions due to absorptions at thermal and resonance energies [24]. Fig. 7.2 shows the schematic process of Pu isotopes build-up and U^{235} burnup where the percentage of nuclides in the fuel is plotted as ordinates and the time of the reactor operation in the range of burnup from 0 to 4500 MW·day/t as abscissae.

3. Reproduction factor. The ratio of newly born fissile atoms to the number of the burnt out atoms out of the whole number of initially loaded atoms is called the reproduction factor F_R . In a thermal reactor, this value can be expressed in terms of the theory presented in Chapter Six. Since a neutron absorption by U^{238} results in Pu^{239} atom, it is the ratio of the number of thermal neutrons absorbed by U^{238} and U^{235} , i.e., $\Phi \Sigma_{a8} / [\Phi (\Sigma_{f5} + \Sigma_{v5})] = \Sigma_{a8} / \Sigma_{a5}$, that determines F_R in the thermal energy region. However, every act of neutron absorption by U^{235} gives birth to η_5 new atoms that are multiplied by the factor of μ in the fast energy

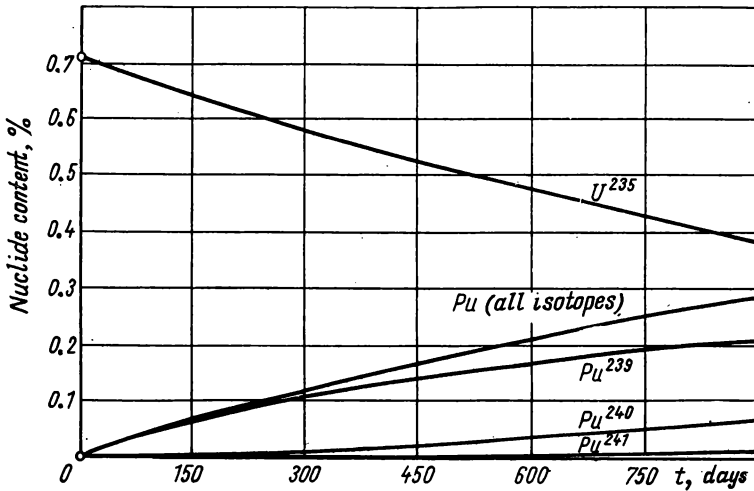


Fig. 7.2. U^{235} burnup and Pu build-up

region and in the process of slowing down $(1 - \phi)$ a part of them is also absorbed by U^{238} with the formation of Pu^{239} . Thus, in the intermediate energy region, $\eta_5 \mu (1 - \phi)$ atoms of Pu^{239} are created per one U^{235} atom burnt out in the thermal energy region. As a result:

$$F_R = \frac{\Sigma_{a8}}{\Sigma_{a5}} + \eta_5 \mu (1 - \phi) \quad (7.3)$$

Expression (7.3) is written without taking account of the neutron leakage which occurs when neutrons are slowed down to the energy of U^{238} absorption resonances. The error is negligible if one deals with a thermal natural uranium or poorly-enriched uranium reactor, where the fraction of neutrons scattered by the core is sure to be small as the value of k_0 is low. Besides, the reactor is assumed

to be critical, i.e., in the case of a bare reactor (7.3) holds if (6.46) is satisfied. It follows from (7.3) that the lower U^{235} concentration, the greater the reproduction factor. Thus, F_R is greater in natural uranium reactors than in enriched-nuclear ones since an enriched-nuclear reactor has a greater reactivity margin which is to be suppressed either by a higher neutron leakage or by useless absorption of neutrons by additional number of shim rods. This results in the decrease of neutron absorption by U^{238} . Furthermore, the smaller φ , the greater F_R , with the condition $k = 1$ being satisfied. The latter, on the contrary, requires the greatest possible φ as it forms part of k_0 (6.7). Thus, F_R of a non-optimum lattice with a reduced φ may be somewhat greater than that of the optimum lattice, but it may be so in case there is some excess of k_0 . The substitution of the known constants from Tables 4.4 and 6.3 into (7.3) and that of the parameters of optimum lattices of a graphite or a heavy water natural uranium reactor ($\mu = 1.03$, $\varphi = 0.9$) results in $F_R = 0.8$. It means that in the process of burning of 10 kg of U^{235} , about 8.5 kg of which results in useful energy, 8 kg of new fissile Pu^{239} is produced as a result of U^{238} decay.

Expression (7.3) is the initial conversion ratio. In the process of the reactor operation and plutonium build-up the number of neutron absorption acts by U^{238} does not make up a net profit of Pu^{239} atoms, since the latter also burn out absorbing neutrons. Pu^{239} burnup results in more portions of Pu^{239} but plutonium breeding in the process of U^{235} and Pu^{239} burning out is essentially equivalent, since U^{235} and Pu^{239} have practically the same value of η (see Table 6.3). Since $F_R < 1$ the number of atoms of secondary plutonium resulting from one U^{235} atom burnt out to form primary plutonium decreases. This gives rise to the decrease of F_R , the decrease being the greater, the deeper the burnup. At the end of the reactor life-time the account is taken of fissionable Pu isotopes only, that is Pu^{239} and Pu^{241} , and U^{235} burnup. In the end, F_R turns out to be less than 0.8 in natural uranium reactors and makes up 0.5—0.6 in enriched uranium reactors.

4. Plutonium-240. At already 3500 MW·day/t burnup, the percentage of Pu^{240} in Pu amounts to 20% and such Pu is not considered to be a pure fissionable material. Fission by fast neutrons is more easy with Pu^{240} than, say with U^{238} or Th^{232} . The Pu^{240} fission threshold is about 0.2 MeV. Beginning with $E = 1$ MeV, the Pu^{240} fission cross-section is even larger than σ_f of U^{235} . In case Pu is used as fuel for fast neutron reactors, its contamination with Pu^{240} isotope is not considered to be a defect. But it is absolutely inapplicable for military purposes since the presence of Pu^{240} in fissile material causes a preliminary thermal explosion in nuclear weapons, the moment supercriticality is reached.

The thing is, that Pu^{240} has a short life-time as compared with other heavy nuclides in spontaneous fission. $T_f = 1.2 \cdot 10^{11}$ years that corresponds to spontaneous fission of 460 000 nuclei per second per kg of Pu^{240} without preliminary neutron absorption. Every act of fission results in 2.2 neutrons, i.e., each 1 kg of Pu^{240} per second produces about one million neutrons. Furthermore, although a self-sustained chain reaction is not possible if $k < 1$, the number of neutrons resulting from exterior sources is sure to multiply if $k \neq 0$. Each portion of the neutrons gives birth to damping circuits, the neutrons of damping circuits sum up with those of newly produced ones (in case of a continuous source) and the overall number of neutrons ($k \neq 0$) proves to be $1/(1 - k)$ times as great as it would have been in the absence of multiplication (the source being the same). The same phenomenon is observed in the absence of Pu^{240} , since the fission of all heavy nuclides is spontaneous, the number of neutrons of other nuclei due to spontaneous fission being, however, negligible (see Table 3.2). The total number of neutrons at any moment in supercritical state and, correspondingly, the number of fission acts depend both on the excess of k over 1 and the original number of neutrons n_0 (6.3). The greatest part of fissionable atoms becomes involved in the reaction before they get scattered by the power of thermal explosion. The greatest number of original neutrons is produced (in case it is possible) after k reaches its maximum. In the process of combining subcritical parts of nuclear weapons k varies continuously from $k < 1$ to its maximum value. In the presence of some excess Pu^{240} , the increase of k is accompanied by such rapid neutron multiplication that thermal explosion occurs before k reaches its maximum, the part of the material consumed in the reaction and the power of explosion decreasing.

In reactors of atomic power plants Pu is produced in the process of deep burnup, contains a relatively large portion of Pu^{240} and is suitable only for peaceful purposes as nuclear fuel for atomic power reactors. A low burnup provides a low percentage of Pu^{240} in Pu, but because of frequent refueling the cost of power generated by such a reactor increases. Furthermore, the fraction of Pu^{240} increases with the increase of the temperature of the reactor core because of the increase of the temperature of thermal neutrons (4.43), the value of α of Pu^{239} increasing with the neutron energy approaching the first resonance energy of Pu^{239} at 0.3 eV. The reactor efficiency depends upon the temperature of the core: the lower the temperature, the lower the efficiency of the power reactor. Thus, operating conditions of the reactor intended for peaceful and military purposes turn out to be different. However, in modern reactors producing conditioned plutonium for military purposes

it is justifiable to obtain some amount of electric power so that it could reduce the cost of plutonium.

5. Temporary increase of k_0 . The absorption cross-section of thermal neutrons of Pu^{239} is 1.5 times as large as that of U^{235} . Therefore, even when $F_R = 0.8$, plutonium build-up results in the increase of neutron absorption by fissile nuclei as compared with that by U^{238} . Since η_9 is not smaller than η_5 (see Table 6.3), it implies an increase of secondary neutrons η (6.12) and (6.13) per neutron absorbed by fissile material. This can be compared with the increase of the efficiency of fissile U^{235} in natural uranium when neutrons slow down to thermal energy region where U^{235} neutrons have a much larger absorption cross-section than those of U^{238} . The substitution of Pu^{239} for a portion of U^{235} still more increases the efficiency of fissile nuclides in neutron absorption. Along with the increase of η one observes a slight increase of θ as the absorption cross-section of the fuel material also increases as a result of the cross-section increase of one of its constituents. In the end, plutonium build-up gives rise to k_0 increase and results in some excess reactivity margin. In the process of reactor operation and fissile nucleus burnup, the excess reactivity gets exhausted. The reactor core, however, may be designed in such a way as to equalize the excess reactivity value and the life-time of fuel elements with uranium metal in a graphite reactor. Such a reactor requires a minimum initial reactivity margin which enables it to operate on account of the temporary excess of reactivity, actually in the course of the reactor life-time.

6. Breeding. If $F_R > 1$, the reproduction of nuclear fuel conversion is called breeding and this process is qualitatively new. Natural uranium contains only about 0.7% of fissile U^{235} , only half of which is used in thermal neutrons reactors. It follows that most of natural uranium meant for energetic purposes is spent. It is true that a part of U^{238} is reprocessed into Pu^{239} which can be extracted from uranium by chemical means. It is evident, however, that if $F_R < 1$, the amount of U^{238} to be reprocessed into Pu^{239} turns out to be always small and comparable to the amount of U^{235} . Even considering that $F_R = 0.8$ and taking no account of possible Pu^{239} losses in the process of its extraction from uranium, one will have 0.8 of Pu^{239} per unit U^{235} burning which results in $0.8 \times 0.8 = 0.64$ of Pu^{239} , etc., the figure that, in the long run, will amount to $1/(1 - 0.8) = 5$ units. In other words, besides 0.35% of U^{235} $4 \times 0.35 = 1.4\%$ of U^{238} of the total amount of uranium will prove fissile. But in case $F_R > 1$, the above progression will prove divergent meaning that the total amount of U^{235} , that of U^{238} and that of Th^{232} , the second by-product, can be reprocessed into fission products with a corresponding release of energy. Therefore, the virtues of breeding are evident.

Breeding in thermal reactors with U^{235} or Pu^{239} fuel proves, however, impossible, because of a great deal of radioactive neutron capture by these nuclei. It is the excess of fission neutrons left over those used for maintaining a chain reaction that makes breeding possible. Since fission process and radiative capture are competitive, it takes $(1 + \alpha)$ neutrons from the number ν (see Table 3.4) released in the previous fission to excite each subsequent fission. The same will be true assuming that there is no radiative capture, but each fission yields η neutrons (6.10). It takes one neutron to maintain a chain reaction, the excess being $\eta - 1$. The excess is wasted off in neutron leakages, in the absorption by the moderator and other exterior materials and by U^{238} itself. Assuming the leakage and other losses to be negligible, $\eta - 1$ turns out to be the limiting value of the reproduction factor. It may be concluded from the data tabled in 6.3 that U^{235} and Pu^{239} cannot provide breeding on thermal neutrons, for 0.2-0.25 of a neutron from $\eta - 1$ of the excess neutrons makes up the above-mentioned inefficient losses. But if U^{233} is used as fuel in a thermal reactor, breeding is feasible. A useless radiative neutron capture by U^{233} nuclei is small and, provided inefficient losses are reduced to minimum, F_R may be but slightly over unity.

It is only in fast neutron reactors that the reproduction factor may considerably exceed unity. In case the core is unmoderated, neutrons get absorbed before they have time to considerably decrease their energy while colliding with nuclei. For fast neutrons (see Fig. 7.1), all fissile nuclides have a smaller α factor than in the thermal energy region with the effect that η in the fast energy region increases approaching ν , as a result, the neutron excess $\eta - 1$ considerably exceeds unity. The number of neutrons lost due to leakage and absorption by exterior materials does not exceed that in a thermal reactor. Hence, the increase of the neutron excess $\eta - 1$ implies that of the reproduction factor. The decrease of radiative capture in the process of absorption of neutrons by fissile nuclei maintaining the chain reaction is the main reason for the increase of the reproduction factor in fast reactors. There are some other reasons that still further facilitate the increase of the reproduction factor. It is pointed out in Sec. 6.6 that the number of fast U^{235} fission neutrons due to U^{238} fission increases by 28% in the limiting case of the medium made up primarily of U^{238} . There are no moderators in fast reactors with the effect that the portion of U^{238} atoms in their volume proves to be considerable. Because of this, the number of U^{238} fission acts may make up about a quarter of the whole number of fission acts involved in the reactor. In the first place, it means that the burnup of unfissionable material is responsible for a quarter of the reactor power, that is profitable in itself, and in the second place, that U^{238} fis-

sion multiplies the number of U^{235} or Pu^{239} fission neutrons, thus increasing breeding. In principle, F_R may outgrow $\eta - 1$, that is, the limiting value of the reproduction factor with no account of U^{238} fission. Finally, since a moderator is lacking, there is a lesser fraction of parasitic neutron absorption in fast reactors with the effect of some greater increase of F_R as compared with F_R of a thermal neutron reactor.

7. A fast reactor. In case the reactor is unmoderated, it is only on enriched uranium that a self-maintaining chain reaction is possible (Sec. 6.1-8). Since a fast neutron reactor is a power reactor, the greatest possible amount of heat is to be removed from the core. An effective heat removal requires that the surface of fuel elements be well developed and a considerable part of the volume be occupied by the coolant. The surface of fuel rods is the larger, the smaller their diameter, a decrease of the diameter meaning an increase of the cladding portion, a layer of which providing the integrity of the fuel elements is of some fixed thickness. Besides uranium and plutonium, there may be other constituents of the fuel providing a higher stability of the fuel element against fission product build-up. As a result, a self-sustaining nuclear reaction in a fast reactor proves possible in case there is about 20-25% of fissile nuclide in uranium. Despite a high concentration of fissile atoms, critical masses of fast reactors are great amounting to hundreds of kg (only pure fissile material being meant). The cost of a fast neutron reactor core turns out to be much higher than that of a thermal one. This is the main defect of a fast neutron reactor, the virtue being the breeding of nuclear fuel.

A neutron leakage from the core of moderately small size being great, breeding proves possible with the reflectors of raw materials being applied, e.g., natural or spent uranium. The reflector (the screen) turns back a part of scattered neutrons to the core and absorbs the rest to form plutonium, the reflector being called the reactor blanket. It is in the reactor blanket that the greatest amount of Pu piles up while F_R of the core is less than unity because of the high concentration of the fissile material. The reproduction factor of commercial plant may amount to 1.5 and even more provided plutonium, which has the greatest ν (see Table 3.4), is used as the primary fissile material. In the experiment with a pure plutonium core and an infinite reflector of U^{238} there was obtained the value of $F_R = 2.5$ that is to be regarded as the actual limiting value of the reproduction factor of a fast neutron reactor.

The value of F_R depends upon the hardness of the neutron spectrum in the reactor, since the greater the energy of neutrons absorbed by fissile nuclei, the smaller the value of α and the closer η to ν . The energy neutron spectrum in a fast neutron reactor

does not coincide with the fission spectrum because of some moderation of neutrons even in the absence of moderating materials. Fast neutrons are effectively moderated by heavy nuclei in the case of inelastic scattering (Secs 4.5-1 and 4.5-7), in the first place, by U^{238} nuclei which have the greatest absolute value of the cross-section σ_n (see Fig. 4.14). Elastic scattering also affects the neutron spectrum. Provided UO_2 or UC or PuC serve as materials for fuel elements, the presence of light atoms in the core also facilitates neutron moderation. Sodium is used as coolant in fast neutron reactors, it is a poor neutron moderator due to its rather appreciable atomic mass ($A = 23$). Nevertheless moderation due to elastic scattering by sodium nuclei effects the neutron spectrum. Thus, the smaller the amount of U^{238} , sodium, and other light materials in the core, the harder the neutron spectrum, and the greater the value of F_R in a fast neutron reactor. If the maximum of the fission neutron spectrum corresponds to the energy of about 1 MeV, the maxima of neutron spectra in fast neutron reactors shift to the range of 0.1-0.5 MeV.

8. Economic aspect. The cost of the primary equipment of a nuclear power plant (N.P.P.) is much higher than that of a fossil power plant. However, in the process of burning up nuclear fuel used in N.P.P. releases $2.5 \cdot 10^6$ times as much energy as one obtains from burning the same mass of high quality coal. It is fuel expenditures on energy production that speak in favour of nuclear power plants. But, as mining, refinement and handling of uranium for use in nuclear reactors are very expensive the evident virtue of nuclear fuel does not pay off. The necessity of enriching nuclear fuel with fissile nuclides makes the cost of it still higher. However, with the development of applied nuclear energetics, the perfection of manufacturing technology of materials and equipment and gained experience, the expenditures on both nuclear primary equipment and fuel fabrication are reduced.

This resulted in that atomic power plants with thermal reactors proved by late 1960's to have been economically efficient. A number of nuclear power plants under construction now will generate energy the cost of which does not exceed that of the energy generated by coal power plants. Through the same considerations a fast neutron reactor atomic plants are expected to have become economically efficient by 1980. After that, fast power reactors are to replace all other power plants and, in the first place, those operating on coal, oil, gas, etc. Taking into account the man's growing requirements in electric power, chemical fuel is to be exhausted within the nearest 200-100 years for its supplies are far from being unlimited, to say nothing of the fact that chemical fuel is indispensable stuff for chemical industry, whereas uranium can be used only as reactor fuel.

Thermal neutron reactors burn only a small part of natural uranium and it is more justifiable to use fission plutonium in fast breeder-reactors. Although the earth crust contains rather large amounts of uranium, it is scattered and there are practically no rich deposits of uranium. Ores with some tenths of % of U_3O_8 are considered to be rich. However, such deposits are rather few and the uranium extracted from them (the cheapest one) is to have been exhausted as thermal reactor fuel by as early as 1990, after which the price of natural uranium will grow fourfold. Assuming this to be the case, it will result in a considerable deterioration of economic indices of atomic power plants. By this time nuclear power breeders are to have been put into everyday use. There will have been sufficient amount of spent uranium piled up in thermal neutron reactors to serve as raw material for plutonium breeding for dozens of years ahead. Fuel is to be provided by fast neutron reactors themselves. U^{235} occurring in nature may be used for fast neutron reactors but only at the very start of nuclear energetics, with time, it is to be replaced by U^{238} fission plutonium, F_R in reactors with plutonium used as primary fuel being greater than that with U^{235} since $\nu_9 > \nu_5$ (see Table 3.4).

Since plutonium is produced in the course of reactor operation, it is only in case fast neutron reactors will yield new fuel at such a rate as to be able to outrun the requirements of the society in energy, that they will grow into the basis of future energetics. The economical index of a fast reactor is characterized by fuel time doubling (T_2), i.e., the period within which a reactor can operate with double efficiency at the expense of fission plutonium already fissioned in the first run. An annual increase of energy consumption in the world makes up about 5%, i.e., energy consumption doubles every 15 years, as for energy production it doubles every 10 years. Hence, for the reactor to be efficient, its T_2 is to be less than 10 years. Reactors reprocessing U^{238} into plutonium (uranium-plutonium fuel cycle) turn out to meet the requirement.

9. Fuel time doubling. The piling of fission products in fissionable material limits the life-time of fuel elements in the reactor. As soon as the reactor life-time is over, the fuel is unloaded from the reactor core and subjected to reprocessing, fission products are removed by chemical means, plutonium is extracted, and new fuel elements are fabricated. Because of high initial radioactivity of fission products in spent fuel elements the reprocessing is rather time-consuming, lasting for months. Radiolysis of water molecules and that of other chemical agents used for fuel refinement results in the production of parasitic substances hindering the separation of chemical elements. Therefore, it is only after prolonged conditioning of fuel elements in special storages for the reduction of radioactivity to the allowable level (reactor cooling)

that proper fuel refinement is to be performed. Thus, nuclear fuel for fast reactors is recycled within the period of time made up of T_{op} and T_p (operating cycle and reprocessing one, respectively). By the end of the reactor life-time the number of fuel elements increases, plutonium, however, is built up only within T_{op} (operating cycle). Suppose, the total amount of plutonium available at present is G_0 , then the ratio of G (the total amount available in the reactor cores) to G_0 is:

$$\frac{G}{G_0} = \frac{T_{op}}{T_{op} + T_p} \quad (7.4)$$

The faster the amount of plutonium available burns up and the more the reproduction factor exceeds unity, the greater the rate of plutonium build-up. Suppose, all the reactors available consume ΔG of plutonium within the reactor life-time, in this case the amount of plutonium produced will make up $F_R \Delta G$, the net profit being $(F_R - 1) \Delta G$. The reactor life-time being T_{op} , the profit per unit time is $(F_R - 1) \Delta G / T_{op}$, thus the period of the increase of plutonium is:

$$\tau = \frac{G_0}{(F_R - 1) \frac{\Delta G}{T_{op}}} \quad (7.5)$$

The build-up of plutonium with time means the increase of operating reactors in number and that of generated power. Therefore, N (the number of operating reactors available) increases with time as:

$$N(t) = N_0 e^{\frac{t}{\tau}} = N_0 \cdot 2^{\frac{t}{T_2}} \quad (7.6)$$

where T_2 is fuel time doubling related to τ by (3.7) (with τ being the period of plutonium build-up). The facilities for the reprocessing of the materials of fuel elements are assumed to be of unlimited capacity and the necessary number of fast reactors are supposed to be always at hand and ready to be fed with fuel so that fuel cycle does not exceed $T_{op} + T_p$.

Formula (7.5) holds true for the case of continuous fuel cycle through reactors and reprocessing plants, i.e., when fuel is delivered to reprocessing plants and reactors in small portions. Actually, fuel is unloaded in batches and it is only after there is enough plutonium in the reactor to make up its critical mass that it is put into operation. Only in case there are a lot of operating reactors fuel cycle approaches a continuous one. However, the time of fuel cycle in the core is different from that in the reactor blanket. Besides, the part of the fuel of the reactor blanket located over and under the core is unloaded together with the fuel of the core since

uranium lumps in the end part of the reactor blanket are designed so that they are in contact with the fuel elements of the core. This complicates the equation for τ or T_2 . By means of equation (7.5) it is, however, possible to account for the trends of T_2 change in the function of the parameters in (7.5). T_2 must be inversely proportional to $(F_R - 1)$ and to the rate of the plutonium burning $\Delta G/T_{op}$. However, the greater the rate of burning, the greater the specific energy release in the cores Q . More intensive energy release necessitates more effective heat removal which requires an extra dilution of the core with the coolant and cladding material to avoid overheating of the fuel element over the temperature allowable for the material involved, which may occur because of the decrease of the diameter of the fuel element.

Such changes in the composition of the core bring about a more effective neutron moderation resulting in T_2 increase (with α increasing and F_R decreasing). Besides, the dilution of the core with the coolant and structural material requires an extra enrichment of fissionable material with fissile nuclides to maintain the reactor in its critical state, especially if one wishes to retain the total power core which cannot continuously vary with the variation of the physical parameters of the core. In the case of the increase of the specific energy release Q , to retain total power means to reduce the volume of the active core. The smaller the volume, the greater the neutron scattering. The above additional enrichment is necessary to compensate for it.

Since there is a limit to fission product piling per nuclear material unit volume, the maximum permissible amount of fission products builds up within a shorter time than it does in the case of a lower plutonium content in a fissionable material provided one wishes to obtain, with higher enrichment, the same or greater power per fissionable material unit mass. This brings about the reduction of T_{op} reactor life-time and, provided (7.4) is observed, the decrease of plutonium amount G available in the reactor core, i.e., the reduction of plutonium piling rate and the increase of T_2 . Thus, with the increase of plutonium burning rate, T_2 is affected in two opposite ways. As a result, T_2 has a minimum which is essentially governed by T_p . Thus, G is the greater (7.4) and T_2 is the smaller, the less time it takes to reprocess fuel elements. Hence, fuel time doubling T_2 is determined not only by the peculiarities of fast reactors themselves but of the fuel cycle as a whole.

$\Delta G/T_{op}$, in case ΔG is expressed in the units of fission power, is the thermal power of any reactor due to Pu fission. Besides, there is some additional power due to U^{238} fission. The ratio of the total power to the overall volume of the reactor cores is Q , i.e., power in MW per m^3 of the core. It is evident that the greater Q , the smaller the reactor volume with a pre-set power and the lower

the capital investments into its construction. It is in the function of Q that fuel doubling period T_2 may be determined. For uranium-plutonium cycle, when $Q = 450\text{--}500 \text{ MW/m}^3$, in about 1000 MW thermal power $\text{UO}_2 + \text{PuO}_2$ or $\text{UC} + \text{PuC}$ ceramic-type thermal nuclear reactors with about 10% of heavy nuclei burnout, i.e., 100 000 MW·day/t, F_R proves to be equal to 1.4–1.6, fuel doubling time being $T_2 = 6\text{--}8$ years, that is less than the above mentioned 10 years. It is noteworthy, that in the fuel cycle $\text{Th}^{232}\text{--U}^{233}$ (3.56) where $F_R = 1.3$ on the account of a low value of ν_3 (see Table 3.4) the fuel doubling time period is 15–20 years. Such reactors will fail to provide the necessary rate of breeding and the necessary increase of power. This does not mean, however, that natural thorium supplies cannot be used for the purposes of energetics. A mixed cycle can be effected with U^{233} , Pu^{239} , and U^{238} used in the core and with thorium in the blanket. This may result in $T_2 \approx \approx 10$ years, Th^{232} being consumed 10 times as much as U^{238} , that is, it is essentially thorium that will be used as fuel in this type of reactors with permissible T_2 . The cost of power generation is the domineering factor in determining power plant efficiency. Though, there may be variations of fast reactor parameters so as to reduce the cost of power generation within the limits of the minimum value of T_2 .

7.3. Radioactive Poisoning of the Reactor

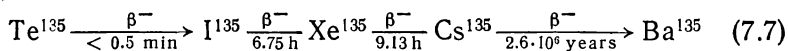
1. **Poisoning.** Fission product build-up results in a decrease of η (6.13) in a heterogeneous medium or that of θ (6.15) in a homogeneous one. Thermal neutron absorption cross-section averaged for all fission products being known [24], one can estimate the corresponding decrease of k_0 . Mass numbers of most of fission product nuclei are over a hundred. Such nuclei, with the exciting energy being of the order of that of neutron bond energy in the nucleus, have a short interval between nuclear energy levels (Sec. 2.7-3) measuring in dozens of eV and the first resonance at the same value of the kinetic energy of neutrons. Low-lying resonances are capture resonances (Sec. 4.5-8). It follows from the above-said that, like U^{238} , fission products also participate in the absorption of intermediate neutrons. But, due to a very low concentration of fission atoms as compared with U^{238} concentration, their effect upon ϕ probability is negligible [24]. Fission products make the greatest contribution to the resonance energy absorption in intermediate reactors, where practically all the neutrons get absorbed before reaching the thermal energy region.

2. **Xenon poisoning.** Some fission products have the first resonance near the thermal region or directly in it, hence, their thermal neutron absorption cross-section values are very high. Of the

greatest importance among them is Xe^{135} having resonance at the energy of 0.084 eV and cross-section $3.15 \cdot 10^6$ barns averaged for Maxwell spectrum (4.44) at normal temperature. An unusually large Xe^{135} cross-section even when the concentration is negligible which is typical of short-lived fission products gives rise to essential neutron absorption. The fact that Xe^{135} is formed through intermediate fission products I^{135} results in the instability of the reactor operation and great negative reactivity after high flux thermal neutron reactor shut-down.

The Xe^{135} effect on the neutron balance is called radioactive poisoning since radioactive Xe^{135} vanishes, in the long run, after the chain reaction stops.

3. The maximum permissible concentration. Xe^{135} is one of the links of the following chain of radioactive transformations of fission products:



half-life for each element of the chain being shown. Mass number 135 approaches the maximum yield of heavy elements (see Fig. 3.14), that is why the yield of this particular chain is rather high $y = 6.34\%$. If one estimates atomic concentration in terms of its production and disintegration rate, one can evaluate the contribution of every fission product to neutron absorption (Sec. 3.1-6). In case Σ_f is a macroscopic fission cross-section, fission rate per unit volume with neutron flux Φ is $\Phi \Sigma_f$, the rate of Te^{135} production being $y \Phi \Sigma_f$. Te^{135} half-life being very small, I^{135} is produced almost at once. The balance of atomic numbers of $\text{I}^{135} - \text{I}$ and $\text{X}^{135} - \text{X}$ per unit volume is expressed by equations

$$\frac{dI}{dt} = y \Phi \Sigma_f - \lambda_i I \quad (7.8)$$

$$\frac{dX}{dt} = \lambda_i I - \lambda_x X - \Phi \sigma_x X \quad (7.9)$$

The first term in the right-hand part (7.8) is the production rate, the second being the I^{135} disintegration rate (3.9). Although fission process itself gives birth to a small amount of Xe^{135} ($y_x = 0.24\%$), taking into account that I^{135} $y_i = 6.1\%$, one can conclude that almost all xenon atoms are the iodine decay products with $y = y_i + y_x$. In this case, the Xe atom balance is the same as that of N_2 in (3.12) with λ_i and λ_x being decay constants of iodine and xenon, respectively, the only correction made is that a new term for Xe^{135} vanishing is introduced in the case of the operating reactor. If σ_x is Xe^{135} thermal neutron absorption cross-section, $\sigma_x X$ is its macroscopic cross-section (1.55), the third term in (7.9) showing the rate of the transformation of Xe^{135} into Xe^{136} in the process

of neutron absorption. Since all substances are neutron-absorbers, equation (7.8) for I^{135} must contain this term as well. However, with the fluxes typical of thermal neutron reactors, this term would not essentially affect the number of I^{135} atoms at any instant of time since the I^{135} absorption cross-section is very small. On the contrary, $Xe^{135}\sigma_x$ absorption cross-section is so large that the coefficient at X in the third term of (7.9) may be even greater than in the second one, i.e., $\Phi\sigma_x > \lambda_x$.

Provided I^{135} and Xe^{135} concentrations equaled zero before the reactor start-up, then the solution for (7.8) is (3.10) with $Q = y\Phi\Sigma_f$, the solution for (7.9) being:

$$X(t) = \frac{y\Phi\Sigma_f}{\lambda_x + \Phi\sigma_x} \left[1 - \frac{\lambda_x + \Phi\sigma_x}{\lambda_x + \Phi\sigma_x - \lambda_i} e^{-\lambda_i t} + \frac{\lambda_i}{\lambda_x + \Phi\sigma_x - \lambda_i} e^{-(\lambda_x + \Phi\sigma_x)t} \right] \quad (7.10)$$

Expression

$$X_s = \frac{y\Phi\Sigma_f}{\lambda_x + \Phi\sigma_x} \quad (7.11)$$

is the saturation concentration of the order of some half-lives of Xe^{135} attainable in the reactor with constant power. Just as in the other cases (3.10), the saturation concentration is the ratio of the rate of radioactive product build-up to the number of fission acts in unit time, but in the running reactor along with the natural decay xenon burnup occurs under the action of the neutrons. Hence it turns out that the number of channels of xenon escape is two which is expressed by the sum of the two terms in the denominator of (7.11). Both the efficiency of the xenon escape through the second channel and the power of the source $y\Phi\Sigma_f$ depend on the neutron flux Φ . The latter is responsible for the maximum permissible concentration X_m in the operating reactor. Although with the increase of the neutron flux Φ the rate of Xe^{135} production increases proportionally, Xe^{135} concentration with the infinitely great flux cannot be higher than:

$$X_m = \frac{y\Sigma_f}{\sigma_x} \quad (7.12)$$

This holds for $\Phi\sigma_x \gg \lambda_x$ which is feasible in a thermal reactor. In fact, with the flux $\Phi = 10^{18} \text{ (m}^2 \cdot \text{s)}^{-1}$, the product $\Phi\sigma_x = 10^{18} \text{ (m}^2 \cdot \text{s)}^{-1} \cdot 3.15 \cdot 10^6 \text{ barn} \cdot 10^{-28} \text{ m}^2/\text{bn} = 3.15 \cdot 10^{-5} \text{ s}^{-1}$ (1.45) whereas $\lambda_x = \ln 2/T = \ln 2/(9.13 \cdot 3600) = 2.11 \cdot 10^{-5} \text{ s}^{-1}$ (3.7). Neutron fluxes in thermal reactors are in the range of $10^{17} - 10^{18} \text{ (m}^2 \cdot \text{s)}^{-1}$. It is essential that although xenon concentration approaches its limiting value (7.12), I^{135} concentration does not. I^{135} cross-section is small and with the actual values of the

flux Φ its saturation concentration $I_s = y\Phi\Sigma_f/\lambda_i$ does not have the second term in the denominator since, as $\Phi\sigma_i \ll \lambda_i$, it always increases in proportion to Φ . Hence, in case Xe^{135} concentration approaches its limiting value (7.12), the amount of I^{135} in the reactor will be unproportionally great as compared with that of Xe^{135} .

4. Poisoning of the operating reactor. The ratio of the number of parasitic absorptions by Xe^{135} neutrons to that by uranium is called poisoning of the reactor P . Since Xe^{135} is produced as a result of uranium fission, it is uniformly mixed up with uranium and poisoning is:

$$P = \frac{\Sigma_x}{\Sigma_{aU}} = \frac{\lambda \Sigma_x}{\Sigma_{aU}} \quad (7.13)$$

Equilibrium poisoning is effected in the operating reactor after the substitution of (7.11) into (7.13), as for a high flux thermal neutron reactor, where $\Phi \gtrsim 10^{18} \text{ (m}^2 \cdot \text{s)}^{-1}$, the maximum permissible poisoning is

$$P_m = y \frac{\Sigma_f}{\Sigma_{aU}} \quad (7.14)$$

High-flux thermal neutron reactors are highly enriched uranium nuclear reactors. Let pure uranium be considered as fuel, then $\Sigma_{aU} = \Sigma_{a5}$ and

$$P_m = y \frac{\sigma_{f5}}{\sigma_{a5}} = 0.0634 \frac{582}{683} = 0.054 \quad (7.15)$$

Fluxes of about $10^{18} \text{ (m}^2 \cdot \text{s)}^{-1}$ are virtual ones for both natural-uranium and heavy water-moderated reactors. In the case of natural uranium, maximum permissible poisoning at the initial U^{235} concentration is (see Table 4.4):

$$P_m = y \frac{\Sigma_{fU}}{\Sigma_{aU}} = 0.0634 \frac{4.18}{7.68} = 0.035 \quad (7.16)$$

The value of poisoning expresses the ratio of the neutron absorption by Xe^{135} to that by uranium. At the same time the value of poisoning may be taken to be that of negative reactor reactivity ρ_x due to Xe^{135} production. By the definition of reactivity (6.95)

$$\rho_x = \frac{k_x - k}{k_x} \quad (7.17)$$

where k_x is the neutron multiplication factor in a xenon poisoned reactor, and $k = 1$ is the multiplication factor of a critical reactor not yet poisoned by Xe^{135} . By means of analytic expression k in a bare critical reactor (6.46), one can obtain the actual value of ρ_x .

Xenon absorbs only thermal neutrons and affects the values expressed by thermal cross-sections θ , η , L^2 . Neglecting the effect of Xe^{135} upon thermal neutron leakage, i.e., considering L^2 to be invariable, (7.17) will take the form:

$$\rho_x = \frac{(\theta\eta)_x - (\theta\eta)}{(\theta\eta)_x} \quad (7.18)$$

In a homogeneous reactor, fission product build-up is allowed for by θ_{hom} factor (6.15), q_f of which denotes poisoning (7.13), provided other fission products, besides Xe^{135} , are ignored. Other fission products being essential, one can always estimate the adend q_x as a part of Xe^{135} equal to P . In any case

$$\rho_x = \frac{\theta_x - \theta}{\theta_x} = -P\theta \quad (7.19)$$

where θ is (6.14) and θ_x is (6.15). In a heterogeneous reactor the effect of Xe^{135} upon reactivity is allowed for by η_{het} (6.13), θ_{het} being affected as well because of the increase of the macroscopic absorption cross-section of the materials contained in uranium lumps and owing to a greater relative reduction of the thermal neutron flux in the lump (see Fig. 6.4). If one does not consider the change of the flux, ρ_x according to (7.18) will be equal to $P\theta_{het}$, where θ_{het} just like in the case for (7.19) is a thermal utilization factor of an unpoisoned reactor. In case one does consider the change of the flux, the absolute value of $P\theta_{het}$ will be increased by $d\theta_{het}/\theta_{het}$ where $d\theta_{het}$ is the increment of θ_{het} in the first approximation due to the change of the flux only, i.e., with the cross-sections being unchanged. Since θ may be approximated to 1 ($\theta = 0.8-0.9$) poisoning may be taken to be negative reactivity due to Xe^{135} . Its value, especially the limiting one, is sufficiently great, (7.15), (7.16), and the reactor control system must have a sufficient reactivity margin to suppress negative reactivity due to Xe^{135} .

5. Xenon pit. With the burnout lacking, one would estimate Xe^{135} saturation concentration by radioactive I^{135} equilibrium. This equilibrium does not hold in the neutron flux, where X^{135} has the maximum permissible concentration (7.12) and I^{135} build-up is unlimited. When the reactor is shut down and the neutron flux is equal to zero, the equilibrium between I^{135} and Xe^{135} atoms is re-established, I^{135} concentration reducing at once, Xe^{135} concentration increasing for some time. Assuming I_0 and X_0 to be I^{135} and Xe^{135} concentrations at the moment of the reactor shutdown, the number of I^{135} atoms will change with time by the law (3.2), that of Xe^{135} ones by the law (3.13), N_{20} being equal to X_0 , $N_{10} = I_0$, $\lambda_1 = \lambda_i$ and $\lambda_2 = \lambda_x$. X_0 being negligible as compared with I_0 , the behaviour of $X(t)$ will prove to be similar to that of N_2'' in Fig. 3.3

when the maximum concentration is reached within the time interval $t_m = [\ln(\lambda_i/\lambda_x)]/(\lambda_i - \lambda_x) = 11.3$ h. The time interval reduces as I_0 and X_0 are getting equal and, when the neutron flux in the operating reactor is $10^{18} \text{ (m}^2\cdot\text{s)}^{-1}$, makes up 10.5 h.

The increase of Xe^{135} concentration implies the growth of the reactor poisoning. The absolute value of the maximum poisoning depends upon $I_0 - X_0$ ratio, i.e., upon the flux, in the operating reactor (Fig. 7.3). When Φ is $\lesssim 10^{17} \text{ (m}^2\cdot\text{s)}^{-1}$, the maximum permissible concentration of Xe^{135} is not reached and after the reactor

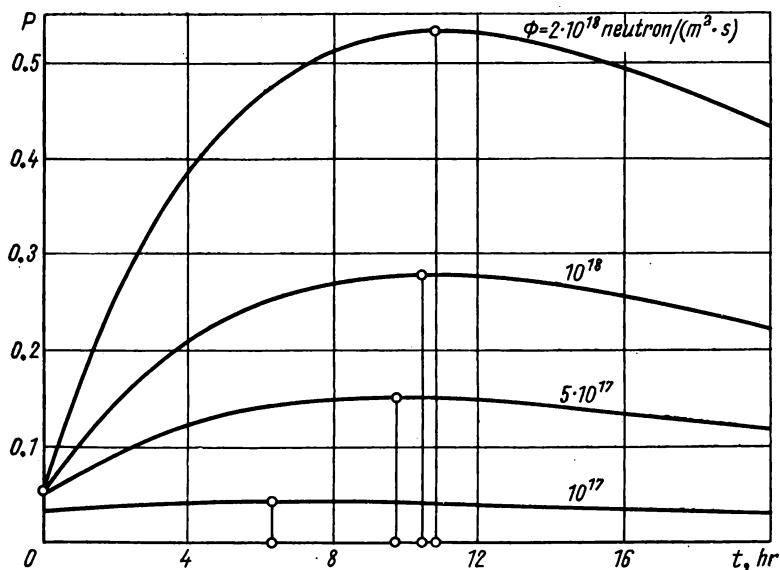


Fig. 7.3. Reactor poisoning upon shutdown

shutdown, the increase of poisoning is not observed. But already in case $\Phi = 10^{18} \text{ (m}^2\cdot\text{s)}^{-1}$, poisoning in 10.5 hours after the reactor shutdown exceeds its maximum permissible value by the factor of 5. Not every reactor has reactivity margin $\Delta\rho = 0.27$ even at the beginning of the reactor life-time so that the reactor cannot be started in some time after its shutdown, i.e., till xenon poisoning exceeds the reactivity margin available. With still higher neutron fluxes, the poisoning maximum rapidly grows, correspondingly increasing the time during which the reactor cannot start operating. This time amounting to scores of hours is called a xenon or iodine pit. After Xe^{135} decay, the reactor spontaneously resumes its operation. As soon as the reactor resumes its operation, Xe^{135} concentration readily reduces to the equilibrium value (7.11) owing to

Xe^{135} burning due to neutrons. Because of a high degree of poisoning after the reactor shutdown the neutron flux $10^{-18} \text{ (m}^2\cdot\text{s)}^{-1}$ is the natural limiting value of fluxes in a thermal reactor. Fortunately, with this neutron flux, the rate of energy release turns out to be too high, and the limitations on the overheating of the reactor core make its operation with the neutron fluxes in the range of $10^{18} \text{ (m}^2\cdot\text{s)}^{-1}$ impossible.

6. Instability of reactor with a high neutron flux. The neutron flux in the reactor with Xe^{135} concentration close to the maximum permissible one has a positive xenon reactivity feedback. A random increase of the flux reduces Xe concentration since the burnup is immediately affected by the change in the flu. As for Xe^{135} build-up, it lags behind on account of the intermediate fission product I^{135} . A reduction in the Xe^{135} concentration brings about the release of a part of reactivity bound by it, with the effect that the reactor becomes supercritical that results in the progressive increase of both the flux and the positive reactivity. If the positive reactivity is suppressed by the control system this will immediately result in the excess build-up of I^{135} in the reactor, which is accompanied by the production of an excess amount of Xe^{135} at a high flux. The increase of the rate of Xe^{135} production will give birth to negative reactivity resulting in the progressive decrease of both the flux and reactivity. Such a reactor, even with balanced flux and Xe^{135} concentration is unstable, being always on the verge of either increasing or decreasing both the flux and the reactivity with the period depending upon the value of Φ . If $\Phi = 10^{18} \text{ (m}^2\cdot\text{s)}^{-1}$ it makes up about a minute. The temperature increase of the medium accompanying the increase of the neutron flux hinders the development of the process because of the negative temperature reactivity coefficient, which is especially high in liquid moderated reactors (Sec. 7.4). As a result, the periods of spontaneous accelerating or decelerating of a chain reaction increase by scores of times. Nevertheless, a high flux thermal reactor due to heavy Xe^{135} poisoning turns out to be unstable in operation if its negative, to say nothing of its positive, temperature reactivity coefficient, is not sufficiently high.

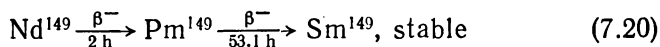
7. Xenon waves. Local overheating of the reactor core moving throughout the pile with the period of about a day occurred in the reactors with the sizes of the core exceeding the critical ones to such an extent that they may be imagined as consisting of several critical reactors, with the overall energy control liberated over the whole reactor volume. The phenomenon was called that of xenon waves since overheating arose from a local decrease of xenon concentration. Large reactors with the system of control rods affecting the whole reactor volume prove to be unstable with respect to the distortion of the neutron field in the reactor volume.

A random increase of the neutron flux in some part of the reactor gives rise to the energy release and a reduction in Xe^{135} concentration. Since this part of the reactor may act as a critical reactor by itself, the processes are developed there independent of those in the other parts of the reactor. However, the control system responds to the energy increase in a part of the reactor in such a way as if the whole reactor became supercritical.

The suppression of the process of the increase of the neutron flux does not immediately stop the process, as it stimulates the decrease of the flux in the other parts of the reactor and the development of the opposite process in them, since the control system is supposed to provide a constant power level in the reactor volume as a whole. It is realizable but at the cost of the distortion of the neutron field resulting from the overheating of one part of the reactor where the flux is higher than the normal one and the underheating of the other one with the flux lower than that. A deeper Xe^{135} burnup in one part is simultaneously accompanied by excess I^{135} build-up subsequently giving birth to excess amount of Xe^{135} which results in the decrease of the flux. A reduction in the burnup in the other part results in the decrease of the Xe^{135} source and, as a consequence, in the increase of the neutron flux. As a result, in about a day period one may observe the displacement of the zones of overheating and underheating. Temperature reactivity coefficient substantially affects the spread of xenon waves over the reactor volume.

It is the function of the proper control system to suppress the formation of xenon waves in high flux neutron reactors. Since the distortion of the neutron field may occur only in case the reactor is so large that in the absence of shim rods its parts can act as self-independent critical reactors, the power control is to be independent in every part of it and corresponding control rods are to come into operation irrespective of the control rods of the other parts.

8. Samarium poisoning. Sm^{149} is another fission product which is a good thermal neutron absorber. Sm^{149} is given birth to as a result of:



The yield of the chain $y = 1.13\%$, the Sm^{149} cross-section in the thermal energy region $\sigma_{\text{sm}} = 4.08 \cdot 10^4$ barn. The half-life of Nd^{149} is short as compared with that of Pm^{149} . And like Xe^{135} , Sm^{149} may be considered to be produced through one intermediate product. Therefore, Sm^{149} build-up is similar to that of Xe^{135} , the only difference is that Sm^{149} radioactive constant decay is equal to zero.

Let S be the concentration of Sm atoms, then:

$$S(t) = y \frac{\Sigma_f}{\sigma_{\text{Sm}}} \left[1 - \frac{\Phi \sigma_{\text{Sm}}}{\Phi \sigma_{\text{Sm}} - \lambda_{\text{Pm}}} e^{-\lambda_{\text{Pm}} t} + \frac{\lambda_{\text{Pm}}}{\Phi \sigma_{\text{Sm}} - \lambda_{\text{Pm}}} e^{-\Phi \sigma_{\text{Sm}} t} \right] \quad (7.21)$$

where $\lambda_{\text{Pm}} = 3.63 \cdot 10^{-6} \text{ s}^{-1}$, i.e., Pm^{149} radioactive decay constant. Sm^{149} saturation concentration $S_s = y \Sigma_f / \sigma_{\text{Sm}}$ is always the maximum permissible one (7.12) and poisoning at saturation is always a maximum (7.14). The maximum permissible poisoning does not depend upon the value of the cross-section of the neutron-absorber, it does depend only upon the yield of the chain (y). Hence, not only good neutron-absorbers but any other stable fission products can, in principle, cause reactor poisoning determined only by the yields of the chain. However, the time it takes considerably exceeds the reactor life-time. The period of the half-build-up of the limiting value, when $\Phi \sigma_{\text{Sm}}$ is at least $\ll \lambda_{\text{Pm}}$ and when the first exponent factor in (7.21) is approximately zero, is $T_{\text{Sm}} = \ln 2 / \Phi \sigma_{\text{Sm}}$ (3.7). Sm^{149} is the only neutron absorber whose period is sufficiently small. When the flux is $\Phi = 10^{17} (\text{m}^2 \cdot \text{s})^{-1}$, $T_{\text{Sm}} = 20$ days and, assuming five periods are necessary for reaching saturation, Sm^{149} will get saturated within 100 days. If $\Phi \sigma_{\text{Sm}} \gg \lambda_{\text{Pm}}$, Sm^{149} build-up is described by the exponent $e^{\lambda_{\text{Pm}} t}$ that is characteristic of Pm^{149} (7.21), since Sm^{149} cannot get saturated before the saturation of its precursor. Five periods of Pm^{149} half-life make up 11 days and with any high flux Φ , it takes exactly 11 days for Sm^{149} poisoning to reach its limiting value (7.14) which, when $y = 0.0113$ equals 0.0096 in the case of pure U^{235} and 0.006 in the case of natural uranium used as fuel which is much less than Xe^{135} poisoning.

After the reactor shutdown, samarium poisoning increases due to its precursor Pm^{149} decay. As distinct from the case of Xe^{135} , this poisoning, however, does not reduce with time, since Sm^{149} is not radioactive. The concentration of Pm^{149} atoms in the operating reactor reaches saturation before that of Sm^{149} does and it is equal to $y \Phi \Sigma_f / \lambda_{\text{Pm}}$. In eleven days all the atoms decay into Sm^{149} atoms and the overall poisoning is

$$P_{\text{Sm}} = y \frac{\Sigma_f}{\Sigma_{\text{aU}}} + \frac{y \Phi \Sigma_f}{\lambda_{\text{Pm}}} \frac{\sigma_{\text{Sm}}}{\Sigma_{\text{aU}}} = y \frac{\Sigma_f}{\Sigma_{\text{aU}}} \left(1 + \frac{\Phi \sigma_{\text{Sm}}}{\lambda_{\text{Pm}}} \right) \quad (7.22)$$

when the flux is sufficiently high $\Phi = 10^{18} (\text{m}^2 \cdot \text{s})^{-1}$ the ratio $\Phi \sigma_{\text{Sm}} / \lambda_{\text{Pm}} = 1.125$ and, after the whole amount of Pm^{149} decays, Sm poisoning doubles amounting to $P_{\text{Sm}} = 0.02$, the value still retains after Xe^{135} vanishes. However, high-flux neutron reactors always have sufficiently great reactivity margin to suppress this kind of poisoning.

7.4. Temperature Coefficient

1. **Core temperature.** Under the steady state operating conditions the rate of the energy release is equal to that of the heat removal from the core. The rate of the heat removal depends upon the temperature difference between the heat source and the coolant, i.e., the environment. Hence, the higher the temperature of the core, the greater the thermal power of the reactor. But, what is important is that the efficiency of a heat engine also increases with the increase of the temperature of the heat source. Therefore, the temperature of the nuclear reactor core is to be as high as possible and its actual temperature value is determined by the stability of its structural materials. Unlike thermally-excited processes (Sec. 6.1-1), a chain reaction can develop at any temperature. Nevertheless, the absolute value of the neutron multiplication factor depends upon the temperature of the multiplying medium through secondary effects. As a rule, the reactivity of the multiplying medium decreases with the decrease of temperature.

2. **Temperature reactivity coefficient.** The value of the reactivity increase with the increase of the medium temperature by 1°K or 1°C $d\rho/dT$ is the temperature reactivity coefficient. The fact that reactivity changes with temperature is due to many reasons. Most of which are accounted for by either the expansion of materials when heated, or by a change in the neutron cross-section in the process of interacting with atomic nuclei moving with high velocities in a heated medium. According to this temperature coefficients fall into two groups: temperature density coefficients and nuclear temperature coefficients.

3. **Temperature density coefficient.** The density of all substances decreases when heated as a result of their being expanded. The decrease of density is the loss of atoms per unit volume (1.2), i.e., macroscopic cross-sections (1.55). The greater the characteristic lengths describing the movement of neutrons in space in the process of diffusion, L^2 (5.59) and τ (5.68), the higher the neutron leakage from the nuclear reactor core. Each of these values is twice inversely proportional to the macroscopic cross-section. Hence, it increases with increasing temperature which means the decrease of ω in (6.49) and reactivity reduction. Thus, the expansion of substances when heated is accompanied by the increase of neutron losses and the emergence of negative temperature effect. On the basis of known linear thermal expansion coefficients and volume thermal expansion coefficients one can estimate the corresponding reactivity coefficient [24, 26]. Since liquids are most of all subjected to expansion when heated liquid moderated reactors have high absolute temperature coefficient values amounting to 10^{-4} $1/^\circ\text{C}$. This means that the heating of the reactor core by every 65°C

brings about negative reactivity equal to β_5 (Sec. 6.10-9). Solid moderated reactors have density coefficients of 10^{-6} $1/^\circ\text{C}$ since solids are less subjected to expansion when heated than liquids.

4. Maxwell spectrum hardening. Most of the nuclear temperature effects occur due to the increase of the energy of the neutrons that are in thermal equilibrium with the medium. An increase of temperature of the medium gives rise to that of the neutron gas (4.43) and the shift of the energy distribution of thermal neutrons (4.44) towards high energy values. Since absorption cross-sections decrease with the increase of energy, the decrease being proportional to $1/v$ in the absence of resonances close to the thermal energy region (Sec. 4.5-5), neutron absorption decreases with the increase of the temperature of the medium. σ_a decrease results in additional L^2 increase (5.59), i.e., still higher thermal neutron leakage. On the other hand, the increase of the mean energy of thermal neutrons brings about a slight reduction of the slowing-down period, decreasing the neutrons age (5.68) with a slight positive temperature effect.

Besides characteristic lengths affecting the leakage, an increase of temperature also causes the change in the factors in the four-factor formula (6.7). Factors θ and η are expressed through cross-sections in the thermal energy region. However, since they are the ratio of the absorption cross-sections and, assuming all cross-sections undergo a change by the law $1/v$, an increase of temperature does not affect them. It is the absorption cross-sections of moderators, those of construction materials and that of natural uranium that follow the law $1/v$. Therefore, θ_{hom} (6.14) would not change. θ_{het} (6.17) increases with the increase of temperature owing to $\overline{\Phi}_{mod}/\overline{\Phi}_U$ decrease (Sec. 6.4-4). The smaller the cross-section of a uranium block, the more uniformly the neutron flux is distributed over the volume of the unit cell (see Fig. 6.4). Thus, a temperature increase positively affects the neutron distribution and, as a result, θ_{het} increase brings about a positive temperature effect. Xe^{135} and Sm^{149} reactor poisoning is also responsible for a positive temperature effect since their cross-sections rapidly decrease with energy and η_{het} (6.13) or θ_{hom} (6.15) increases.

U^{235} absorption cross-section does not quite follow the law $1/v$, the deviation from the law in Pu^{239} being especially great since its resonance is very high at the energy of 0.3 eV (see Fig. 4.20). Although decreasing with the increase of temperature, the ratio of Pu^{239} cross-section to those of other substances increases. Therefore, the effect of plutonium buildup in uranium with increasing temperature may give rise to the $\eta\theta$ increase, that is, to a positive temperature effect. The effect manifests itself to a smaller extent as factor α_9 increases (6.11) in the transition from the thermal energy region to that of the first resonance of Pu^{239} , and as the

number of secondary neutrons due to the absorption of a thermal neutron by plutonium decreases. Nevertheless, the positive temperature coefficient due to the plutonium build-up in uranium-graphite reactors with a low negative temperature density coefficient may surpass all the negative components. In this case, the total temperature reactivity coefficient develops into a small positive one of about $+10^{-5}$ $1/^{\circ}\text{C}$.

5. **ϕ decrease.** The probability of avoiding a resonance capture is feasible in the intermediate energy region and is not affected by the energy distribution of thermal neutrons. However, the heating of the medium causes ϕ decrease due to Doppler effect (Sec. 4.5-4). As temperature increases resonance cross-section peaks get wider (see Fig. 4.13), but the total area limited by the resonance curve does not change. This means that the actual resonance integrals (6.22) do not undergo a change. However, it is not the actual resonance integrals but the temperature dependent effective ones that are responsible for ϕ probability. The physical reason for the increase of the resonance neutron absorption with temperature is as follows: in case uranium nuclei have a low concentration in the homogeneous medium a neutron, within the interval between two collisions with moderating nuclei, interacts only with one uranium nucleus. The absorption probability is determined only by the energy dependence of the cross-section and by the distribution of the relative nucleus and neutron energies, i.e., by the actual resonance integral I_a . It does not depend upon temperature, I_a being not temperature dependent.

In the case of a high nucleus concentration, a neutron, before it gives off its energy as a result of the collision with a light nucleus, approaches some uranium nuclei and, since its energy corresponds to that of the resonance tail, it is not absorbed at a low temperature since all the nuclei have small cross-sections. However, it may be absorbed at a high temperature when the energy of motion of some of the nuclei relative to the neutron corresponds to a large value of the cross-section. The decrease of the cross-section at the exact resonance energy does not essentially affect the number of absorbed neutrons, for, as the resonance cross-section is great, practically all the neutrons with the exact resonance energy get absorbed in the very first collision with a uranium nucleus. In the heterogeneous medium a resonance capture occurs in the material of uranium blocks where uranium atom concentration is high, therefore, ϕ_{het} is always temperature dependent. The temperature dependence of ϕ gives birth to a negative temperature coefficient which is a nuclear one, for it is affected by the change of interaction cross-section $I_{a\text{eff}}$ (6.26), (6.29).

6. **The structural deformation.** Reactivity may also be affected as a result of the deformation of the core structures when heated.

The lattice of thermal neutron reactors is usually optimal or close to that with respect to k_0 . In such reactors, deformation causes a negative temperature effect. The lattices of fast neutron reactor are not optimal with respect to k_0 . The rods highly enriched with fissile material are divided by the coolant with the purpose of efficient heat removal only; therefore, the displacement of the coolant that takes place when the rods come in contact is accompanied by k_0 increase. A positive temperature effect and the deterioration of the heat removal because of the deformation may result in the fact that a part of the core may be overheated and get destroyed. Hence, the reactor core is to be of such a design that would exclude the possibility of deformation due to thermal expansion.

7. Boiling. In boiling-water nuclear reactors, a part of the core contains vapour-water mixture instead of water that acts as moderator. Vapour causes a decrease of the density of the moderator and an increase of neutron leakage and besides it gives birth to negative reactivity that is allowed for in vapour reactivity coefficient. In case vapour content exceeds a particular limit, vapour reactivity coefficient may cause auto-oscillations in a boiling-water nuclear reactor. The increase of the neutron flux due to some reason brings about the increase of vapour content and the decrease of reactivity. The damping of the chain reaction which follows decreases the energy release with the effect that the reactor core gets filled with non-boiling water, reactivity increases and so does the power of the reactor. The amplitude of auto-oscillations, beginning from some particular value of vapour content, increases and may result in the overheating of the core. Hence, there exists the limiting value of vapour content in water in a stable boiling-water nuclear reactor. The vapour content being low, a part of the reactor power has to be removed with hot water and the additional amount of vapour is supplied by the heat exchanger. The increase of non-boiling water gaps between fuel element plates makes it possible to reduce the leakage of neutrons and to increase vapour content in water by up to 25%. The vapour for the heat-generating unit is produced only in the reactor core, thus making unnecessary the heat exchanger.

The unexpected boiling of the coolant in a fast reactor with non-optimal, with respect to k_0 , lattice may bring about positive reactivity. Sodium, to some extent, moderates neutrons and decreases the value of η (6.10). Voids that are formed inside the reactor core due to sodium boiling up hardens the neutron spectrum and increases η , in other words, causes the increase of reactivity. Reactivity proves negative provided boiling takes place all over the reactor core because of the increase of the neutron leakage through the voids between the rods. Nevertheless, as the chain reaction would start in the vicinity of the core centre, where the neutron flux and

the energy release are maximum, reactivity may prove positive within the initial period that is fraught with the start of uncontrollable development of the chain reaction before boiling extends all over the reactor. Hence, the reactor core must not be very high so that the voids formed in the core centre should immediately result in the increase of the neutron leakage through both the upper and the lower supports to suppress the positive reactivity involved.

8. Power reactivity coefficient. The value of reactivity increase with the increase of power by unity dp/dW is power reactivity coefficient. Although the temperature of the core increases with the increase of power, the temperature and power coefficients are not proportional in heterogeneous reactors with forced heat removal, since temperature distribution over the unit cell is non-uniform. The coolant has the minimum temperature, the regions of fuel elements most removed from the heat-exchange surfaces and from those of the moderator, in case it is not specially cooled, have the maximum temperature. Temperature gradients are not the same at various power levels, especially during transients. The greatest part of fission energy is released in the material of the fuel element (see Table 3.3) and with the increase of power it gets heated more readily than the moderator and the structural parts of the reactor core. In this connection, the dynamic power reactivity coefficient may differ from the stationary one at one and the same instantaneous value of power. The dynamic power coefficient in the transient is, in the first place, determined by the temperature coefficients of the fuel element, as the increase of power practically momentarily affects the temperature of the fuel elements. The increase of temperature of the coolant lags behind by tenths of a second, that of the moderator by up to 10 s, that of the reflector by scores of seconds (the reactivity of temperature density coefficients being affected within the same period of time).

Since the equilibrium neutron spectrum is developed in the moderator all the temperature effects associated with the change of the neutron spectrum also delay by up to 10 s. On the contrary, the temperature coefficient due to Doppler effect (Sec. 7.4-5) is determined by the temperature of fuel elements only and, therefore, is an instantaneous one. Fortunately, Doppler reactivity coefficient is negative that makes the dynamic power coefficient negative too. In power nuclear reactors, since the temperature distribution over the core volume is inaccessible for measurement, it is power coefficient that is applied as the power of a nuclear reactor is a known value.

9. Reactor stability. A reactor is considered to be stable in operation if it has a negative temperature coefficient, for, in this case, reactivity has a negative feedback with respect to temperature.

Such a reactor is self-controllable since a small increase in its power causes a decrease of reactivity and vice versa. On the contrary, a reactor with a positive temperature coefficient is considered to be unstable. But it is to be pointed out that a low positive temperature coefficient does not make the reactor uncontrollable. Providing the temperature coefficient is not greater than $+0.04 \beta/^{\circ}\text{C}$ (Sec. 6.10-9), a reactor is still successfully controllable. In case U^{235} acts as fuel, the limit in β units amounts to $+2.5 \cdot 10^{-4} 1/^{\circ}\text{C}$ in the absolute units, in the case of Pu^{239} it comprises $+0.8 \cdot 10^{-4} 1/^{\circ}\text{C}$. In the case of a negative temperature coefficient there should be a reactivity margin to suppress it when the temperature of the reactor core increases. Nevertheless, a reactor must have a negative temperature coefficient that would ensure its stable operation. The absolute values of negative temperature coefficients of reactors range from some units of 10^{-5} up to some units of $10^{-4} 1/^{\circ}\text{C}$.

CHAPTER EIGHT

MATERIALS. BIOLOGICAL SHIELDING

8.1. Materials for Nuclear Reactors

1. Thermal scheme of atomic power plants. The nuclear reactor in the atomic power plant is a heat source. Heat is converted into electric power by means of steam turbines as in conventional power stations: water is converted into high pressure steam, and the turbine wheel and the electric generator rotor mounted on its shaft are set in motion due to the expansion work of steam. The substance removing heat from the reactor becomes radioactive under neutron bombardment, the presence of such a substance in the turbine room is inadmissible as the turbine requires servicing by personnel. It is true that water is only slightly activated by neutrons (Sec. 3.5-16) and, therefore, it is possible to supply the turbine with steam produced directly in the reactor because this small level of radioactivity of the steam disappears quickly after the shut-off of its supply. However, if gaseous fission products get into the steam from depressurized fuel elements, radioactivity increases and persists after the turbine shut-down because radioactive substances (not gases) are again produced as a result of the gaseous fission products decay.

Atomic power plants usually employ a two-circuit heat removal scheme (Fig. 8.1) because of the danger of activation and because of the necessity to use other coolants besides water to remove heat from the reactor. Heat from reactor 1 is transferred by the coolant of primary circuit 2 into heat exchanger 3 from where it passes to the coolant of secondary circuit 4, usually water. The heat exchanger is also the steam generator. Steam passes through turbine 5 connected mechanically with electric generator 6 and in condenser 7 is converted into water which is again delivered to the heat exchanger by circulating pump 8. Pump 9 provides for the coolant circulation in the primary circuit. Through the walls of the heat exchanger, the secondary circuit coolant is exposed to γ -rays which are produced (Sec. 3.3-8) due to the β -decay of radioactive atoms of the primary circuit coolant. However, induced radioactivity does not develop, in this case since β -decay γ -rays usually have insufficient energy to induce photonuclear reactions (Sec. 3.5-17) the products of which may be radioactive. Only delayed neutrons emitted

Table 4.5). (Carbon steels are not used because corrosion is inadmissible in reactor cores as the accumulation of its products in process channels deteriorates heat transfer and heat removal.) If a material is a weak absorber of neutrons it must not contain the least admixtures of strong absorbers. Graphite widely used in electrical engineering and metallurgy cannot be used in nuclear reactors because of the traces of boron. Graphite must be exceptionally pure, heavy water must contain only a small portion of a per cent of residue of light water, zirconium must be carefully refined and hafnium present in zirconium ores thus removed, etc.

Nuclear reactor materials are irradiated by powerful neutron fluxes, γ -rays, and the fuel element materials are also irradiated by fission fragments, resulting in such effects as the destruction of molecules and radiolysis in chemical compounds, and in crystalline bodies — microstructure disarrangements. The accumulation of such effects bring about changes in physical properties and cause structural deformation. Water radiolysis is followed by production of an explosive mixture and radiolysis of organic fluids used as coolants is followed by the production of free radicals which after recombination form either gaseous compounds simpler than the original or more complex solid substances which deposit in the process channels and deteriorate heat removal. The fact that materials must be radiation resistant limits the residence time of some materials in the core and also the possibility of using complex chemical compounds. Finally, fission products accumulate in the material of the fuel elements (Sec. 7.1-4). Prolongation of the reactor life-time and improvements in the economy of the plant depend on the number of fission products that can be accumulated in the material of the fuel elements without damage or deformation.

3. Fuel. Uranium metal has good nuclear characteristics. Metal slugs of 3 cm diameter make it possible to obtain small values of effective resonance integrals (Sec. 6.5-6), this ensures the chain reaction when using natural uranium and graphite. The use of metal fuel elements in fast reactors results in the maximum conversion ratio values. However, the burnup value is not high when uranium metal is used. It is limited to 3000-3500 MW·day/t. In graphite moderated natural uranium reactors, this limitation is not so important because the initial reactivity margin is hardly sufficient to reach the prescribed values of burnup. In heavy water reactors or in reactors using enriched uranium, small limiting burnup values are a very serious disadvantage. Besides, uranium is a polycrystalline material with anisotropic properties of crystals. Local heating due to slowing down of fission fragments is followed by recrystallization, which is accompanied by the growth of uranium grains (due to anisotropic properties) and to the corresponding fuel elements deformation. To improve the radiation resistance of

enriched uranium it is alloyed with molybdenum, niobium, zirconium. Uranium and molybdenum form an isotropic solid solution in the regions where fission fragments pass and this prevents the uranium from swelling. Alloyed uranium allows greater burnup values.

Like many crystalline substances, uranium is characterized by polymorphism, that is, it has different crystalline modifications at various temperatures. The modification characteristic for normal temperature, the α -phase, exists up to 662°C and then changes to the β -phase. As modifications have different atomic volumes, approach to the transition point also causes deformation of fuel elements. Therefore, 662°C is the maximum temperature for fuel elements of metallic uranium. Heat sources are distributed throughout the whole volume of a fuel element and the maximum temperature is observed at the greatest distance from the heat removal surface. In solid cylinders cooled from the surface the highest temperature is at the centre. The temperature difference between the centre and the surface is the greater, the greater is the heat flux transferred to the coolant, that is, this temperature difference is the greater, the more powerful is the reactor and the greater the uranium pin diameter because the heat flux is proportional to the temperature drop across unit length in the direction of heat transfer. In the graphite reactor with natural uranium, the diameter of the uranium pin can not be small because it is necessary to have a low value for the effective resonance integral. Therefore, with the limitation of temperature in the centre and the fixed diameter of the uranium pin, the temperature of the fuel element cladding can not be unlimitedly high, even in the absence of temperature limitations on the fuel element cladding material. For example, in gas-cooled uranium-graphite reactors with $d_U = 3\text{ cm}$, the temperature of the fuel element cladding is up to 450°C , the gas temperature in the primary circuit is about 400°C and that of the steam in the secondary circuit $360\text{--}390^\circ\text{C}$, which is much lower than in usual heat-engineering schemes.

The stability of fuel elements as far as burnup levels and temperature are concerned may be considerably increased by using uranium dioxide UO_2 or monocarbide UC . These are porous materials that can accumulate many fission products up to $100\,000\text{ MW}\cdot\text{day/t}$. They have no temperature limitation up to melting-down. Uranium oxide is used in reactors with enriched uranium of different types, monocarbide being considered a promising fuel material for fast reactors. The use of UO_2 and UC means dilution of uranium with the moderator. Although this dilution is insignificant, it increases the effective resonance integral and, hence, decreases k_0 in the thermal reactor. As a result, natural uranium in UO_2 cannot be used in reactors with the graphite moderator. In

a fast reactor neutron moderation in oxygen or carbon somewhat decreases the conversion ratio. However, this disadvantage is compensated by larger burnups. It is more preferable to use monocarbide than carbon dioxide because it has higher heat conductivity and a lower concentration of moderator atoms.

4. Moderators. In thermal reactors, moderators occupy the greater part of the core volume and, therefore, they should absorb neutrons very weakly (Sec. 5.1-7). Admixtures of neutron absorbers are inadmissible. If light water is used as moderator, it is also used as coolant so that the reactor is of the water-water type. The entire core is heated up to 300 °C. Because of the thick walled reactor vessel (vessel variant) it is possible to maintain high pressures which prevent water from turning into steam. If heavy water is used as moderator it is usually kept at a temperature below 100 °C and only the fuel elements and the coolant passing around them have high temperatures. High pressure is maintained by means of coolant tubes (tube variant). In this case the heat energy released in the moderator (see Table 3.3) is not used for generating electric power but is removed at low temperatures through a separate heat-exchanger.

5. Coolants and structural materials. The two-circuit heat-removal allows the employment of any coolant in the primary circuit, that satisfactorily maintains the chain reaction and high temperatures in the core. Finally the choice of the coolant is dictated by the economy considerations, by the possibility of attaining maximum efficiency of the plant. In this respect most important is the amount of power required for pumping the coolant through the core. Evidently, the above-mentioned power requirements are inversely proportional to the quantity of heat removed from the core during one cycle (in other words, they are inversely proportional to the temperature of the coolant). The greater the temperature difference of the heating and heated bodies, the further the heating cycle from the ideal and the lower is the efficiency. Therefore, the less the heating-up of the coolant in the core, the better. Thus for the given power of the plant relative to the efficiency, there are optimum values for the coolant heating-up and flow rate through the core. As for specific thermophysical properties of the coolant, we can say that, other conditions being equal, the greater the heat capacity of the coolant per unit mass and the greater its density [28], the less the pumping power requirements for the coolant. From this point of view water is the best coolant. Liquid metals are somewhat worse than water, organic liquids are still worse and gases are the worst (by 5-6 orders). Therefore, when gases are used as coolants, power requirements of the plant are always high, which, of course, lowers the net efficiency of the atomic power plant. The great difference in the amount of power required for pumping

water and gas in the fuel channels decreases with the increase of flow area and gas pressure, with a more intensive heating of the coolant in the core and a greater temperature difference between the fuel element cladding and the gas.

The absolute value of the coolant temperature at the outlet from the reactor is usually determined by the maximum allowable temperature of the fuel element cladding. The material of the cladding and also of the pipes (which are used to prevent mixing of the coolant and the moderator) should preserve its strength at high temperatures and be corrosion-resistant in the stream of the hot coolant. Therefore, the choice of the coolant should be compatible with the choice of the structural material. Many metals are subject to the active corrosion in water heated to above 300 °C. Hence, if water is used in the primary circuit of the atomic power plant, aluminium can be used as a structural material at temperatures only a little above 200 °C, zirconium, up to 300 °C and at higher temperatures, stainless steel. In this case, the corresponding steam temperatures in the secondary circuit are also rather low: below 200 °C in the case of aluminium and 270-290 °C for zirconium.

A further disadvantage of water is its very high saturated vapour pressure at high temperatures (16 atm at 200 °C, 40 atm at 250 °C and 88 atm at 300 °C). Organic coolants containing hydrogen and similar to water as far as their nuclear properties are concerned (such as diphenyl (C_6H_5)₂) are free from these disadvantages. Low vapour pressure of the coolant and low chemical activity allow the cladding of fuel elements to be heated up to 400 °C when using aluminium as the structural material. When aluminium, being a weak absorber of neutrons (see Table 4.5), is used for the claddings and pipes, it means that natural uranium can be used in the reactor with a graphite moderator and hydrogenous coolants if the film of water (at low temperature) or of an organic coolant on the fuel element surface is not more than 2 mm thick. However, radiolysis of organic coolants followed by polymerization makes it necessary to purify and replenish the coolant continuously during the reactor operation. This limits the application of organic compounds in low-capacity plants which use enriched uranium.

Gases have the worst heat transfer properties. Nevertheless when chemically inert gases (carbon dioxide, helium) are used, the temperature at the outlet from the reactor may be higher than when water is used as a coolant. To improve the transfer of heat to the gas the surface of fuel elements is finned. This increases the quantity of the structural material in the core. In gas-cooled reactors, magnesium alloyed with 0.8% of aluminium can be used, the magnesium absorption cross-section for thermal neutrons being less than that of aluminium (see Table 4.5). The gas temperature at the outlet of the reactor with such cladding of fuel elements is about

400 °C. A very important advantage of gas-cooled power reactors is the possibility of using natural uranium with a graphite moderator. If a graphite moderator, is used pipes for pumping the gas through the core are not necessary. In a heavy-water natural uranium reactor, the temperature of the gas may be higher. In these reactors the better multiplication properties of the medium allow reduction of the uranium pin diameter with a corresponding increase of the fuel element cladding temperature, keeping $k_0 > 1$ in spite of the increase of the resonance neutron capture. The use of berillium cladding in the fuel element allows the temperature of the wall in contact with gas to be as high as 600 °C. However, in this case metallic uranium (Sec. 8.1-3) cannot be employed and the use of UO_2 implies change-over to enriched uranium in the graphite moderated reactor. Carbon dioxide at high temperature has almost the same heat transfer properties as helium and is cheap.

Gas coolants are contemplated to be used in high temperature reactors with the gas to be heated to 650 °C and higher. At such temperatures it is more economical to apply gas turbines. For very high temperatures the following structural materials may be used: graphite, stainless steel, niobium, ceramics and cermets, mixtures of ceramics and metals. Gas and even steam may be employed to remove heat from fast reactors. Although in fast reactors it is not possible to use materials having light atoms, the low density of gases means negligible moderating power (Sec. 5.1-5) and insignificant influence on the neutron energy spectrum.

High parameters of the steam turbine cycle are achieved when heat is removed by liquid metals, sodium being the best. When stainless steel is used for fuel element claddings, the temperature of sodium at the exit of the core is 500 °C but may be higher up to 650 °C. The pressure of sodium vapour is low, which is an important advantage. However, the use of sodium complicates the technological layout of the atomic power plant. As sodium actively reacts with water (water is always present in the steam power circuit) and is strongly activated by neutrons (Sec. 3.5-16) one more intermediate non-radioactive sodium loop is provided to prevent a heat-exchanger accidental failure which may be accompanied by chemical explosion and radioactivity dispersion. In addition, as sodium is a solid substance at normal temperature ($t_{\text{melt}} = 98\text{ °C}$), the atomic power plants must have some means for melting it and for pre-heating the core and heat exchangers before the sodium loops are filled. In reactors with a sodium coolant, it is desirable to discharge the spent fuel assemblies and to charge the new ones without shutting-down or cooling the reactor. All this increases investments in the construction of the plant. Sodium is a weak moderator and is the best coolant for fast neutron

reactors. In reactors with neutron moderation, sodium can be used with graphite or berillium but not with light or heavy water. Sodium absorbs thermal neutrons almost like water and it is incompatible with magnesium or aluminium (above 200 °C). Therefore, when sodium is the coolant in the reactor with the graphite moderator, it means that enriched uranium is used as fuel. Reactors with the berillium moderator always use enriched uranium (Sec. 6.7-2). Lithium could be even a better coolant than sodium. However, it is impossible to use lithium in thermal reactors because of the intensive neutron absorption (see Table 4.5) or in fast reactors because it is a light material ($A = 7$) and consequently is a good neutron moderator.

As all materials weakly absorb fast neutrons, the choice of structural material for a fast reactor is not connected with the absorptive power of the material. However, thermal reactors are very sensitive to this characteristic. The fact that aluminium is incompatible with water and sodium at temperatures above 200 °C and that the absorption of thermal neutrons by stainless steel is relatively strong, made it necessary to search for other materials, zirconium proving to be the best. The absorption cross-section of pure zirconium is less than that of aluminium (see Table 4.5). To improve its mechanical properties in processing zirconium is used as a polycrystalline alloy with composite addition of 0.5-2 per cent by weight of tin, iron, nickel. (This material is called ougenite in the USSR and zircalloy in the USA.) Zirconium is used in boiling water-water reactors and also along with stainless steel in pressurized water and sodium-cooled reactors.

The possibilities of obtaining high temperatures in atomic power plants are limited and because of this saturated steam and specially designed turbines are used in the power circuit. At the same time production of superheated steam with high parameters increases plant efficiency and simplifies turbine design. This is particularly important because powerful superheated steam turbines are used usually in thermal engineering. Nuclear superheating of steam, that is, superheating in the reactor itself was carried out in Beloyarskaya atomic power plant. Part of the fuel channels of the reactor are employed for heating up water and steam production, and the remaining part, for superheating steam. Steam at temperature and pressure of 500 °C and 90 atm, respectively, is supplied to the turbine directly from the reactor. As in other atomic power plants with one-circuit heat removal, the reliability of the hermetic fuel element cladding is of great importance since the entry into coolant of gaseous fission products will increase radioactivity in the turbine room and create difficulties in turbine servicing.

In reactors with two-circuit heat removal superheated steam is produced by heating the coolant of the primary circuit at least up

to 500-600 °C. For this, therefore, stainless steel must be used in the core, and gas or liquid sodium as coolants (Sec. 9.2-8, Sec. 9.5-3).

8.2. Dosimetry and Shielding

1. **Units of measurement.** Moving charged particles lose their energy because of atom ionisation. Uncharged particles also transfer their energy to the substance by means of ionisation but this time through secondary particles; γ -rays, when absorbed, produce electrons with high kinetic energy; fast neutrons, when scattered, produce protons, nuclei and recoil ions; when thermal neutrons are absorbed they produce heavy charged particles or γ -rays as products of the nuclear reactions. As in the case of neutrons, fluxes of any particles are referred to the unit of the intersected area in unit time and are measured in $1/(m^2 \cdot s)$ (this is actually the flux density) (Sec. 1.6-2). If the energy of each particle is E , then the energy transferred by the flux is ΦE (W/m^2), it is called *radiation intensity*. In practice, it is not the transferred energy that is measured but the electric charge being produced by the particles owing to the ionisation of atoms of the media. This charge characterizes the radiation *exposure* dose. For X -ray and γ -radiation with energy up to 3 MeV, a unit exposure dose is taken to be the amount of radiation, which at any time in 1 kg of dry air produces 1 coulomb of electricity of each sign, that is, the exposure dose unit is one coulomb per kilogram (C/kg). The dose rate unit or the dose unit in unit time is one ampere per kilogram (A/kg). From the point of view of injury to the biological objects during radiation the most important factor is the energy transferred to the substance and it is characterized by the *absorbed dose*. Since a definite mean consumption of energy is connected with ion pair production, the exposure dose is compared with the absorbed dose of radiation which is measured by the amount of energy transferred to the mass unit of the substance, i.e., joule per kilogram (J/kg). Correspondingly, unit power of the absorbed dose is watt per kilogram (W/kg).

In the CGS system of units, the exposure dose and absorbed dose units have their own designations which are widely used at present. These units may be used out of the framework of the system. In CGS system the roentgen r is taken as the exposure dose unit (by definition, the amount of X -ray or γ -radiation producing in 1 cm^3 of dry air, at 760 mm Hg and 0°C , one electrostatic unit of charge of each sign or $2.08 \cdot 10^9$ ion pairs). This corresponds to the energy transferred to the air, that is, to the absorbed dose of 0.11 erg/cm^3 or 88 erg/g . In the soft tissues of the human body γ -rays are absorbed somewhat more intensively than in air so that the released energy of $88\text{--}98\text{ erg/g}$ corresponds to the exposure

dose of 1 *r*. The absorbed dose of 100 erg/g was called 1 rad. It is clear that an exposure dose a little larger or equal to 1 *r* corresponds to the absorbed dose of 1 rad. As follows from the given definitions, roentgen and rad are expressed in the SI system (density of air 1.29 kg/m³ at 0°C and 760 mm Hg) as

$$1r = 2.58 \cdot 10^{-4} \text{C/kg} \quad (8.1)$$

$$1 \text{ rad} = 10^{-2} \text{J/kg} \quad (8.2)$$

2. Dosimetry. The extent of injury of living tissue by ionising radiation does not only depend on the amount of energy transferred to the unit mass of tissue but also on the amount of specific ionisation. The greater the number of ion pairs produced in unit path length by the moving particle, the heavier are the consequences of radiation injury on the cells vital activity at one and the same absorbed dose. Therefore, each kind of radiation is characterized by its inherent value of relative biological effectiveness (RBE). As the relative biological effectiveness of radiation depends also on many specific biological factors, the coefficients of relative biological effectiveness are recommended for use only in radiation biology, and in radiation dosimetry the identical concept of the radiation quality coefficient *K* is employed. The radiation quality coefficient as well as the value of relative biological effectiveness increase with specific ionisation.

Medical roentgen-equivalent (roentgen-equivalent-man) is introduced to measure the biological effect of ionisation on the human organism. One and the same dose in terms of roentgen-equivalent-man or equivalent dose *D_{eq}* denotes the same degree of injury by any radiation and therefore different absorbed doses *D* of different kinds of radiation. For the standard is taken the radiation effect of X-ray radiation having energy *E* = 200 KeV for which 1 rad of absorbed dose is also equal to 1 roentgen-equivalent-man of the equivalent dose of biological effect. In this case, the quality coefficient *K* = 1. The quality coefficient is also taken equal to unity for all other kinds of radiation if their specific ionisations are not higher than the given one. If specific ionisation is higher than standard radiation ionisation, then *K* > 1 and the relationship between the medical equivalent dose *D_{eq}* and the absorbed dose *D* is the following:

$$D_{eq} = KD \quad (8.3)$$

Safety regulations stipulate limiting levels of irradiation for people working with radioactive materials [15]. The maximum permissible safe radiation dose (i.e., without harm to the individual) during one year is 5 roentgen-equivalent-man (rem) or 100 mrem per week with continuous external irradiation of the whole body. Ho-

wever, this dose is estimated for the organs most vulnerable to radiation, such as gonads and haemopoietic organs. For the other internal organs the irradiation three times larger is admissible. As for hands, forearms and feet it may be 15 times larger, that is, 75 roentgen-equivalent-man per year.

Table 8.1 shows maximum permissible dose rates, intensities or particle fluxes for a 36-hour working week and different radiations on the basis of the weekly dose of 100 mrem. The quality values of radiation coefficients are also given. The quality coefficient for protons and α -particles is 10 and for multi-charged ions 20.

Table 8.1

**Maximum Allowable Levels of External Radiation
Corresponding to the Dose of 100 roentgen-equivalent-man/week**

| Radiation | Energy, MeV | Quality coefficient, K | Dose rate, intensity or flux for a 36-hour working week |
|-----------------------|--|---|--|
| γ and X -ray | $\left\{ \begin{array}{l} \text{Up to } 3 \\ 3 \cdot 10^1 \end{array} \right.$ | $\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$ | $\left\{ \begin{array}{l} 2.8 \text{ mr/h} \\ 2000 \text{ MeV}/(\text{cm}^2 \cdot \text{s}) \end{array} \right.$ |
| Electrons | Up to 10 | 1 | 20 $1/(\text{cm}^2 \cdot \text{s})$ |
| Neutrons: | | | |
| thermal | $2.5 \cdot 10^{-8}$ | 3 | 750 $1/(\text{cm}^2 \cdot \text{s})$ |
| slow | 10^{-7} | 3 | 550 $1/(\text{cm}^2 \cdot \text{s})$ |
| intermediate | $5 \cdot 10^{-3}$ | 2.5 | 640 $1/(\text{cm}^2 \cdot \text{s})$ |
| | $2 \cdot 10^{-2}$ | 5 | 310 $1/(\text{cm}^2 \cdot \text{s})$ |
| | 0.1 | 8 | 90 $1/(\text{cm}^2 \cdot \text{s})$ |
| | 0.5 | 10 | 33 $1/(\text{cm}^2 \cdot \text{s})$ |
| fast | 1 | 10.5 | 20 $1/(\text{cm}^2 \cdot \text{s})$ |
| | 5 | 7 | 20 $1/(\text{cm}^2 \cdot \text{s})$ |
| | 10 | 6.5 | 20 $1/(\text{cm}^2 \cdot \text{s})$ |
| very fast | $2 \cdot 10^2$ | — | 10 $1/(\text{cm}^2 \cdot \text{s})$ |
| super fast | $5 \cdot 10^2$ | — | 6 $1/(\text{cm}^2 \cdot \text{s})$ |
| | $2 \cdot 10^3$ | — | 3 $1/(\text{cm}^2 \cdot \text{s})$ |
| | $5 \cdot 10^3$ | — | 1 $1/(\text{cm}^2 \cdot \text{s})$ |
| | 10^4 | — | 0.3 $1/(\text{cm}^2 \cdot \text{s})$ |

3. Reactor radiation. A reactor core is a bulky object. Charged particles with small mean free path length cannot leave the reactor. This is possible only for neutrons and γ -rays but they are in most cases absorbed in the core. Small values of k_0 especially in natural uranium reactors make rather large core volumes necessary so that the neutron leakage percentage is kept small as condition (6.1) must always be observed. Factor $1 - w$ shows the percen-

tage of neutrons not absorbed by the material of the core. However, not all neutrons are scattered in the surrounding space, part are absorbed by the control rods and reflector. The greater part of γ -rays are produced in uranium, a heavy material effectively absorbing γ -radiation. The moderator, neutron reflector and other materials also participate in γ -rays absorption and only a small part of γ -radiation is scattered beyond the reflector. Nevertheless, up to 3 per cent of the total fission energy or up to one fourth of the energy associated with the formation of neutrons and γ -rays in the reactor passes out of the core volume beyond the reflector. This fraction of radiation, by absolute value, is of exceptionally high intensity. As a result, it is necessary to have not only biological shielding which reduces neutron and γ -rays fluxes by millions and tens of millions of times but also special shielding of the reactor vessels (thermal shielding), shielding or cooling of the front walls of the concrete biological shield where the greater part of the dissipated radiation is absorbed.

The energy range of neutrons leaving the reactor is very wide, from fission energy to thermal energy. Thermal neutron reactors (except those with a water moderator) dissipate more neutrons with low energy. The cores of light water-moderated and cooled reactors are comparatively small in volume and water is sufficiently transparent for fast neutrons due to the small value of σ_{tr} (5.38) of hydrogen at high energy, that is, it causes increased leakage of fast neutrons. Fast reactors dissipate fast neutrons. Energy of γ -rays leaving the core lies mainly in the range of 0.1-2 MeV. γ -Rays are emitted by excited nuclei produced in the fission, β -decay and neutron radiation capture processes with a resultant energy value of about 20 MeV per act of fission (see Table 3.3). This is the energy of about 10 excited nuclei which de-excite γ -rays either one by one or in cascades of two or more. The excitation energy of fission fragments after emission of neutrons and nuclei (β -decay products) is relatively small but in neutron absorption it can reach 8 MeV or more because in this case it is equal to their bond energy (Sec. 3.5-3). At such great excitation the nuclei often emit high energy quanta close to the total excitation energy. Table 8.2 gives the energy values of hard γ -rays emitted when neutrons are absorbed by nuclei of the atoms of the reactor materials.

De-excitation of some nuclei is accompanied by the appearance (although with small probability) of especially hard γ -rays. Maximum energy of such rays is also presented in Table 8.2. The definite number of captured γ -rays produced is determined by the values of the neutron flux and macroscopic cross-section of nuclei-neutron absorbers, that is, by the number of absorptions (1.56). It is also determined by the γ -rays yields with different energy during

Table 8.2

Hard γ -Rays of Radiation Capture

| Element | Rays with high yield, MeV | Rays with low yield, MeV | Element | Rays with high yield, MeV | Rays with low yield, MeV |
|---------|---------------------------|--------------------------|------------------|---------------------------|--------------------------|
| H | 2.22 | — | Fe | 7.64 | 9.30 |
| Be | 6.81 | — | Zr | 6.30 | 8.66 |
| C | 4.95 | — | Mo | 6.92 | 9.15 |
| Na | 6.42 | — | Cd | 5.94 | 9.04 |
| Al | 7.73 | — | Pb | 7.38 | — |
| Si | 4.93 | 10.59 | Th | 3.53 | 4.92 |
| Ca | 6.41 | — | U ²³⁸ | 4.06 | — |

the transition of excited nuclei into ground energy states [13, 14, 29]. The percentage of high energy γ -rays in the total flux energing from the reactor is not large because only one third of the energy of all γ -rays produced after one fission is emitted during neutron radiation capture, while two thirds of all the captured γ -rays are emitted by U²³⁸ and have energy not higher than 4 MeV. In addition, part of the γ -rays undergo Compton scattering, which reduces their energy before they leave the reactor. As a result, less than 10% of all γ -rays lie in the energy range of more than 2 MeV, and about 1% of γ -rays have energy more than 4 MeV.

4. **Biological shielding.** The shielding material requirements are controversial. It is necessary to use the heaviest possible materials (Sec. 4.4) for protection against γ -radiation. Shielding from thermal neutrons does not present any difficulty as there are many substances which are strong absorbers of thermal neutrons (Sec. 4.5-9). For example, a layer of cadmium 1 mm thick blocks any flux of thermal neutrons. But there are no effective absorbers of fast and intermediate neutrons. It appears that the slowing down of neutrons requires less material than their absorption with high energy, water being the best and the cheapest moderator. As $\tau \gg L^2$ for water (see Tables 5.3 and 5.4) it is not necessary to employ other materials to absorb thermal neutrons being produced because hydrogen nuclei absorb them strongly enough. However, water consists of light atoms and weakly absorbs γ -rays. As a result, biological shielding of a reactor must simultaneously meet two requirements: on the one hand, it must contain the greatest possible amount of light hydrogen atoms for slowing down neutrons and, on the other, it must contain the greatest possible amount of heavy atoms for effective absorption of γ -rays. In addition, as the reactor shield is a massive (bulky) structure it should be made of a material which is not very expensive and used in building.

The shield must be a monolith without any slits transmitting radiation. Therefore, the most practicable material for shielding is concrete in spite of the fact that building concrete consists of light atoms, has the density of 2000-2300 kg/m³ and water content about 200 kg/m³, that is, five times less than in water itself. The composition of building concrete is as follows:

| Element | Composition, % by weight |
|--------------------------------|--------------------------|
| O | 50 |
| Si | 30 |
| Ca | 8 |
| Al | 4 |
| K | 2 |
| H | 1 |
| Na, Mg, S and others | 5 |

However, the shielding properties of concrete can be improved by using instead of sand and gravel other aggregates which increase both water content and the density of concrete. For shielding nuclear reactors, heavy concretes are used with such aggregates as limonite $2\text{Fe}_2\text{O}_3 \times 3\text{H}_2\text{O}$, baryte BaSO_4 , ilmenite FeTiO_3 , the density of which approached 3500-4200 kg/m³. At the same time concrete with the density of 2200-2500 kg/m³ is often used. Shields of experimental reactors or reactors used for transport purposes where the smallest possible thickness or the lowest total weight is required are made of more expensive materials. Paraffin, steel, lead and bismuth walls, concrete with cast iron pig and shot aggregates having the density of 6000 kg/m³ are used. Very often combined shielding is employed. In this case the first layer is water for protection against neutrons and the second layer is concrete for protection against γ -radiation. As the shielding against neutrons is simultaneously the source of γ -rays of radiative capture, the concrete shield must always be placed after the water shield and not vice versa. The first layer of shielding for fast reactors is sometimes made of graphite with a small addition of boron because $L^2 \gg \tau$ for graphite. The thickness of water and graphite shields sometimes reaches 1 m and that of concrete shields 2.5-3 m. The overall mass of concrete biological shielding in a large power reactor amounts to thousands and even tens of thousands of tons.

CHAPTER NINE

REACTORS OF ATOMIC POWER PLANTS

9.1. Water-Cooled Graphite-Moderated Reactors

1. **The world's first atomic power plant.** The world's first atomic power plant was put into operation in Obninsk, USSR, in June, 1954. The heat source of the power plant is a thermal reactor. Designed as an experimental installation, it has a small volume. This is achieved due to enriched uranium metal used as a fuel. Initially the enrichment was 5% and later it was brought to 6%. Graphite is the moderator, water is used as coolant and stainless steel as the structural material. The reactor is a cylindrical assembly of graphite blocks with a total mass of 50 tons, placed in a carbon steel vessel of 3.2 m diameter and wall thickness of 1.5 cm (Fig. 9.1). The maximum temperature of the graphite in the operating reactor is 800 °C. To prevent the graphite from oxidation the vessel is filled with helium or nitrogen.

2. **Core.** The central part of the graphite volume contains uranium and is the reactor core. The remaining graphite acts as a reflector. The effective diameter of the core $D = 1.5$ m, its height $H = 1.7$ m. The graphite blocks of the core are regular hexagonal prisms having axial holes of diameter $d = 65.8$ mm. The spacing between the axes and, correspondingly, between the parallel faces of the hexahedrons is 120 mm. Arranged in the holes are 128 fuel assemblies, 2 safety rods and 18 shim rods. Three channels remain free for carrying out experiments. The reflector accommodates four control rods.

3. **Fuel assembly.** The cylindrical fuel assembly ($d = 65$ mm) is made of graphite with 5 stainless steel tubes passing through it (Fig. 9.2). Water is supplied from above through the central tube (external diameter $d_e = 15$ mm, wall thickness $\delta = 0.6$ mm) into the distributing manifold in the bottom part. From the manifold it flows upwards through four peripheral tubes ($d_e = 9$ mm and $\delta = 0.4$ mm). A fuel element of metallic uranium is fitted over the 1.7 m long section of each of the four tubes. These elements are hollow cylinders having an internal diameter corresponding to the external diameter of the pipes and with an external diameter $d_e = 14$ mm, which includes a stainless steel coating of thickness

$\delta = 0.2$ mm. Heating-up of the coolant takes place in this section.

4. Power data. The total thermal power of the reactor $W_T = 30$ MW. The heat removal scheme is as in Fig. 8.1. The temperature of water at the reactor inlet $t_{inlet} = 190^\circ\text{C}$ and at the outlet $t_{outlet} = 280^\circ\text{C}$. Pressure in the primary circuit is 100 atm, which

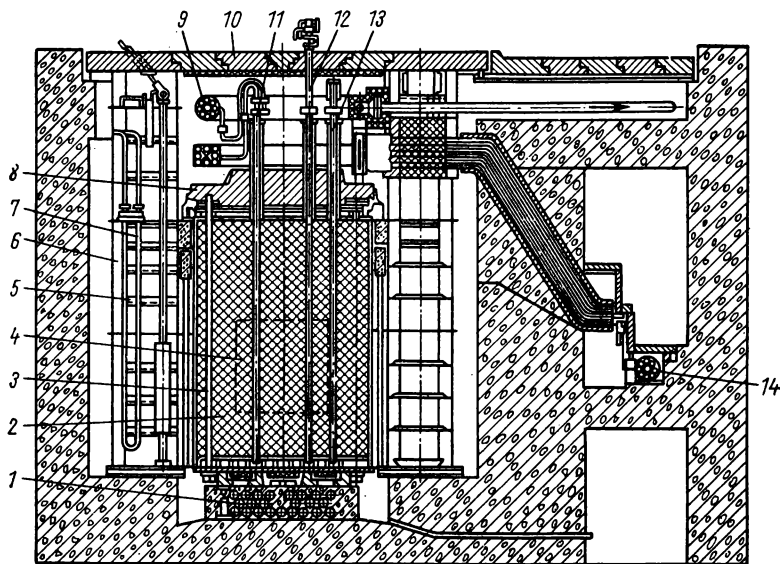


Fig. 9a1. Reactor of the World's first atomic power plant

1—reactor-base cooling tubes; 2—reflector; 3—reflector-cooling tube; 4—core; 5—ionization chamber channel; 6—water shield; 7—water-shield cooling coil; 8—cast-iron plate; 9—collection header; 10—upper shield (cast iron); 11—fuel channel; 12—emergency shut-off rod; 13—control rod; 14—distribution header

allows heat to be removed by water in the liquid state. A circulating pump produces excess pressure of 9 atm at the inlet for pumping water at 4 m/s through the fuel assemblies. Water pressure of the secondary circuit in the heat exchanger is 12.5 atm. Thus, the water in the secondary circuit changes into steam, as it is in contact with the hot water of the primary circuit. Steam at $t = 270^\circ\text{C}$ and pressure 12.5 atm is fed to the 5 MW turbine. In this way, the 30 MW of thermal power are converted into 5 MW of electrical power, thus the efficiency of the atomic power plant equals 16.5%.

5. Power life-time. The total quantity of uranium in the reactor is 550 kg or 27.5 kg of U^{235} at 5 per cent enrichment. The critical mass is 12.9 kg of U^{235} . The uranium excess creates an 11% margin of reactivity, of which 2.7% is spent to compensate for temperature effects, 4% to suppress poisoning by xenon and samarium

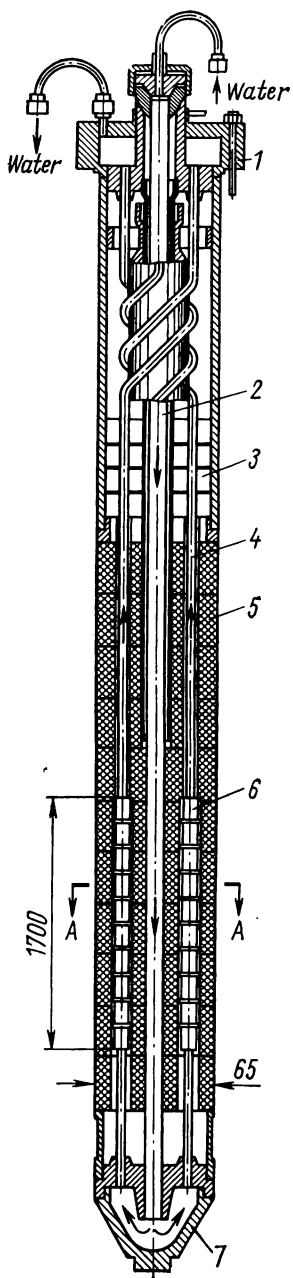
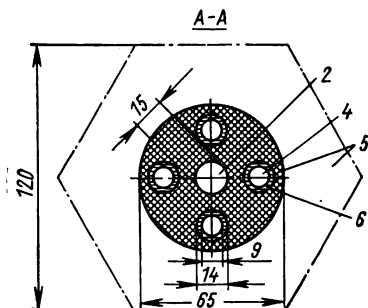


Fig. 9.2. Fuel assembly of the World's first atomic power plant

1—upper end cap; 2—central tube; 3—radiation-shield plug (steel); 4—periphery tube; 5—graphite; 6—fuel element; 7—lower end cap



and 4.3% for burnup. During reactor operation the critical state is maintained by a system of cylindrical shim rods, of 20-mm diameter and 1500-mm length. The rods are made of B_4C and are cooled with water. Later, boron steel tubes with 2.5% B, $d_e = 20$ mm, $d_i = 14$ mm were installed with no cooling. 18 shim rods compensate for 17% reactivity, and 2 safety rods of B_4C ($d = 22$ mm), 18%. The reactor life-time at full power operation with the volume-averaged thermal neutron flux $\Phi_{therm} = 5 \cdot 10^{17}$ 1/(m²·s) is 2.5 months, during which up to 20% of U^{235} atoms burn up. The reactor temperature coefficient is equal to $1.1 \cdot 10^{-2}$ % 1/°C.

6. Shielding. Shielding consists of a one metre thick layer of water and three metres thick concrete of density 220 kg/m³. Overhead shielding is a graphite reflector of increased thickness, 80 cm of cast iron plate, 20 cm of bitum with iron shots and 5 cm of lead. The diameter of the reactor with shielding measures 12 m and the height about 12 m.

7. Beloyarskaya atomic power plant. The low efficiency of the world's first atomic power plant is due to the low steam parameters in the secondary circuit. They are greatly improved in the Beloyarskaya atomic power plant built in 1964 in the Urals. This high capacity power plant is a further development of the first in the world atomic power plant. The reactor of the Beloyarskaya atomic power plant is of approximately the same design but the heat-removal scheme is changed considerably. The higher capacity means a greater core volume ($D = 7.2$ m, $H = 6$ m) with a correspondingly lower uranium enrichment comprising on the average 1.8%. The reactor is graphite moderated. Heat is removed by a steam-water mixture passing through stainless steel pipes in the fuel assemblies of the same design as those in the first atomic power plant. The number of peripheral pipes in the assembly is now six instead of four and the spacing between the moderator assemblies increased to 20 cm. The total number of fuel assemblies is 998. In 730 assemblies, heat is removed by boiling water and the remaining 268 assemblies are used for superheating the steam. The superheated steam from the nuclear reactor is supplied directly to the turbine (Fig. 9.3), which is admissible because the water is only weakly activated by neutrons (Sec. 3.5-16). This heat removal scheme allows superheated steam to be produced at 500°C and 90 atm and the plant efficiency to be considerably increased. With $W_t = 285$ MW, the electric capacity of the first installation of the Beloyarskaya atomic power plant is $W_{el} = 100$ MW, and with the electric energy consumption for the needs of the plant itself (6%) taken into account, the net efficiency amounts to $\eta = 33\%$. The steam superheating and evaporating assemblies alternate in the core. However, its central part

and the periphery comprise evaporating assemblies only, which means that the steam superheating region is an annulus in the cross-section of the core. The total quantity of uranium in the reactor is 67 tons and the reactivity margin 8%. The number of shim rods is 78 (this high number is characteristic of large volume reactors), the number of automatic control rods is 6, that of safety rods is 16. The life-time is two years, fuel burnup is 2300 MW·day/t. The reactor volume with shielding is $20 \times 25 \times 25$ m (height).

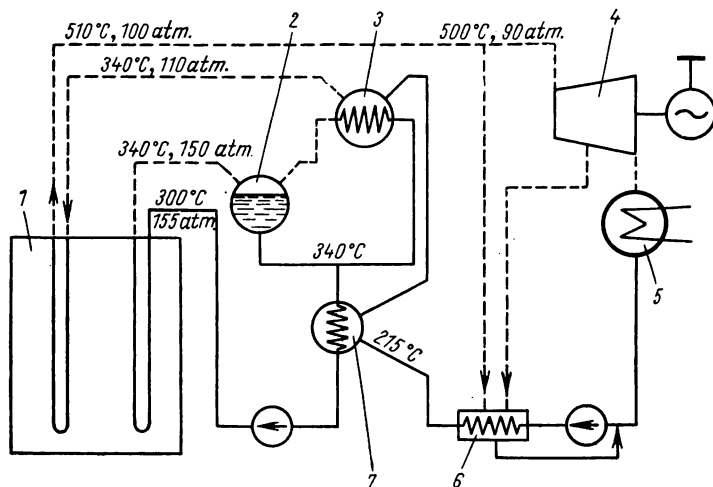


Fig. 9.3. Heat circuit of the first installation of the Beloyarskaya atomic power plant

1—reactor; 2—steam separator; 3—evaporator; 4—turbine; 5—condenser; 6—regenerative heaters; 7—heater

The second installation of the Beloyarskaya atomic power plant (1967) has the same reactor volume as the first installation. However, the enrichment of uranium is 3% and full charge is 50 tons. Heat transfer is still more intensified, thus doubling the capacity of the second installation: $W_t = 500$ MW, $W_{el} = 200$ MW. In the first installation, only the coolant from the steam superheating assemblies is directed to the turbine while the coolant of the evaporating assemblies circulates in a separate circuit, transferring heat to the turbine circuit in the evaporator and heater. In the second installation, a direct cycle of the coolant is employed (Fig. 9.4). The steam taken off in the separator from the steam-water mixture, entering from the evaporating assemblies, is fed directly to the superheating assemblies and the turbine. After regenerative heating the condensate passes to the evaporating assemblies. The capacity increase in the reactor of the same volume

means an improvement in the economy characteristics of the power plant.

8. **РБМК-1000 reactor.** (High-power boiling-water reactor). The long operating periods of the Beloyarskaya atomic power plant reactors and the experience gained determined the design of the modern boiling graphite-moderated reactor. This is a high power boiling water reactor (РБМК) with an electrical power output of 1000 MW (Fig. 9.5). It is a series reactor for a number of two-reactor atomic power plants which are now under construction in

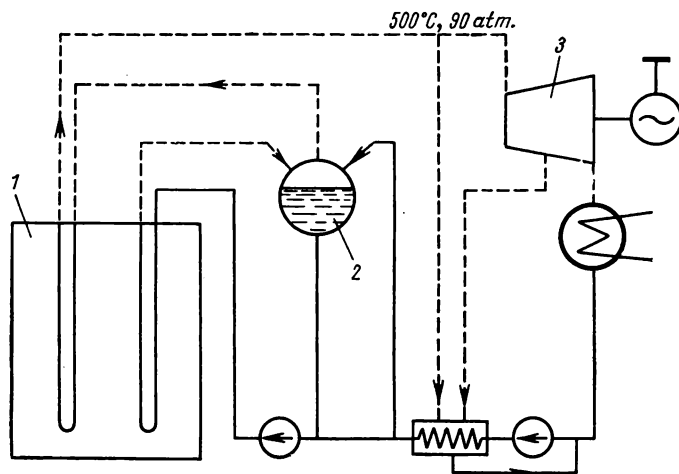


Fig. 9.4. Heat circuit of the second installation of the Beloyarskaya atomic power plant

1 — reactor; 2 — separator; 3 — turbine

the USSR. The first of these plants near Leningrad is being completed.

The РБМК-1000 reactor, like the reactors of the Beloyarskaya atomic power plant, is a channel type. However, the design of the fuel channel is greatly changed (Fig. 9.6). The coolant, water, is supplied from the bottom of the channel, and the steam-water mixture is discharged from the top. Heating and boiling of the water occur in the central part of the channel as the water flows around two fuel element assemblies placed one above the other. The total length of the assemblies is 7 m. Therefore, the height of the reactor core is also 7 m. The part of the channel passing through the core is made of zirconium alloy containing 2.5% niobium, its external diameter is 88 mm and the wall thickness 4 mm. The application of such a structural material in the reactor of the channel type improves neutron economy of the core and promotes

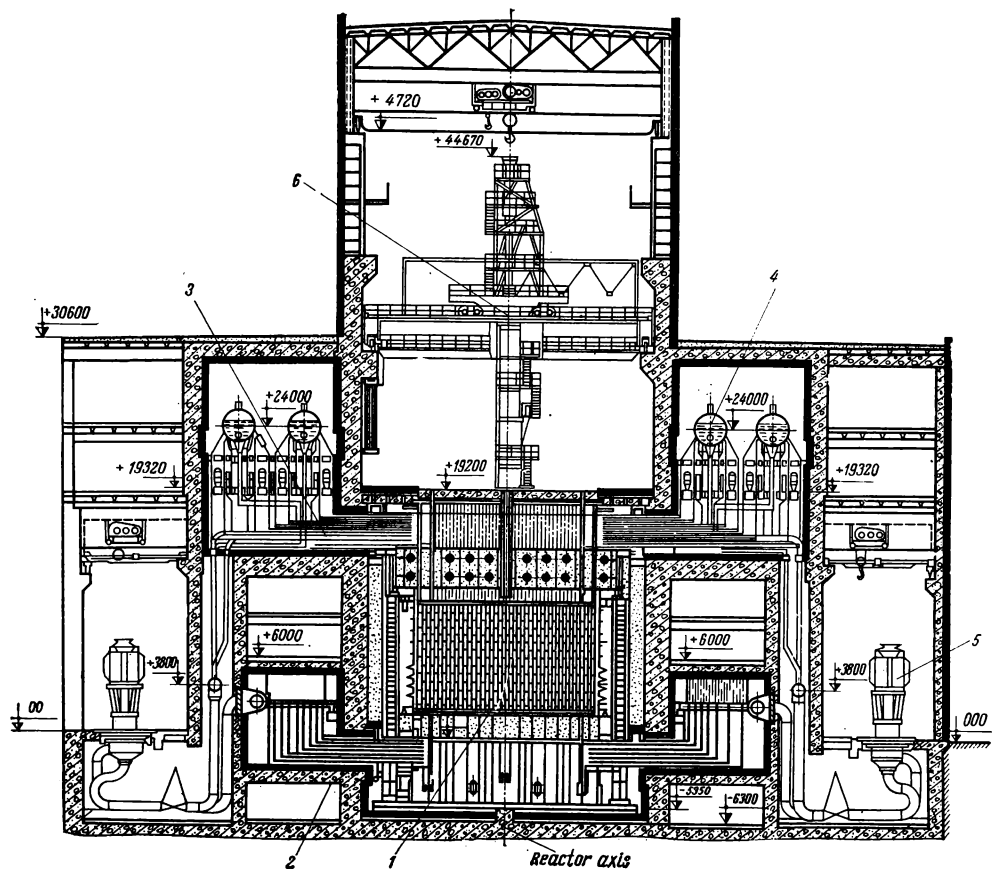


Fig. 9.5. The RBMK-1000 reactor

- 1—core; 2—water inlet utilities;
3—steam-water outlet utilities;
4—separator; 5—pump; 6—recharging (refueling) machine

better use of the nuclear fuel. However, steam cannot be superheated when zirconium is used. Therefore, saturated steam at 284 °C and at 70 atm is produced.

The design of the fuel elements is also changed. In the PBMK-1000 reactor, it is an assembly of 18, 13.5 mm-diameter rods made of UO_2 containing 1.8% U^{235} . The cladding is made of zirconium alloy 0.9 mm thick. The use of UO_2 and zirconium makes it possible to attain burn-up fraction up to 18500 MW·day/t at initial enrichment of 1.8%.

The reactor core has a diameter of 11.8 m, height 7 m and contains 1690 fuel channels with a 25 cm spacing. The uranium charge is 180 tons. The efficiency of this atomic power plant is 31%.

Reactors of the channel type have a number of advantages over the vessel type. The full power of the channel type reactor is in no way limited as the number of channels is in principle arbitrary because of the absence of the high pressure vessel. Reliability is very high because the core operation is controlled in

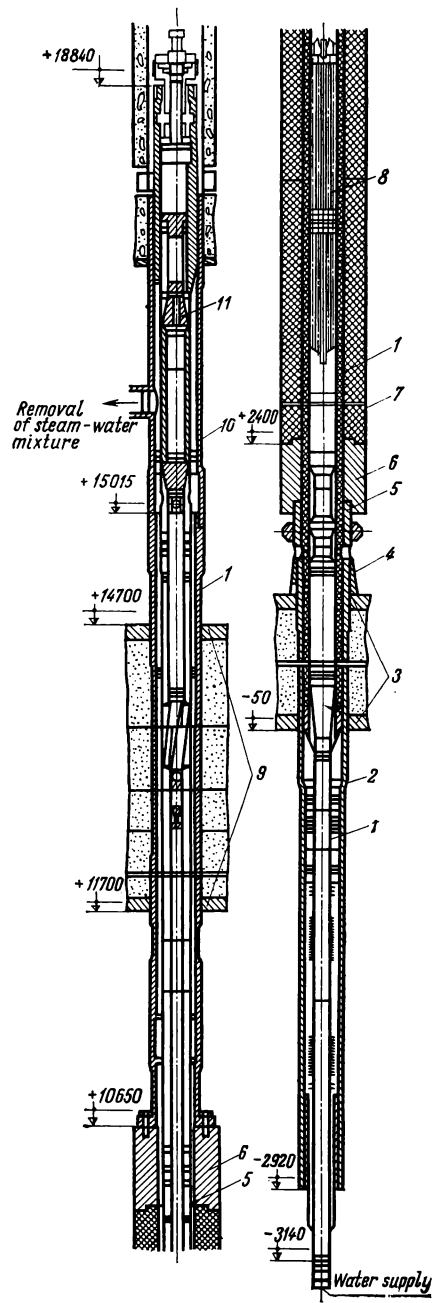


Fig. 9.6. Fuel channel of PBMK-1000

1—pressure tube; 2—channel loop; 3—lower-shield plate; 4—support cylinder; 5—steel-zirconium joint; 6—heat-shield; 7—stacking block; 8—fuel assembly; 9—upper-shield plate; 10—fuel channel cap; 11—fuel assembly suspension

each channel. Replacement of the channels with spent fuel elements is carried out by the refueling machine without shutting down the reactor. In the channel type reactor, it is easy to achieve nuclear superheat of the steam, but stainless steel must be used in the steam superheating channels.

A modification of the РБМК reactor with steam superheating rated at an electrical power output of 20 000 MW, РБМК-КП-2000, has been developed. All these factors show that this type of reactor is competitive with other types of reactors as far as the generation of electrical power is concerned.

9.2. Graphite Gas-Cooled Reactors

1. **Calder-Hall.** The first atomic power plant with a uranium graphite-moderated gas-cooled reactor was built at Calder-Hall, Great Britain, in 1956. The main advantage of such power plants is the use of natural uranium with a cheap moderator along with the attainment of rather high efficiency. The Calder-Hall atomic power plant is a prototype of many other commercial atomic power plants built in Great Britain and some other countries. At the present time, about one third of the power output of all atomic power plants in the world is produced in plants of the Calder-Hall type.

2. **Reactor.** The fuel is natural metallic uranium; the moderator, graphite; the coolant is carbon dioxide and the cladding material for the fuel elements, magnox. In this case, the coolant may enter the moderator. Therefore, no tubes for the coolant are necessary in the process channels. The gas inside the reactor is confined by a steel vessel of diameter 11 m, height 22 m and wall thickness 5 cm. The graphite stack inside the vessel has the following size: height $H = 8.2$ m, diameter $D = 11$ m and mass 1140 t (Fig. 9.7). As carbon dioxide passes through graphite, the temperature of the graphite in the operating reactor is not very high, 336 °C as a maximum.

3. **Core.** Dimensions: $H = 6.4$ m, $D = 9.45$ m, graphite weight 650 t. The remaining graphite is in the reflector. The square lattice is pitched at spacing of 20 cm. A process channel with approximate diameter of 10 cm passes along the axis of each elementary cell. The fuel element with its magnox cladding having a finned surface and longitudinal guiding projections assuring fuel element alignment, is inserted in the process channel. The diameter of the uranium core is 29 mm, its cladding 1.5 mm, and diameter, over the lateral fins, 58 mm. This sufficiently large uranium rod diameter is dictated by the necessity to have a low value of the effective resonance capture integral for U^{238} (Sec. 6.5-6). The use of large diameter process channels and finned fuel elements improve the

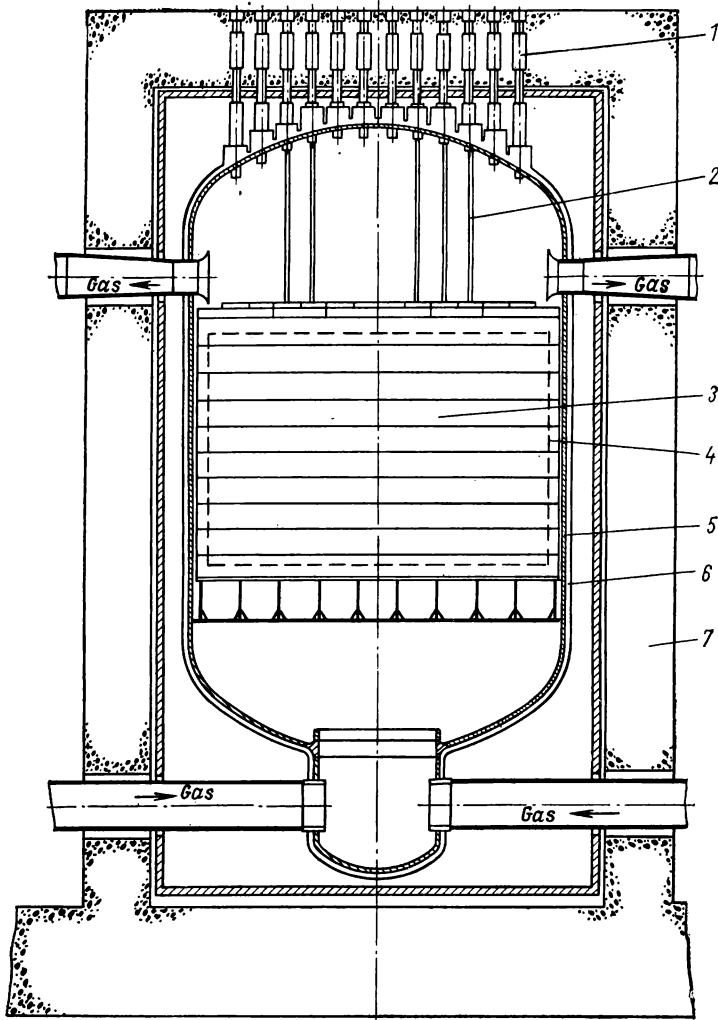


Fig. 9.7. The Calder-Hall reactor

1—loading tubes; 2—control rods; 3—core; 4—reflector; 5—vessel; 6—heat insulation;
7—biological shielding

heat-removal by such a poor coolant as gas. The power consumption for pumping the gas through decreases with the increase of its pressure which in the Calder-Hall reactors, is 7 atm and is determined by the stability considerations of the reactor vessel. The total process channels number 1696.

4. **Reactor power.** $W_t = 225$ MW. Heat-removal system is of a two-circuit type. Maximum temperature of the fuel element surface is 418°C . Gas enters the reactor at $t = 145^\circ\text{C}$ and is heated up to 340°C . High pressure steam (16.2 atm, 315°C) and low pressure steam (4.2 atm, 182°C) are produced in the heat-exchanger. The turbogenerator produces 50 MW of electric power, 20% is, however, consumed for the plant maintenance, and 40 MW is transferred to the electrical power system. Thus, the net efficiency is 18%. The atomic power plant in Calder-Hall includes four reactors which had been successively put into operation by 1959. The total electrical power output of the atomic power plant is $W_{el} = 160$ MeV.

5. **Life-time.** The total quantity of natural uranium is 127 t, and the critical mass 31 t. The 5% excess reactivity is compensated for by 48 shim rods of boron steel. These rods also function as safety rods, four of them being used to control the reactor. Burn-up is 2700 MW·day/t and the life-time is about 150 days. The replacement of spent fuel elements is carried out by loading and unloading machines after shutting down the reactor. This operation is conducted through tubes passing through the vessel cover. One unloading tube services 16 process channels. The reloading procedure takes 25 days.

6. **Physics.** The mean flux of thermal neutrons is $\Phi_t = 5.5 \cdot 10^6$ neutrons/(m²·s), that of fast neutrons, $\Phi_f = 7.1 \cdot 10^{16}$ neutrons/(m²·s). The mean cycle time for a prompt neutron is 10^{-3} s. The multiplication factor is $k_0 = 1.08$, the initial conversion ratio, $K_{con} = 0.85$. After 1000 MW·day/t burn-up, the maximum reactivity increment (2.1%) takes place due to the plutonium build-up (Sec. 7.2-5). The temperature coefficient at the beginning of the life-time equals $-3.7 \cdot 10^{-5}$ 1/°C.

7. **Shielding.** Concrete of minimum thickness 2.13 m and 2600 kg/m³ density.

8. **Development of graphite gas-cooled reactors.** The characteristics of atomic power plants built later are considerably improved. The gas pressure is increased up to 20 atm. The temperature of the fuel element magnox cladding is raised up to 450°C , with a corresponding increase of the gas temperature up to 410°C at the reactor outlet, which produces steam at $t = 380$ - 390°C with an efficiency of 28% and even 33%. Further increase of the coolant temperature, however, becomes impossible because of the phase transition danger in the metallic fuel element (Sec. 8.1-3) or of the damage to the magnox cladding. Utilization of uranium dioxide and some other structural materials excludes the use of cheap natural uranium.

Nevertheless, due to the intensified heat transfer the gain in capital costs per kilowatt of the installed power proved to be much

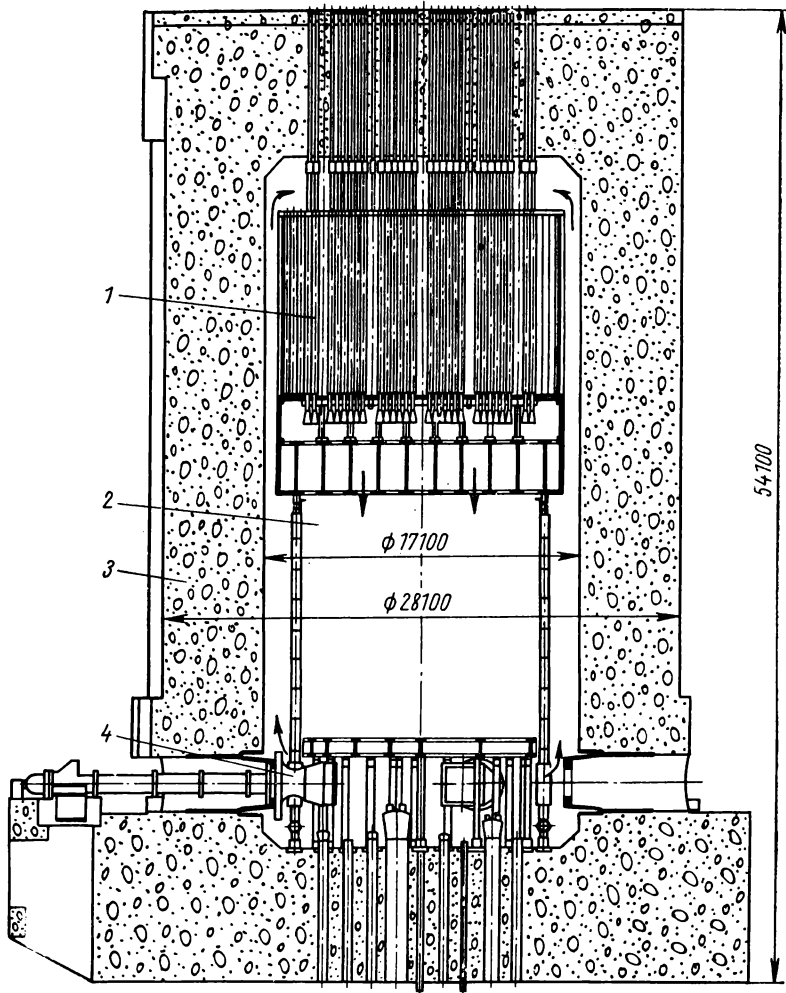


Fig. 9.8. General view of the EDF reactor.

1—reactor; 2—heat-exchanger steam-generator; 3—vessel and biological shielding made of prestressed concrete; 4—gas-blower

higher than the loss in the uranium cost. Therefore, the second generation of the British graphite gas-cooled reactors are enriched uranium reactors.

A commercial prototype of an advanced graphite reactor (AGR) with electrical power output 33 MW has been in operation at Windscale, Great Britain, since 1963. At present, two-reactor atomic

power plants, $W_{el} = 1320$ MW, using this type of reactor are being put into operation. Because of the enrichment, metallic uranium in these reactors is replaced by uranium dioxide. The 3 cm fuel element is replaced by a fuel element assembly 1.45 cm in diameter and the fuel element cladding is of stainless steel. As a result, the temperature of carbon dioxide at the reactor outlet reaches 675°C , the parameters of the superheated steam are 565°C and 163 atm, and the efficiency 41.5%, which is a very high value for an atomic power plant. Uranium enrichment varies from 1.5 to 2.5% under different operating conditions and in different parts of the core, the burn-up approaches $20\,000\text{ MW}\cdot\text{day/t}$.

The core of the 660 MW AGR reactor measures 8.2 m in height and 9.4 m in diameter. The square lattice spacing is 40 cm. Assemblies of 36 fuel elements are distributed along the 19 cm-diameter process channel. The carbon dioxide is held at 35-40 atm. The reactor vessel is of prestressed concrete. If the cost of electric power produced in atomic power plants with the first generation of graphite gas-cooled reactors is somewhat higher than the cost of electric power produced by coal power plants, the second generation of reactors produces cheaper energy than the coal power plants.

Graphite gas-cooled reactors with natural uranium as fuel were also developed in France. The scheme of the EDF-3 reactor is shown in Fig. 9.8. An important design feature of the reactor is its vessel made of concrete reinforced by steel cables. This vessel withstands the gas pressure and simultaneously acts as the biological shield. The gas pressure in the vessel made of prestressed concrete may be increased to 40 atm. All the equipment of the primary circuit may be accommodated in this vessel, thus solving the communication problem for the high pressure radioactive gas. Only the steam fed to the turbine passes from the vessel. All modern graphite gas-cooled reactors have prestressed concrete vessels.

9.3. Heavy-Water Reactors

1. Economy characteristics. Because of weak absorption of thermal neutrons by deuterium the reactors using heavy water as moderator have a large initial reactivity margin. Even natural uranium may, therefore, be used as UO_2 . Fuel assemblies can be made of small diameter rods, and burn-up values may be large. All this creates indisputable advantages for heavy water reactors. The high cost of heavy water, however, adds greatly to the capital expenditures when constructing an atomic power plant. Nevertheless, the very low fuel constituent makes the heavy-water reactors effective agents for the generation of electrical power.

2. Reactors in Canada. The main contribution to the development of heavy-water power reactors is made in Canada where some atomic power plants with a total capacity of more than 1500 MW have been built. In these reactors the heat is removed by heavy water, either at high-pressure or boiling, or by ordinary boiling water. All the reactors are of the channel type. The heavy water used as moderator is isolated from the coolant and is kept at low temperature.

In the first reactor, with heat-removal by heavy non-boiling water, the coolant at 100 atm flows in pipes made of zirconium alloy. The fuel assemblies of 19 fuel elements with $d_U = 14.5$ mm are arranged inside these tubes. The diameter of the assembly is about 8 cm; the lattice spacing of the assemblies in the core is 23 cm, the number of the process channels (zirconium-alloy pipes) in the reactor tank is 306. Natural uranium is used in the form of UO_2 . The fuel element claddings are of zirconium alloy. The maximum temperature of the fuel element wall is 302°C , D_2O has 293°C at the outlet; the temperature of the water steam in the secondary circuit is $t = 250^\circ\text{C}$, its pressure, 41 atm. The capacity of the atomic power plant is $W_t = 690$ MW, $W_{el} = 200$ MW, the net efficiency is 29%, specific heat removal, 4.8 MW/m³. The coolant mass in the stainless steel reactor tank (a horizontal cylinder 5 m long and 6 m in diameter) is 144 t; the temperature of the moderator is not higher than 55°C , the pressure in the tank is atmospheric. The large reactivity margin of the heavy-water reactor and the use of UO_2 as fuel make it possible to obtain burn-up values up to 10 000 MW·day/t, that is, to accumulate up to 10 kg of fission products per ton of uranium, while the content of U^{235} in natural uranium is only 7 kg per ton. This implies the effective use of U^{235} , the content of which is very low in spent uranium. It also indicates that U^{238} after conversion into Pu^{239} transforms to a great extent into fission products.

3. Czechoslovakian reactor. In Czechoslovakia, an atomic power plant is now under construction. It has a D_2O moderator, CO_2 as coolant, and aluminium and Mg—Be alloy as the structural material. The core has 196 aluminium process channels. Fuel assemblies occupy 156 process channels and control rods, the remaining 40. A cylindrical assembly contains 160 fuel elements of natural metallic uranium with 4 mm diameter each, and with the Mg—Be alloy cladding without fins on the surface. The temperature of the fuel element wall approaches 500°C and that of the gas at the outlet from the reactor is 425°C . The water steam entering the turbine has temperature and pressure of $t = 400^\circ\text{C}$ and 29 atm, respectively. The capacity of the atomic power plant is $W_t = 590$ MW, $W_{el} = 150$ MW, gross efficiency 25.5%, specific power of the core is 10.9 MW/m³. The gas pressure of 60 atm is held in

Mg—Be alloy tubes arranged with clearances inside the aluminium channels. The D₂O moderator is kept at a temperature not higher than 90 °C in an aluminium tank.

9.4. Light-Water Reactors

1. **Design characteristics.** If ordinary water is used as moderator its volume fraction in the reactor core is exceedingly small (see Table 6.5). The amount of the moderator appears to be approximately equal to the amount of the coolant required to remove heat. Therefore, in light-water reactors, the water serves as coolant and there is no separation between the moderator and the coolant (water-cooled and water-moderated reactor). Since the water is heated up to 300 °C, the entire core is under high pressure maintained by the vessel which is the main structural part of the light-water reactor. Thick-walled vessels of large diameters (up to 3-4 m) are made of stainless steel or of carbon steel lined on the inside with stainless steel. Reloading of the core is carried out after stopping the chain reaction and removing the vessel head. Light-water reactors are the most compact because of the small amount of the moderator in the core. Specific heat removal per unit volume of the core is much higher than that in graphite and heavy-water reactors and is 30 to 45 MW/m³, with small uranium enrichment which leads to low capital costs per kilowatt of the rated power of the atomic power plant. Compactness of the light-water reactors makes them most suitable for use in marine power plants.

Heat-removal by non-boiling water assures strict temperature conditions at the fuel element surface. High pressure (100-140 atm) is, however, needed to keep the water in a liquid state. A two-circuit heat removal scheme is used. The pressure of the steam in the turbine is much lower than that in the reactor vessel. Such reactors are called pressurized light-water reactors. Since the turbine is water-steam operated, the steam may be produced directly in the core of the light-water reactor. Such reactor is called a boiling water-cooled and water-moderated reactor. The pressure of the saturated steam produced is determined by the temperature attained in the core and is 35-100 atm. This is the pressure of the steam supplied to the turbine. The greatest advantage of the boiling reactor is the one-circuit heat-removal. The absence of a heat-exchanger makes capital costs per kilowatt of the rated power especially low in comparison with capital costs at power plants working on coal.

2. **Reactor BBÖP-210.** The water-cooled and water-moderated power reactor for the power output of 210 MW, BBÖP-210, has been in operation in Novo-Voronezhskaya atomic power plant since 1964. Heat is removed by pressurized water. The reactor vessel

is cylindrical of external diameter 3.8 m, height 12 m and wall thickness 10 cm (Fig. 9.9). It is made of carbon steel and clad with stainless steel. The pressure is 100 atm, water temperature at the inlet is 250°C, at the outlet, 275°C, the temperature of the fuel element wall approaches 300°C, and at the centre of the UO_2 fuel element it is up to 2200°C. The reactor has six circuits, each having its own pump for pumping the coolant into the reactor, and its own heat-exchanger. From the heat-exchangers, the steam is fed to three turbines. The steam parameters are 230°C and 29 atm. Heat capacity of the first installation is $W_t = 760$ MW, the gross electrical power output is 210 MW. The consumption of the electric energy for the needs of the installation itself is 6.7%, so that the net efficiency is 26%. The specific power of the core is 43 MW/m³.

3. **The core.** The core of effective diameter $D_{eff} = 3$ m, and height $H = 2.5$ m, consists of hexagonal fuel assemblies, plates (Fig. 9.10), made of zirconium alloy. The assembly includes 90 UO_2 fuel elements with 2% enrichment, each of 10.2 mm diameter together with the zirconium alloy coating 0.6 mm thick. The fuel elements

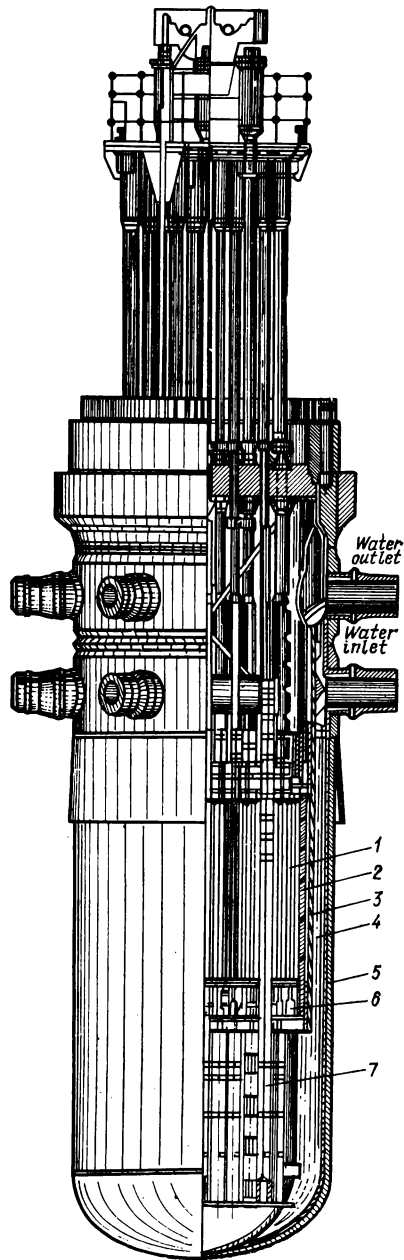


Fig. 9.9. The BBEP-210 reactor

1—core; 2—thermal shielding; 3—support cylinder; 4—water neutron-reflector; 5—reactor vessel; 6—lower grid; 7—shim rod

are spaced by zirconium spacer rods. The coolant, which is at the same time the moderator, passes through the assemblies from the bottom to the top. The thermal shield of the reactor vessel around

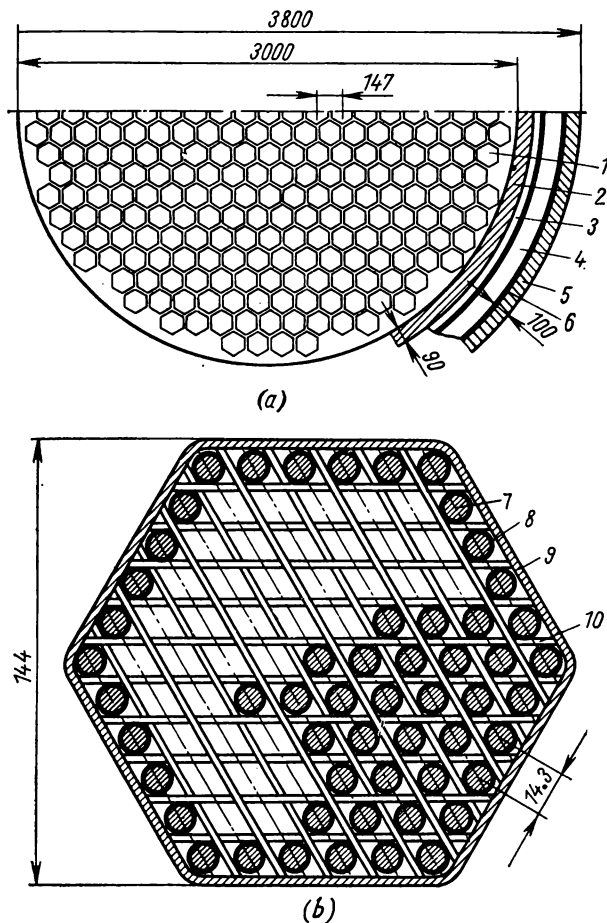


Fig. 9.10. Cross-sections of the core (a) and of the fuel assembly (b) of the Novo-Voronezhskaya atomic power plant reactor

1—grid assembly; 2—heat shielding; 3—support cylinder; 4—reflector 20 cm of water; 5—vessel wall; 6—stainless steel layer; 7—uranium dioxide; 8—fuel-element zirconium spacer-rod

the periphery of the core is a steel cylinder 9 cm thick, it absorbs the greater part of the core radiation energy. Water layer 20 cm thick is fed to the core and passes from the top to the bottom between the thermal shield and the reactor vessel. This water layer serves as a neutron reflector.

4. Life-time. Uranium charge is 40 t, or 600 kg of U^{235} . The number of fuel element assemblies in the reactor total 343, of which 31 are movable and serve as control assemblies. In the control assemblies, the absorbing material is placed in tandem with the fuel. This allows the reactivity to be increased first by the withdrawal of the absorber and then by the insertion of a fissionable material. There are six safety rods. The burn-up value of 3000 MW·day/t is attained in approximately 160 days. Reloading of the fuel assemblies takes about a month, which is considered to be good for a vessel type reactor. It is planned to increase burn-up to 10 000 MW·day/t. The temperature coefficient of reactivity is $2 \cdot 10^{-4}$ 1/°C. The shielding consists of 1 m of water and 3 m of concrete, as in the world's first atomic power plant.

5. The second and third installations. The second installation reactor, BBЭP-375, has the same vessel as BBЭP-210 but its power output is almost twice as large, 375 MW, due to the increased uranium enrichment, up to 3%, the modernization of the core and forced operation. The third installation reactor, BBЭP-440, with power output of 440 MW, is the final modification of the BBЭP-type reactors. As compared to the BBЭP-210, the vessel of the BBЭP-440 type reactor is increased in height, the dimensions of the core being unchanged. The uranium enrichment is increased to 3.5%, the fuel element diameter reduced to 9.1 mm and water heating in the core is by 31 °C. The steam parameters in the turbine are somewhat improved and a high efficiency value of the plant is obtained (gross efficiency is 32%). The burn-up value has been doubled and is 28 600 MW·day/t, which greatly improves the economical characteristics of the atomic power plant. The BBЭP-440 reactor is a prototype for a number of atomic power plants under construction in the USSR and is exportable.

The vessel of a light-water reactor is its most critical part and must be made under factory conditions. Limitations connected with transporting large-calibre loads restrict the power output of the vessel-type reactors. Nevertheless, it is planned to make light-water reactors with power output up to 1000 MW. The design of the BBЭP-1000 type reactor has been developed.

Pressurized-water reactors have been perfectly developed and at present are widely used in the power generating industry of the USA. The first atomic power plant of this type, with power output of 60 MW, has been operating since 1958 in Shippingport (USA).

6. Other power plants. Pressurized light-water reactors are not used only in stationary atomic power plants. Owing to their compactness and comparatively small mass per kilowatt of the installed capacity, they are used in all nuclear power plants of merchant and naval ships, including submarines. The same reactors are

installed in the atomic ice-breaker "Lenin" built in the Soviet Union in 1958. In 1961 a low-capacity power plant, TЭC-3 (a pressurized light-water reactor, $W_{el} = 1.5$ MW) mounted on four automotive caterpillar platforms was built and tested in the USSR to be used in some regions which are difficult of access. The transportable power plant "Arbus", with $W_{el} = 0.75$ MW, built in 1963, is a modification of this type of the power plant. It consists of 19 blocks, each of mass not greater than 20 t and can be assembled in 2 to 3 months in a $28.5 \times 12.3 \times 6.3$ m building. In the "Arbus" power plant reactor, organic fluid is used instead of water as moderator and coolant. As the boiling point of the organic fluid is high, the pressure in the reactor vessel is as low as 6 atm at 243°C while the absence of corrosion due to the use of the organic fluid allows the employment of carbon steel for the reactor vessel and piping, and aluminium in the core.

7. Boiling-water reactors. Boiling water-cooled and water-moderated reactors are developed in the USA. The 184 MW Dresden atomic power plant with this type of reactor has been in operation there since 1960. The coolant cycle is of the one-circuit type with no heat-exchanger. Water enters the core from the bottom and begins boiling in the upper part. The steam-water mixture passes to the separator, from where the steam is directed to the turbine and water returns to the reactor. At present, in the USA, the number of atomic power plants with boiling-water reactors grows as rapidly as that with pressurized-water reactors. The absence of heat-exchangers in atomic power plants with boiling water reactor reduces capital costs per kilowatt of the installed capacity almost to the level of coal power plants, which makes the advantage of such atomic power plants. The somewhat poorer physical and thermal characteristics make both boiling and pressurized water-cooled and water-moderated reactors almost equal as far as their competitiveness is concerned. In the USA, such reactors are considered the most economical of all nuclear power installations. However, the competitiveness of a nuclear reactor in the production of electric power is not only determined by its physical characteristics and technical perfection, but also by the industrial basis created in the country in the course of development of nuclear power engineering as a whole. Therefore, each country has its own economical designs of atomic power station reactors. Nevertheless, although in the late sixties, almost half the power output of atomic power plants was produced by graphite gas-cooled reactors, mainly in Britain, nowadays, more than half is produced by light-water reactors, mainly in the USA.

A power plant with the boiling water 50 MW reactor BK-50 has been in operation in Melekes, Ulyanovsk region, USSR, since

1965. The reactor vessel and the core are similar to those of BBЭP-210. The reactor is used for investigating the boiling coolant over wide ranges of temperatures and pressures.

9.5. Fast Reactors

1. **Shevchenko atomic power plant.** Advantages and disadvantages of fast reactors were discussed in (Sec. 7.2-6 to Sec. 7.2-9). At present, fast reactors are still under investigation, experimental study and perfection. However, high-power units are already under construction. A dual-purpose atomic power plant is being built in the USSR on the basis of the experience acquired from the operation of several small reactors. This atomic power plant is being built in Shevchenko, on the banks of the Caspian sea for generation of electrical power and sea water distillation. The distillation unit uses the steam from the power circuit of the atomic power station, which reduces the power output but is economically advantageous for regions with insufficient fresh water. In 1972, the physical process of the nuclear reactor was implemented, that is, the self-sustaining chain nuclear reaction obtained.

2. **БН-350 reactor.** The БН-350 reactor is a fast reactor with the 350 MW power output. The reactor vessel is of a complex configuration (Fig. 9.11). It is filled with 165 m³ of liquid sodium. The core is in the lower part of the vessel. Its dimensions are $D_{ef} = 1.5$ m, $H = 1.06$ m and the volume is about 2 m³. The blanket, 60 cm thick, is outside the core. Above the reactor vessel, there is a steel dome ($D = 9.5$ m, $H = 10.4$ m) to guard against unforeseen escape of radioactivity through the vessel head. The core includes the fuel and the coolant, the volume fraction of the latter being 39%. The fuel is either uranium dioxide with 23% enrichment or a mixture of plutonium and U²³⁸ dioxides with 19% of plutonium. The critical mass is 950 kg of U²³⁵ or 780 kg of Pu²³⁹. The 5 mm diameter fuel elements clad in stainless steel 0.4 mm thick are gathered in assemblies of 217 pieces. The assemblies are 96 mm hexagonals. In the core the assemblies number 211. The upper and the lower end portions of the core assembly are at the same time the assemblies of the blanket.

The blanket consists of depleted uranium dioxide. Since energy release in the blanket is much lower than in the core, the diameters of the fuel elements are larger, $d = 12$ mm, and the thickness of the stainless steel cladding $\delta = 0.4$ mm in the upper or in the lower blanket and $d = 14.2$ mm, $\delta = 0.5$ mm in the lateral blanket, where the middle portions of the assemblies do not form part of the core. The number of fuel elements in all the blanket assemblies is 37 and the number of the lateral blanket assemblies is 500. Storage and cooling cells are placed round the blanket periphery.

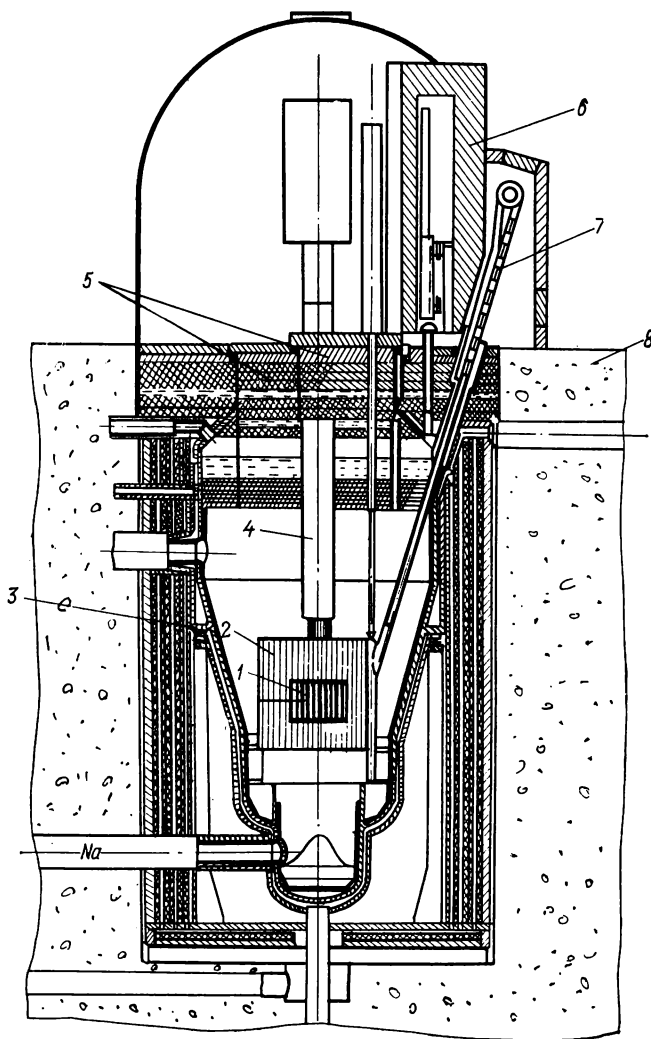


Fig. 9.11. The BH-350 reactor

1—core; 2—blanket; 3—vessel; 4—control column with drives of CSS; 5—revolving plugs; 6—discharge box; 7—discharge elevator; 8—biological shielding

In this section new assemblies are placed before use and spent assemblies are stored until the energy release, due to the β -decay of fission products, reduces.

3. **Power.** The reactor heat output is $W_t = 1000$ MW, its electric power output is $W_{el} = 350$ MW, and the efficiency of the atomic power plant is 35%. The electric power output is $W_{el} = 150$ MW

when supplying steam to the distillation plant which has an output of 150,000 tons of water per day. Six circuits are provided to remove heat from the reactor. The temperature of sodium at the inlet is 300 °C and 500 °C at the outlet. The heat-removal system is of the three-circuit type (Sec. 8.1-5). Non-radioactive sodium is used in the second circuit and water is used in the third. The use of sodium as the coolant allows steam to be produced with high parameters: 430 °C, 50 atm. The specific power of the core is also very high (500 MW/m³) and is achieved due to the good heat conductivity of sodium passing through the core at 8 m/s.

4. Life-time. Burn up equals, 50,000 MW·day/t in 300 days. The conversion ratio of the core is 0.62 and that of the whole reactor is $K_{con} \approx 1.5$. To compensate for the burn up, six fuel assemblies having reactivity margin $\Delta\rho = +1.4\%$ are used. The temperature coefficient is $2.4 \cdot 10^{-5}$ 1/°C and the power reactivity coefficient is $(0.5-0.7) \cdot 10^{-5}$ 1/MW. Temperature effect compensation is achieved by bringing an ordinary assembly out of the core to the upper blanket. This assembly contains, instead of fuel, boron carbide, B₄C, enriched by B¹⁰ up to 60%. The efficiency of the compensator is 1% with 0.7 kg of B¹⁰. Automatic control is accomplished by two B₄C rods each with reactivity margin 0.2%, clad in stainless steel. The emergency protection consists of three assemblies of B₄C rods with 4.5% total reactivity margin. Because of the small values of the absorption cross-sections in the fast region, the neutron flux is approximately equal to 10^{19} (m²·s)⁻¹. The lifetime of a prompt neutron is about 10^{-7} s.

5. Shielding. At the edge of the blanket the neutron flux is still large and equal to $5 \cdot 10^{17}$ (m²·s)⁻¹. The shield inside the vessel is a sandwich of 12 cm thick steel, 50 cm of sodium and 6 cm of steel. Nevertheless, the energy release in the vessel wall due to the radiation absorption is 0.1 MW/m³. The shield outside the vessel consists of 15 cm of steel, a one metre thick layer of iron oxide and 2 metres of building concrete. The protection from above is: sodium above the core, a steel plate in sodium, layers of iron and graphite.

6. Further development. A fast reactor, BH-1000, with electric power output 1000 MW is under design in the Soviet Union. It is planned to use uranium and plutonium monocarbide as fuel, not uranium dioxide. This will increase the conversion ratio to 1.75 and the conversion will practically compensate for the burn-up. This will simplify the requirements to the reactor control system. It is planned to reach burn-up values of 100-150 thousand MW·day/t and to obtain, in the turbine loop, modern steam parameters of 580 °C, 240 atm. Such an installation must have good economical characteristics and be competitive as far as the generation of electric power is concerned.

An experimental reactor БОР-60 was built in Melekes in 1968. It is used for testing materials, fuel elements and equipment at very high temperatures and high burn-ups. Heat output is $W_{th} = 60$ MW, maximum temperature of the fuel element cladding is 800°C , the temperature of sodium at the outlet is up to $630\text{--}650^{\circ}\text{C}$, mean specific power of the core is 900 MW/m^3 , and turn-up is greater than $100,000\text{ MW}\cdot\text{day/t}$. The plutonium carbide fuel elements are being tested at the experimental БР-5 fast reactor built earlier.

Power plants with fast reactors are not yet economical, therefore, their numbers are few. Nevertheless, in the next ten years their economical characteristics should be significantly improved. This will some time allow fast reactors to occupy a leading position in nuclear power engineering. The main prerequisite for this is the breeding of nuclear fuel in the fast reactor.

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INDEX

- Absorbed dose, 370
 - unit rate of, 370
- Activation of water, 155
- Activity unit, 118
- Age equation, 260
- Albedo, 255
- Alpha-particles, 13
 - ranges of, 176, 178
- Annihilation, 47, 54
 - of electrons, 55
 - of nucleon pairs, 70
 - of positrons, 55
- Antineutrino, 60
- Antineutron, 71
- Antiparticle, 47
- Antiproton, 69
- Antisubstance, 72
- Atomic electricity, 11
- Atomic mass, 9, 80
- Atomic mass unit, 80
- Atomic number 17, 21
- Atomic radius, 11
- Atomic size, 11
- Atom recoil, 35
- Auger electrons, 141
- Avogadro number, 10

- Barion charge, 50
- Beta-decay, 56
- Beta-spectrometer, 130
- Biological shielding, 374
- Bohr magneton, 18, 53
- Boson, 54
- Bragg-Cleaman rule, 178
- Breit-Wigner formula, 206, 212
- Bremsstrahlung, 182
- Burnup depth, 334

- Chain reaction
 - absorption, 266
 - linear, 42, 189
- Coefficient
 - diffusion, 248
- Coefficient
 - dynamic power, 360
 - mass absorption, 187
 - power reactivity, 360
 - quality, 371
 - radiation quality, 371
- Composition of building concrete, 375
- Compound nucleus mechanism, 143
- Compton effect, 193
- Concentration,
 - maximum permissible, 349
- Control rods, 332
- Core lattice, 272
- Coulomb barrier, 93
- Coulomb forces, 89
- Critical equation, 300
- Critical mass, 269, 270
- Critical size, 269
- Critical volume, 269
- Cross-section
 - boron, 220
 - bound nucleus, 230
 - cadmium, 223, 224
 - Compton scattering, 195
 - compound nucleus production, 209
 - fission, 209, 216, 224
 - inelastic scattering, 216
 - effective interaction, 38
 - pair production, 198
 - photoeffect, 192
 - potential scattering, 209, 213
 - radiative capture, 206, 208
 - thermal absorption, 225, 226
 - Thomson scattering, 194

- De Broglie postulates, 24
- De Broglie wavelength, 25
- Decay constant, 112
- Decay law, 112
- Delayed neutrons, 135, 157, 163, 170, 320
- Diffusion current, 247
- Diffusion equation, 248
- Diffusion length, 253

- Direct interaction, 147
- Direct interaction mechanism, 147
- Doppler effect, 358
- Doppler width, 211
- Dose rate, 370
- Effective resonance integral, 285, 287, 289
- Electric quadrupole moment, 78
- Electron, 12
- Electron bond energy, 80
- Electron charge, 12
- Electron path, 186
- Electron radius
 - classical, 17
- Electrons of internal conversion, 140
- Elementary particle, 47
- Emergency rods, 332
- Energy
 - bound, 33
 - free, 33
 - kinetic, 28
 - standard thermal, 221
 - threshold, 36, 37
 - total, 33
- Energy level width, 28, 110
- Energy regions, 214
- Exchange interaction, 97
- Excitation energy, 143
- Exposure dose, 370
- Extrapolated boundary of the body, 251
- Factor
 - fast(-neutron) multiplication, 275, 292
 - linear attenuation, 42, 190
 - multiplication, 268, 274
 - reproduction, 337
 - thermal utilization, 276
 - statistical, 31
- Faraday constant, 12
- Fermion, 54
- Fissile materials, 270
- Fissile nuclides, 159
- Fissionable nuclides, 161
- Fission energy, 164
- Fission fragments, 162
- Fission neutrons, 167
- Fission products, 163, 166
- Fission width, 207
- Flux density, 251
- Forces
 - nuclear, 51
 - weak, 51
- Four-factor formula, 276
- Fuel burnup, 334
- Fuel channel, 273
- Fuel element, 273
- Fuel time doubling, 344
- Fusion,
 - thermonuclear, 92, 265
- Geiger—Nuttall rule, 121
- Half-life, 113
- Heat removal, 379
- Ionization, 172
- Isobar, 76
- Isomer, 138
- Isotone, 76
- Isotope, 76
- Isotopic spin, 50
- K-capture, 128
- Klein-and-Nishina formula, 195
- Lattice spacing, 272
- Law of energy conservation, 147
- Law of mechanical moment conservation, 150
- Lepton charge, 50
- Loshmidt number, 10
- Magic numbers, 104
- Mass defect, 85
- Mass formula, 103
- Mass number, 21
- Mass
 - nuclear, 80
 - relativistic, 32
- Mass-spectroscopy, 81
- Mean diffusion time, 263
- Mean life-time, 113
- Mean-square translation, 251, 254, 261
- Moderating power, 238
- Moderation length, 261
- Moderation ratio, 238
- Magnetic moment, 78
- Magnetic moment of a neutron, 53
- Magnetic moment of a proton, 53
- Mechanical moment, 29
- Momentum
 - angular, 28, 29
 - specific, 35
- Moseley law, 74
- Mu-meson, 62

- Natural uranium, 271
- Neutrino, 56
- Neutron, 20
- Neutron activation of water, 155
- Neutron age, 260, 262
- Neutron cycle, 267, 275
- Neutron mobility, 250, 256
- Neutron path-length, 252
- Neutrons
 - fast, 215
 - intermediate, 215
 - thermal, 215
- Neutron temperature, 219
- Nuclear charge, 74
- Nuclear drop model, 100
- Nuclear energy spectrum, 108
- Nuclear fission, 158
- Nuclear forces, 92
- Nuclear fuel, 334
- Nuclear interactions, 67
- Nuclear isomerism, 138
- Nuclear magnetic moment, 78
- Nuclear magneton, 53
- Nuclear photoeffect, 156
- Nuclear potential, 93
- Nuclear power plant, 343
- Nuclear reaction energy, 38
- Nuclear reactor, 266
- Nuclear shell model, 103
- Nucleon, 21
- Nuclide, 76
- Nuclide abundance, 91

- Organic coolants, 367

- Pair production, 47, 54, 69
- Pair production conversion, 142
- Pair production process, 197
- Parahydrogen, 31
- Parity, 50, 132
- Path, 171
- Path length, 43, 171
- Photoeffect, 190
- Photon, 33
- Polymorphism, 365
- Positron, 54
- Positronium, 55
- Prompt neutrons, 162
- Proton, 19

- Quanta hypothesis, 17

- Radiation intensity, 370
- Radiative capture, 146, 152, 203
- Radiative capture cross-section, 206, 208
- Radiation quality coefficient, 371
- Radioactive families, 124
- Radiolysis, 364
- Radius
 - nuclear, 76
 - of action of nuclear forces, 51
- Range, 180
 - extrapolated, 187, 188
 - of fission fragments, 180
 - of neutrons, 187, 188
- Reaction yield, 151
- Reactor, 266
 - boiling-water-cooled-water-moderated, 390
 - fast, 395
 - graphite-moderated-gas-cooled, 384
 - graphite-moderated-water-cooled, 377
 - heavy-water, 388
 - heterogeneous, 272
 - homogeneous, 272
 - intermediate, 215
 - light-water, 390
 - organic-cooled, 394
 - pressurized-light-water, 390
 - steam-superheating, 376
 - thermal, 379
- Reactor control and safety system, 333
- Reactor cooling, 344
- Reactor core, 268
- Reactor life-time, 333
- Reactor poisoning, 347
- Reactor reflector, 313, 317
- Reactor savings, 314
- Reduced width, 212
- Relative biological effectiveness, 371
- Relativistic velocity, 32
- Reproduction factor, 337
- Resonance absorption integral, 284
- Resonance escape probability, 242, 275, 282
- Resonance integral, 284
- Resonance neutrons, 207
- Rutherford-Bohr atom, 17

- Saturation activity, 115
- Scattering
 - coherent, 225
 - elastic, 202, 231
 - inelastic, 202, 231
 - resonance, 202
- Shell model, 106
- Shim rods, 332
- Slowing-down density, 241
- Slugging, 347
- Spent uranium, 344

- Spin, 52
 - of excitation level, 111
- Statistical weight, 31
- Statistics, 53
- Steam superheating, 379
- Strangeness, 50
- Structural materials, 273, 366
- Thermal shielding, 373
- Thomson atom, 12
- Threshold heavy nuclides, 161
- Threshold particle production, 36
- Threshold reaction, 37
- Total width, 207
- Transport length, 245
- Uncertainty relation, 26, 27
- Virtual particles, 27
- Velocity,
 - relativistic, 32
- Weak interaction constant, 131
- Wavelength, 23, 24
- Wave vector, 22
- Width
 - partial, 207
 - radiative, 207
- Xenon pit, 351
- Xenon poisoning, 347
- Xenon waves, 353

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